A TEXT-BOOK

OF

PHYSICS

BY

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PREFACE

THIS volume is designed primarily for use as a text in courses offered to engineering and technical students. With this use of the book in view, particular emphasis is given to the practical aspects of the science of Physics. Illustrations of physical laws are drawn as far as possible from familiar phenomena, and physical principles are exemplified by numerous important applications.

Particular emphasis is placed upon the subject of Mechanics, for the reason that, notwithstanding its fundamental importance, this part of the general subject is perhaps the least attractive, and consequently the least apt to receive proper consideration at the hands of the beginner. It has seemed desirable, therefore, to emphasize the importance of this part of the work, and to fix the student's attention and interest upon it to such an extent as to enable him to secure the necessary grasp of its fundamental principles. The topics of vector analysis, force and torque, accelerated motion, work, energy transformations, moment of inertia, and the kinetic energy of rotating masses have been given special prominence.

It is expected that the book will be used as a basis for class-room work and that it will be supplemented by a course of experimentally illustrated lectures and suitable laboratory exercises. For this reason the descriptions of many illustrative experiments, which might otherwise have been included, have been omitted from the text. For the same reason but small space has been given to a description of laboratory methods, it being supposed that this phase of the subject will be covered by the accompanying laboratory course.

In determining the scope of the text the author has been guided by the belief that an elementary discussion of general

physics should include a description of every phenomenon and an exposition of every experimental law that may be regarded as contributing directly to the logical development of the general subject, and, on the other hand, should be kept free from digressions and such descriptions of unimportant and distantly related phenomena as are not really necessary to such development.

The order in which the various branches of the subject are presented and, in smaller measure, the treatment of the subject matter of such subdivisions has been determined by a desire to present each subject in such manner that nothing is taken for granted which has not been demonstrated or proven in some preceding section. It is believed that such an arrangement is of the very highest importance, and that this, rather than any other consideration, perhaps, should determine the sequence of the various subdivisions of the general subject.

To read the text readily, the student should have a knowledge of algebra and geometry and the trigonometric functions. An effort has been made to make all demonstrations as elementary as possible, and the mathematical aspect of the subject has been given but little prominence. At the same time, care has been taken to make exact statements of law and fact and to make each discussion as complete as possible, consistent with the nature of the work. As a matter of course, it is always necessary in an elementary text to avoid, in large measure, those subtle distinctions and more elaborate discussions without which a more advanced treatise would be regarded as incomplete.

L. B. S.

Ames, Iowa,
September 20, 1911.

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PART I

MECHANICS

LENGTH, MASS, AND TIME

CHAPTER I

THE METRIC SYSTEM

1. In the determination of weights and measures many different systems of units are employed. For measurements in scientific work the **metric system** of units has come into almost universal use. The units and the multiple and submultiple units of this system bear a decimal relation to one another. This system, therefore, has the advantage that any quantity expressed in units can be reduced to multiple or submultiple units, or *vice versa*, by simply moving the decimal point of the expression. From the scientific standpoint this system offers other important advantages, some of which will appear in the discussions which follow.

UNIT OF LENGTH

2. In this system the unit of length is the **centimeter**, which is defined as the 100th part of the standard meter. The meter is defined as the distance at the temperature of melting ice between the ends of a certain platinum bar which is in the possession of the French Government. It is a familiar fact that bodies expand and contract as they are heated and cooled. At a fixed temperature, however, a given body always has the same length. Thus the length of the platinum bar referred to, which is known as the standard meter, is, under the given conditions, a fixed and definite quantity. Copies of this standard have been made with great care and placed in the possession of the various nations in which this system is used, and have become the standards of those nations.

The unit of length employed by English-speaking people for ordinary measurements is the yard. This unit is defined in

terms of the length of a certain standard bar in the possession of the British Government. In the following discussions the centimeter will be employed as the unit of length. It is, however, desirable to express the relation between the meter and the yard, and the centimeter and the inch, in order that one who is unaccustomed to the metric system may more quickly become familiar with the centimeter as the unit of length. The relation between the two systems of units is as follows:

1 meter = 39.37 inches (very approximately)
1 yard = 0.914383 meter

The unit of area is the square centimeter. A **square centimeter** is the area of a square each side of which is one centimeter in length.

The unit of volume in the metric system is the cubic centimeter. A **cubic centimeter** is the volume of a cube each edge of which is one centimeter in length.

THE UNIT OF MASS

3. The unit of mass in the metric system is called the gram. The **gram** is defined as the 1000th part of the mass of a certain piece of platinum which is in the possession of the French Government, and which is known as the standard kilogram.

A unit of mass commonly employed for ordinary measurements is the pound. The **pound** is defined as the mass of a certain piece of platinum in the possession of the British Government. The relation between these two units of mass is as follows:

1 kilogram = 2.2 pounds (very approximately)
1 pound = 453.6 grams

THE UNIT OF TIME

4. The unit of time employed for both scientific purposes and for ordinary use is the second. The **second** is defined as $\frac{1}{86400}$ mean solar day ($86{,}400 = 60 \times 60 \times 24$), the mean solar day being defined as the average time throughout the year which elapses from noon to noon as measured by the sundial.

THE MEASUREMENT OF LENGTH

5. In the ordinary process of measuring length a standard of length, for example, a meter stick, is made use of, and is compared in length with the object to be measured. Depending upon the size of the divisions into which the meter stick is divided (for example, tenths, hundredths, or thousandths), the length of the body may, by this means, be more or less accurately determined. If, however, it is desired to measure a length with extreme accuracy, it is found that some other method must be resorted to. If, for example, we wish to determine the length of a body to within a thousandth part of a centimeter, it is found impracticable to make use of a scale in the usual manner, because a scale having 1000 lines to the centimeter could not be used in the ordinary way by the eye unaided, on account of the minute size of the divisions. Under such circumstances recourse is had to one of the following devices.

THE VERNIER CALIPER

6. The vernier caliper consists essentially of two parts as shown in the diagram, Figure 1. The part A is divided, as indicated in the figure, into small divisions as in the ordinary meter stick. The part B, which slides upon A, is provided with a vernier which enables the scale to be read to within a small fraction of one of its divisions.

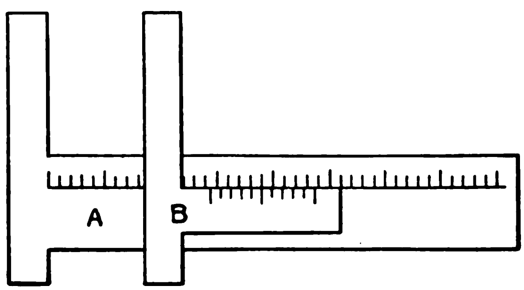

Fig. 1. — The Vernier Caliper.

The principle of the vernier will be understood from the following figure and discussion. In Figure 2 let A be an ordinary scale divided into equal parts as indicated. Let B be a similar but shorter scale and equal in length to nine of the divisions on the scale A. Further let it be assumed that the length of B is divided into

ten equal parts so that each of its divisions is nine tenths of one of the divisions on the scale *A*. Assume also that the scale *B* is in the position shown in the figure so that its zero coincides with the zero on the scale *A*. Then from the preceding statement it will be evident that **mark 1 on the scale *B* will be out of coincidence with mark 1 on the scale *A*, by one tenth of one division of the scale *A***, since a division on *B* is nine tenths of a division on *A*. In the same way mark 2 on the vernier will be out of coincidence with mark 2 on the scale by two tenths of a division on scale *A*. Mark 3 will lack three tenths of a division of coincidence, mark 4, four tenths, etc. That is, **the number of the mark on the vernier, counting from that mark on the vernier which is in coincidence with a mark on the scale, indicates how much that mark lacks of coincidence** with the corresponding mark on the scale. The scale *B* is called a **vernier.**

In the measurement of the length of a body the vernier is made use of in the following manner. Let *O*, Figure 3, represent an object whose length is to be determined by the use of the scale and the vernier. It is placed in the position shown so that one edge coincides with the zero of the scale. The vernier is placed against the scale in the manner shown so that its zero coincides with the other end of the object *O*. Glancing along the scale, we observe that mark 3 on the vernier coincides with

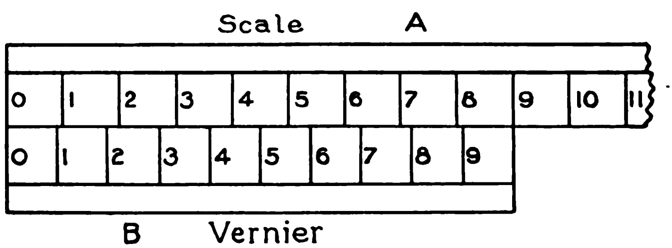

FIG. 2.—The Principle of the Vernier.

a mark on the scale. Applying the rule above given, it will be evident that the zero mark of the vernier lacks three tenths of a scale division of being in coincidence with mark 1 on the scale. The length of the object *O* is therefore one and three tenths scale divisions.

If it is desired to measure by this means with a still greater degree of accuracy, the vernier might be so constructed that in length it is equal to 99 divisions on the scale and be divided into 100 equal parts. One of its divisions would therefore be

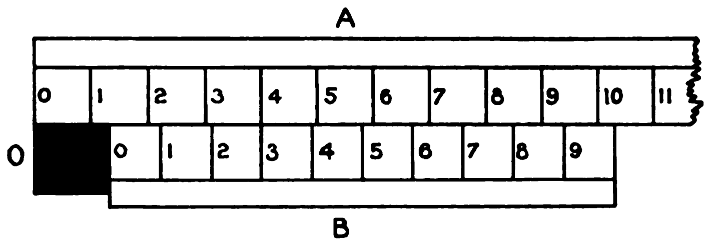

Fig. 3. — The Use of the Vernier.

equal to ninety-nine one-hundredths of one of the divisions on the scale. It would be possible with such an arrangement to measure to within one one-hundredth part of a scale division.

MICROMETER CALIPER

7. The **micrometer caliper** consists essentially of a rigid U-shaped piece of metal as shown in Figure 4 and a screw fitted into one of the extremities of the U-shaped body as indicated. The micrometer caliper is used in the following manner. The object whose length is to be determined is placed between the parts *E* and *F* and the screw is turned until it lightly clamps the object. The object then being removed, the screw is turned in such direction as to close the gap *EF*, care being taken to count the number of turns necessary to entirely close this gap. This being known

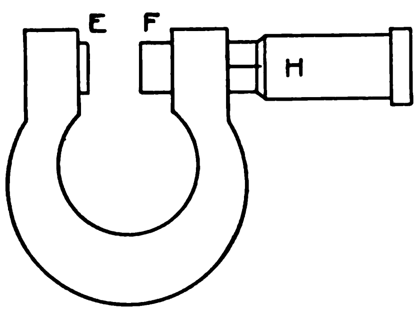

Fig. 4. — The Micrometer Caliper.

and the "pitch" of the screw having been determined, the length of the object becomes known at once. The pitch of a screw is the distance from one thread to the next, measured parallel to

the axis of the screw, that is, in the direction *EF*. For example, let it be assumed that the pitch of the screw is one twenty-fifth of an inch; then for every complete turn of the screw, the end *F* moves toward, or away from, the part *E* one twenty-fifth of an inch. Such being the case, it will be evident that a half turn of the screw will advance the point *F* one fiftieth of an inch and a quarter turn of the screw will advance the point *F* one one-hundredth of an inch. Let it be assumed further that the head of the screw *H* is divided in its circumference into 40 equal parts so that it is possible to turn the screw one fortieth of a turn with accuracy. Such a turn will advance the point *F* one one-thousandth of an inch. With such an arrangement it would therefore be possible to determine the distance *EF* to within the thousandth part of an inch.

MEASUREMENT OF MASS

8. The mass of a body is defined as the quantity of matter which it contains. This definition of mass assumes that **the quantity of matter is determined by the effect of force upon it.** This is equivalent to saying that the way to determine the quantity of matter in a body is to take hold of it and move it. A body which is free to move and is easily moved about is a body of small mass. A body which is moved with difficulty is a body of large mass. It is understood, of course, that in making tests of this kind the body examined must be kept free from all other force actions while being subjected to the test.

Let it be imagined, for example, that we have two boxes of merchandise the masses of which are to be compared. Every one knows that a simple way to determine which has the larger mass is to take hold of the boxes and overturn them. But this operation is really a test of their weights, that is, in this experiment we compare the force with which the earth pulls upon the two boxes and their contents. Another test which might be made is to drag them over a horizontal surface, and by an experiment of this kind one can easily satisfy himself as to which has the larger mass; but here again the test is an indirect one because the difficulty or ease with which the box is moved in such a test depends both upon the weight of the

object moved and the character of the surface over which it is caused to slide. As a matter of fact, if the boxes could be placed upon a truly horizontal surface against which they would bear with absolutely no friction, then their masses might be compared by determining how much force is required to start each one into sudden motion over this surface. That is to say, if two bodies can be placed so that the force of gravity and all friction effects are eliminated from the experiment, the best possible way to compare their masses is to compare the forces required to bring about the same change of motion in each.

The simplest and most common method of comparing masses is that of weighing. This method depends upon the fact that **at any given place upon the surface of the earth the weights of bodies are proportional to their masses,** so that bodies having equal masses will, at the same place, have equal weights.

The weight of a body is defined as **the force with which the earth attracts that body.** This force depends upon the distance between the body in question and the center of the earth. If the position of the body with respect to the center of the earth is changed, its weight is changed. The weight of a body is therefore not a definite thing unless the place at which it is weighed is specified. The nearer a body is to the center of the earth, so long as it is outside of the earth's surface, the greater is its weight. A body will therefore weigh more in northern latitudes than it will near the equator for the reason that, because of the flattened form of the earth, the body is nearer the center when near the north or south pole than it is when in the region of the equator.

Thus, while the weight of a body is a variable quantity, and its mass is a constant quantity, we are enabled to compare masses by weighing them, that is, by comparing the attraction of the earth for them, **providing the determination is made at some one specified point upon the earth's surface.**

Attention should also be called to the fact that in the process known as "weighing" the weight of the body is really not determined at all, but instead the mass of the body, as indicated by the discussion just given. In the use of the bal-

ance, or platform scale, or any similar weighing machine, the adjustment is made by adding to one side of the system a number of standard masses until the beam indicates that the forces with which the earth attracts the body weighed and the standard masses are equal. When such adjustment has been made, the balance or scale indicates that the weight of the body is equal to the weight of the standard masses, **but it does not indicate what the weight of either really is.** If the arms of the balance or scale are unequal, the balanced weights are inversely proportional to the lengths of the supporting arms.

MEASUREMENT OF TIME

9. Time intervals are measured or compared by means of clocks or watches. The essential parts of a clock or watch are,

1st. The pendulum or balance wheel.
2d. The driving train.
3d. The recording device.

The function of the pendulum or balance wheel is to divide passing time into equal intervals. The property of the pendulum or balance wheel which is taken advantage of is that, when swinging through small angles, each is constant in its period, that is to say, the time required for one swing is always the same. The function of the driving train is to keep the pendulum or balance wheel in motion. If it were not for the driving train the pendulum or balance wheel would, after a few vibrations, come to rest, because of the opposing forces of friction. The recording device simply records the number of vibrations made by the pendulum or balance wheel. The hands of the clock or watch are simply the terminals of the recording mechanism, which is so adjusted that one of these hands makes one complete revolution in twelve hours, while the other completes a revolution in sixty minutes.

Problems

1. What is the value of a square meter in square yards? In square feet?

2. How many square centimeters are there in a square foot? In a square inch?

3. A man can walk 4 mi. in one hour. How many kilometers can he walk in 5 hr.? (1 km. = 1000 m.)

4. How many cubic centimeters are there in a cubic foot?

5. How many grams are there in an ounce avoirdupois?

6. If coal is worth \$5.00 per ton, how much is it worth per kilogram?

7. A man is 6 ft. tall and weighs 180 lb. What is his height in centimeters and his weight in kilograms?

8. The divisions on a certain scale are half inches. It has a vernier $7\frac{1}{2}$ in. long, which is divided into 16 equal parts. What is the smallest fraction of an inch that can be measured with this scale and vernier?

9. The vernier of a vernier caliper is so divided that 20 divisions on the vernier equal 19 divisions on the scale. What is the smallest fraction of a scale division that can be read?

10. If on a caliper 19 divisions on the vernier equal 20 divisions on the scale, would the vernier read the same as that of the caliper of Problem 9?

11. A micrometer caliper has 25 threads to the inch. The head has 40 equal divisions. What is the smallest fraction of an inch that can be read directly from the instrument?

12. A micrometer screw has a pitch of $\frac{1}{50}$ in. Its head is divided into 200 equal parts. What is the smallest fraction of an inch that can be measured with this screw?

VECTORS

CHAPTER II

DEFINITION OF A VECTOR

10. A vector quantity or **vector** is one which **has both magnitude and direction.** Force is a good example of such a quantity. No one can tell what the effect of a force will be without knowing its **direction** as well as its **magnitude** or size. The velocity of any body, *e.g.* the velocity of the wind, is another example.

Quantities which have magnitude only are called **scalars. Mass** and **volume** are examples of scalar quantities.

REPRESENTATION OF A VECTOR BY A STRAIGHT LINE

11. Since for the complete specification of a vector quantity both its magnitude and direction must be given, it can be seen at once that a vector quantity may be represented completely by a straight line. In representing a vector quantity in this manner the length of the line is made to represent the magnitude of the quantity, and the direction in which the straight line extends is made to represent the direction of the vector. An arrowhead is placed upon the line to indicate the sense in which it is to be considered. In representing the magnitude of a vector quantity in this manner use is made of what is known as a scale number. Let it be desired, for example, to represent a force of 100 units acting from left to right. A line on this page to represent such a force would be drawn in a horizontal direction, would have an arrowhead on the right, and would be of such length that interpreted according to the scale number, **its length in units of length would represent the**

magnitude of the force in units of force. For example, it might be found convenient to represent such a force by a line 5 centimeters in length. If such length of line were employed, it is evident that each centimeter of length in the line would stand for 20 units of force. This number 20 is called the scale number. If the line were drawn to a length of 10 centimeters, the scale number would evidently be 10.

THE ADDITION OF VECTOR QUANTITIES

12. Vector quantities are added (combined), not by the ordinary rules of addition and subtraction, but **by a method which takes account of direction as well as magnitude.** To illustrate, let it be imagined that a certain body, free to move, is acted upon by two forces. The one force urges it toward the north, the other toward the east. The body upon which these forces are acting will evidently move off in a northeasterly direction. The combined effect of the two forces is not determined by adding the separate effects of these forces numerically, but in the following manner. After the elapse of a certain period of time the body will be found as far north of the starting point as it would have been had the force urging it north been the only one acting. At the same time it will be found as far east of the starting point as it would have been in the same length of time had the force urging it east been the only one acting. It is evident, therefore, that it will lie at the end of the diagonal of a rectangle whose sides represent the distances east and north which the body would have traveled under the influence of the forces acting singly. This is commonly known as the **parallelogram law.**

A consideration of the following problem will make the principle more clear. Let it be desired to find the direction in which a boat moves over the surface of a stream under the combined action of the current, which of course tends to carry the boat downstream, and the efforts of the oarsman which, let us say, tend to carry the boat across the stream at right angles to the current. Let *a*, Figure 5, represent the velocity of the stream, *b* represent the velocity with which the boat tends to move across the stream under the impulse of the oars.

Applying the principle just given, the boat will move along the diagonal OP, arriving at the point P in the same length of time that it would pass from O to S under the action of the current alone, or from O to T under the influence of the oars were the boat in still water. It is evident, therefore, that while the value of OP, which is called the **resultant** velocity, is larger than either component velocities a and b, its value is not given by the numerical sum of a and b, but it is determined by the relation

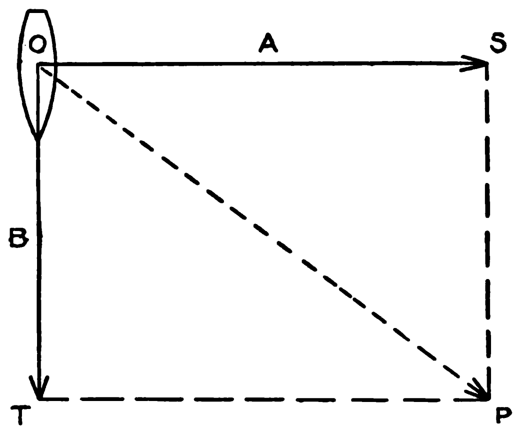

FIG. 5.—Motion of a Boat acted upon by Two Forces.

$$OP = \sqrt{a^2 + b^2}$$

this relation holding of course only when the components a and b are at right angles to one another. When the angle between a and b is other than 90°, we may still obtain a value for the resultant in terms of its components by making use of the following trigonometrical relation between the diagonal and the sides of a parallelogram:

$$D^2 = a^2 + b^2 + 2\,ab \cos (aob)$$

in which D is the diagonal, a and b the sides, and aob the angle between a and b.

GRAPHICAL METHOD

13. Referring to Figure 5, the value of the resultant may be determined as follows: assuming that OS has been drawn to scale to represent the downstream velocity and that OT has been drawn to the same scale to represent the velocity across the stream, the value of the resultant is determined at once by measuring OP and interpreting its length according to the scale number made use of in drawing OS and OT. For example, let it be assumed that the downstream velocity is 4

miles per hour and the cross-stream velocity is 3 miles per hour. Make the line OS 8 centimeters long. We have, therefore,

$$8 \text{ centimeters} = 4 \frac{\text{miles}}{\text{hour}}$$

from which it is seen that

$$1 \text{ centimeter} = \tfrac{1}{2} \frac{\text{mile}}{\text{hour}}$$

This number $\frac{1}{2}$ is the "scale number." Making use of the same scale number, the line OT must have a length of 6 centimeters, since it is to represent a velocity of 3 miles per hour. After completing the parallelogram we find the length of the diagonal OP to be 10 centimeters. This line, as we have seen, represents, **to the same scale** as that employed in drawing OS and OT, the resultant velocity. Interpreting its length, 10 centimeters by the scale number given above, we see that the actual velocity of the boat in the direction OP is 5 miles per hour. **This method of laying off the components to scale and determining the value of the resultant by measurement** and interpretation according to the scale number, is called the **graphical method** of solving vector problems.

In the use of this method great care must be taken in the measurement of the lines and the determination of the angles which separate them; otherwise, the accurate solution of a problem is not possible. To this end **the vector diagram should always be made as large as is consistent with the dimensions of the sheet of paper or the blackboard, as the case may be, upon which the figure is drawn.**

THE VECTOR POLYGON

14. In the statement of the law for combining vector quantities as given above, use has been made of the parallelogram constructed upon the component vectors as sides. Referring again to Figure 5, it will be evident that two sides of the parallelogram might have been dispensed with in the determination of the resultant OP. For example, the line OS might have been drawn representing the downstream velocity and then from the arrowpoint S, the line SP representing the

cross-stream velocity. The point P would have been just as fully determined as by the completion of the parallelogram. The lines OT and TP might therefore have been omitted. The rule for vector summation is as follows: Draw a line to scale representing one of the vectors in magnitude and direction. From the arrowpoint of this line draw a second line representing the second vector in magnitude and direction, and so on, until all of the component vector quantities are represented. Finally **a line joining the starting point with the arrowhead of the last line represents in magnitude and direction the resultant of the several vectors.** The polygon so constructed is known as the vector polygon. It should be noted that the rule as stated is general, and by its application the resultant of any number of vector quantities may be determined.

THE CLOSED VECTOR POLYGON

15. In the problem discussed in Section 12, it is evident that a person moving across the deck of the boat in the direction PO at the rate of 5 miles per hour would, to an observer on the shore, appear stationary. This is evident from the following considerations. Such a person has three velocities: first, a velocity of 4 miles per hour downstream represented by the line OS; second, a velocity across the stream of 3 miles per hour represented by the line SP; third, a velocity in the direction PO of 5 miles per hour represented by the line PO. Applying the rule for the graphical solution of problems of this character after having drawn the three lines OS, SP, and OP, the final step is to join the starting point with the last arrowhead, the line joining these points representing the resultant velocity as pointed out above. Evidently in this case, since the last arrowhead falls at the starting point, the resultant of the three velocities is 0. The lines drawn in this manner would have their arrowheads all pointing in the same direction around the polygon. This, then, is the criterion for a zero resultant of several different vector quantities. The rule may be stated to advantage in the following terms: **When a number of vector quantities are so related in magnitude and direction that the lines representing them to scale form a**

closed polygon with the arrowheads all extending in the same sense around the figure, their resultant is zero.

The converse of this proposition is, of course, equally true; that is to say, whenever a body which is under the influence of several forces acts as if no force were present, the vector sum of the forces acting is zero, and the lines representing these forces to scale would form a closed polygon. This principle is often used in the solution of problems. An example of its application is given in the following section.

APPLICATION OF THE PRINCIPLE OF THE CLOSED POLYGON

16. A device which is commonly used for the lifting of heavy objects and known as a "crane" is represented in outline in Figure 6. It consists of a pole PO hinged against a vertical

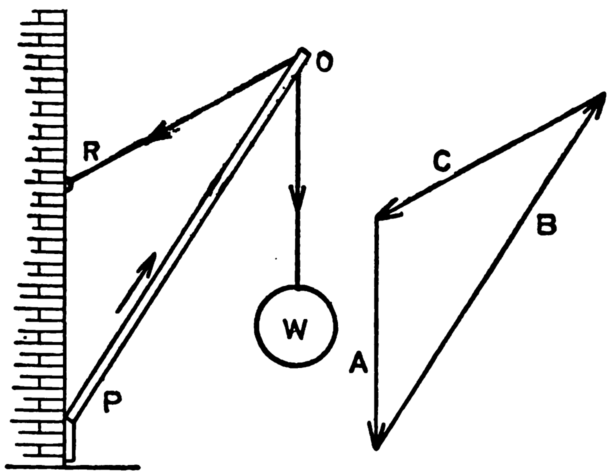

FIG. 6. — Equilibrium of Forces acting on a Crane.

wall at the point P, the upper end of which is supported by a rope RO attached to some point on the same wall. The heavy object W which is supported by the crane hangs from the point O. Let it be assumed that the crane supporting its load is stationary. The end of the pole O is then acted upon by three forces: first, the **weight** of the body W; second, the **pull** of the rope RO; third, the **push** of the pole P; the direc-

tions of the three forces being indicated by the arrowheads. Let it be assumed that the force OW is the weight of a ton, that is, the weight of 2000 pounds, and let it be required to find the values of the forces in the rope and the pole. The solution of this problem is very simply reached by the application of the principle of the closed polygon. The principle is applied as follows. Draw a line A representing to scale the weight of the body W in magnitude and direction. Through the arrowpoint of A, draw a line B parallel to PO and indefinite in extent. Through the other end of the line A draw C parallel to RO. The intersection of C with B determines the lengths of both of them. Measuring their lengths and interpreting them by the scale numbers to which A was drawn, the values of the forces PO and RO are obtained ($A = 2000$, $B = 3700$, $C = 2200$).

Note carefully the assumption made in the application of this principle, namely, **that the end of the pole upon which the three forces act is stationary, that is, it behaves as if no force action were present.**

THE KITE PROBLEM

17. A kite floating on a steady breeze affords an illustration of three forces in equilibrium. The forces acting are represented in Figure 7, their relative values being given by the lines P, S, and W of the closed polygon. The effective pressure of the wind is perpendicular to the face of the kite and acts, of course, over its entire surface ; its effect, however, is that of a single force P acting through the center of the kite as shown. W represents the weight of the kite. Here, also, the several downward pulls corresponding to the weights of the various parts of the kite are equivalent to a single force W assumed to act through the center. The pull of the string S is the third force. In order that the three forces may be in equilibrium, S must pass through the center, and its magnitude and direction must be that given by the line S in the closed polygon.

If at any instant the system of forces is unbalanced, the resultant will tend to move the kite up into the wind, or down the wind, as the case may be, until the condition of equilibrium is reached. Evidently if the lower end of the string is fixed, a

motion of the kite either with or against the wind will change both P and S in magnitude and direction.

So long as the condition of equilibrium is maintained the kite will remain stationary in position.

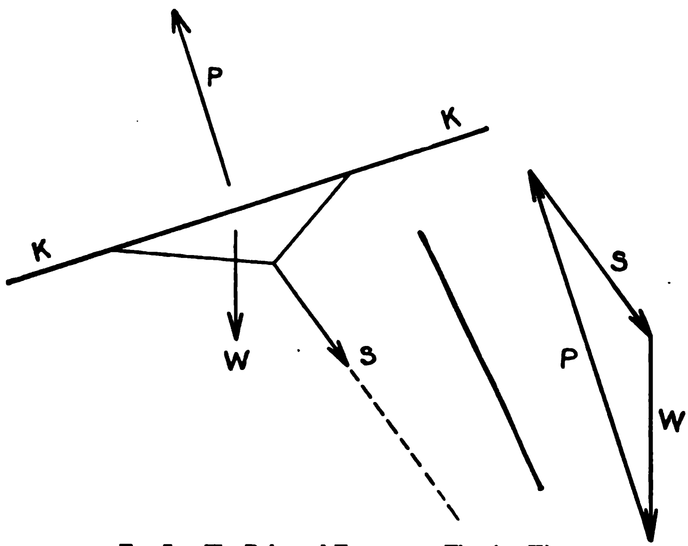

FIG. 7.—The Balanced Forces on a Floating Kite.

PROBLEM OF THE AËROPLANE

18. The three forces acting on an aëroplane are the resultant **air pressure** P, the **weight** W, and the **push** of the propeller E. The directions and relative magnitudes of these forces are given in Figure 8.

Let it be assumed that the aëroplane is headed into the wind and the push of its propeller is just sufficient to prevent it from moving with the wind. Under these conditions the aëroplane floats like a kite, the push of the propeller of the aëroplane taking the place of the pull of the kite string. As a matter of fact, **the same relation of forces holds when the aëroplane is moving forward in still air without increase or decrease of speed**; that is, the forces E, P, and W are so related that they form a closed polygon. If the speed of the aëroplane is changing, the vector polygon is not closed.

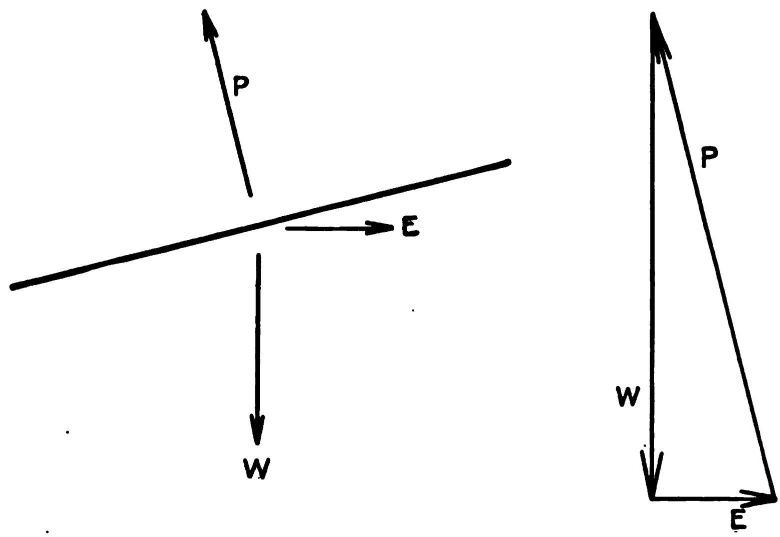

FIG. 8. — Relation of Forces acting on an Aëroplane.

THE RESOLUTION OF A VECTOR QUANTITY

19. There is a process of vector analysis which is the converse of the process of Section 12. This is known as the resolution of a vector quantity into components in specified directions. As an example of its application consider the following problem. Let Figure 9 represent a sled coasting downhill. It is required to find the force which is urging the sled down the hill. Of course it is the weight of the sled which causes the motion. It is evident, however, that only a part of the weight, which is a force extending in a vertical direction, as indicated by the arrow *CD*, is effective in producing motion down the hill. To be wholly effective in producing motion the force *CD* would have to act in a direction parallel to the direction of motion. One of the effects of the weight is to hold the sled against the hillside, and one part of the force is to be thought of as accomplishing this result and as being inoperative so far as the motion of the sled is concerned. To find that part of the weight which is effective in producing motion, proceed as follows: Draw *CD* to scale to represent the weight of the sled. Through the upper end of the arrow

CD draw a line **parallel to the hillside** and indefinite in extent. Through the arrowpoint of the arrow *CD* draw a line **perpendicular to the plane** *AB* and also indefinite in extent. The intersection of these two lines determines the point *E* and the lengths *CE* and *ED*. *CE* then represents to scale that part of the weight which causes motion in the direction *AB*. *ED* represents that part of the weight which holds the sled against the hillside and has no tendency to cause motion either up or down the hill. *CE* and *ED* are called the components of *CD*.

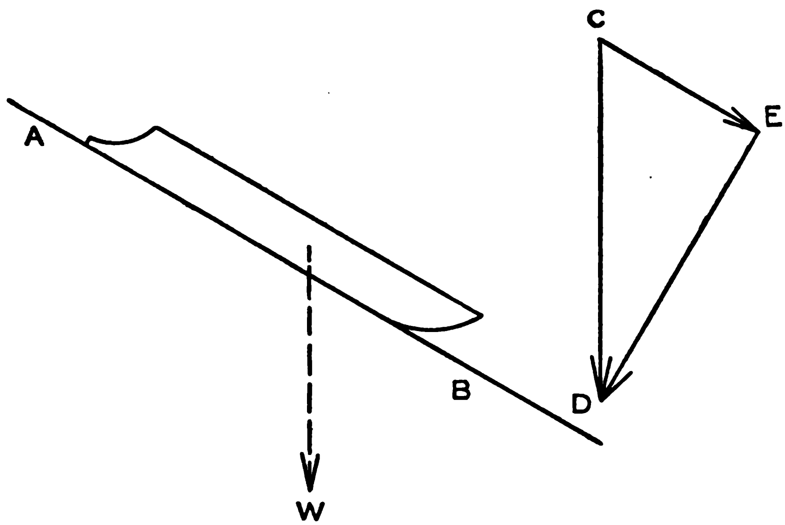

Fig. 9.—Only a Part of the Weight of a Sled tends to give it Motion down the Hill.

It is evident that under the action of two forces whose magnitudes and directions are given by the lines *CE* and *ED*, the sled would move exactly as it moves when acted upon by the force *CD* alone. This consideration determines where the arrowheads of the components *CE* and *ED* should be placed.

The rule for resolving a vector into components in given directions may be stated as follows: **Through one end of the line representing the vector draw a line in one of the given directions. Through the other end of the line representing the vector draw a line in the other given direction. The intersection of these two lines determines their lengths and therefore the magnitudes of the components.**

THE WIND AND THE SAIL

20. A problem similar to that of the last section is the problem of the wind and the sail. Let Figure 10 represent a boat fitted with a sail SS. Let W represent the direction and magnitude of the wind pressure. A part only of the wind pressure will tend to move the boat. That part or **component** which is parallel to the sail will exert no pressure on the sail, and that part only which is perpendicular to SS will operate to move the boat. The wind velocity W must therefore be **resolved** into two components I and E parallel and perpendicular respectively to the sail. The component E gives that part of the wind velocity which is effective in producing pressure on the sail. In other words, the wind W is equivalent to **two winds** I and E acting together. The effect of W upon the boat is the same as that of E acting alone.

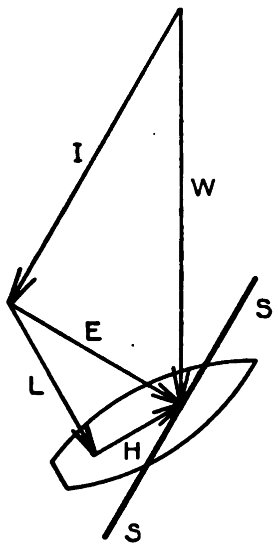

FIG. 10. — Resolution of Wind Pressures on the Sail of a Boat.

Now E must be resolved into components H and L parallel and perpendicular to the keel of the boat. The effect of L is to produce a slight sidewise motion (leeway) of the boat. H is the effective component in response to which the boat moves forward.

VECTOR DIFFERENCE

21. The difference between two vectors is obtained by **reversing the quantity to be subtracted** and then adding according to the rule for vector summation.

Consider the vectors represented by a and b, Figure 11. The sum of these is given by the line OA. The **vector difference** $a - b$ is given by the line OB, and the **vector difference** $b - a$ is represented by the line OC. Evidently $a - b$ is numerically equal to $b - a$ and exactly opposite in direction.

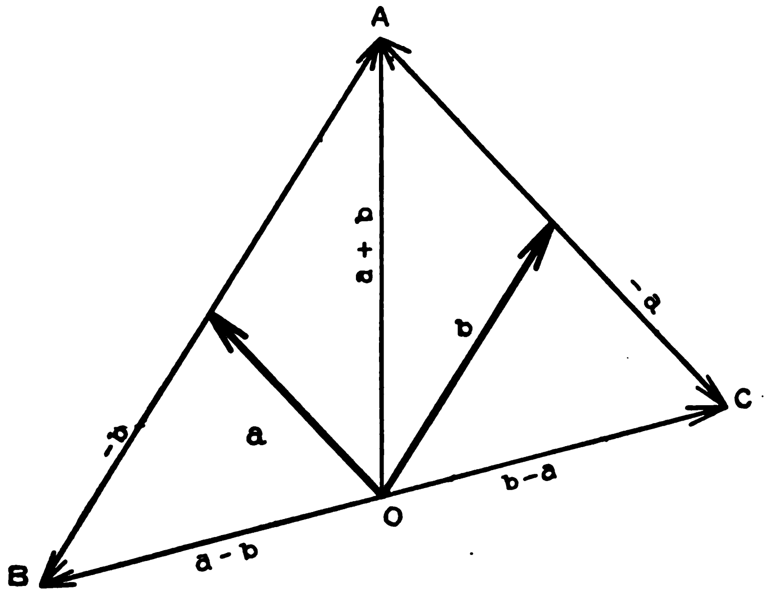

FIG. 11. — Illustrating Vector Difference.

PROBLEM OF THE TWO SHIPS

22. As an illustration of the application of the principle outlined in the preceding section consider the following problem. Two ships *A* and *B* sail from port at the same time. After a certain time *A* has traveled a distance represented by the line *OA*, Figure 12, and *B* a distance represented by the line *OB*. It is required to find **the direction and**

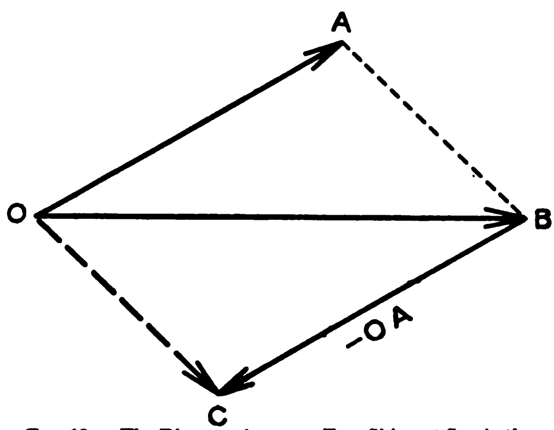

FIG. 12. — The Distance between Two Ships at Sea is the Vector Difference between their Distances from a Common Starting Point.

distance of B from A. Had A remained in port B's distance from A would be given by OB. But inasmuch as A has also sailed, evidently A's distance from port must be subtracted from B's distance from port in order to get B's distance from A. Inverting OA and adding, we obtain the line OC, which represents B's distance and direction from A.

Problems

1. A man walks 4 mi. north, then 6 mi. northeast, then 3 mi. east, then 8 mi. south. Determine by diagram his distance and direction from the starting point.

2. A man can row a boat 6 mi. per hour in still water. If he wishes to go due east across a river which flows south at the rate of 3 mi. per hour, in what direction should he row?

3. A boat is sailing due north at a velocity of 12 mi. per hour. A wind of 7 mi. per hour is blowing from the east. What is the apparent direction and velocity of the wind to a person on the boat?

4. To a man on the deck of a steamer the wind appears to blow from the northeast and to have a velocity of 30 mi. per hour. The steamer is moving due north at the rate of 16 mi. per hour. What is the actual direction and velocity of the wind?

5. Two vessels, A and B, pass in midocean. A is traveling southeast 16 mi. per hour. B is moving southwest 20 mi. per hour. What is B's distance and direction from A 3 hr. after they pass?

6. Three forces A, B, and C act upon a body. $A = 5$ units, directed eastward, $B = 7$ units, directed northward, and $C = 10$ units, directed northeast. Find $A + B + C$ and $A + B - C$.

7. A rope is stretched horizontally between two hooks which are 20 ft. apart. When a force of 500 lb. weight is applied at the center, the rope sags 2 ft. Find the magnitude and direction of the force acting on each hook.

8. Find the vertical and horizontal components of the forces acting on the hooks in problem 7.

9. A chain hangs between two hooks. The hooks are in the same horizontal plane and the chain sags so as to make an angle of 30° with the horizontal at each hook. What is the stretching force in the first link at each end of the chain in terms of the total weight W of the chain?

10. In the crane shown in Figure 6, assume the length of the pole is 20 ft. and that the angle between the pole and the wall is 30°. What must be the distance PR in order that the pull on the rope RO may be a minimum?

MOTION

CHAPTER III

UNIFORM AND ACCELERATED MOTION

23. Linear motion may be either uniform or accelerated. **A body has uniform motion when it moves over equal distances in equal times. A body has accelerated motion when it moves over unequal distances in equal times.**

VELOCITY AND ACCELERATION

24. The **velocity** of a body is the rate at which it passes through space. Velocity is usually determined by dividing the distance passed over by the time required by the moving body to pass over that distance. For example, if a train moves uniformly from one station A to another station C in 3 hours, the distance A to C being 90 miles, the velocity is given by:

$$\begin{aligned} \text{velocity} &= \frac{\text{distance}}{\text{time}} \\ &= \frac{90 \text{ miles}}{3 \text{ hours}} \\ &= 30 \frac{\text{miles}}{\text{hour}} \end{aligned}$$

or 30 **miles per hour.**

If the motion of the body is accelerated, the quotient of distance divided by time gives the **average velocity** of the moving body. Thus if a rifle ball fired vertically into the air rises to a height of 3000 feet in fifteen seconds, its **average velocity during that time** is,

$$\begin{aligned} \text{average velocity} &= \frac{3000 \text{ feet}}{15 \text{ seconds}} \\ &= 200 \frac{\text{feet}}{\text{second}} \end{aligned}$$

or 200 **feet per second.**

Acceleration is defined as the rate of change of velocity. It may be determined by dividing the change in velocity by the time in which that change takes place. By way of illustration, let it be assumed that a train starting from rest acquires a velocity of $30 \frac{\text{miles}}{\text{hour}}$ in 15 minutes. The gain or change in velocity is $30 \frac{\text{miles}}{\text{hour}}$, and this has taken place in 15 minutes, therefore,

$$\text{acceleration} = \frac{\text{change in velocity}}{\text{time}}$$

$$= \frac{30 \dfrac{\text{miles}}{\text{hour}}}{15 \text{ minutes}}$$

$$= \frac{2 \dfrac{\text{miles}}{\text{hour}}}{\text{minute}}$$

or **2 miles per hour per minute.**

UNIFORMLY ACCELERATED MOTION

25. A body is said to have uniformly accelerated motion when the acceleration is constant, that is to say, when the gain in velocity is the same for each unit of time. A familiar example of uniformly accelerated motion is that of a body falling freely under the action of gravity.

Uniform motion is distinguished from uniformly accelerated motion as follows: The inertia of a body tends to keep it in motion once it has been put in motion; and were it not for the fact that any moving body encounters frictional and other resistances, it would continue in motion indefinitely, moving over equal distances in equal times, that is to say, it would have uniform motion. It is therefore evident that **the criterion for uniform motion is that no force shall be acting upon the body,** or what amounts to the same thing, that all forces acting are balanced, the combined effect being zero. On the other hand, in **uniformly accelerated motion an unbalanced force is necessarily present,** this kind of motion being impossible without a steadily acting unbalanced force. In the example

above referred to it is evident that while the body is falling freely through space the force which is urging it downward is at each and every instant present in unchanged magnitude. The weight of a body is practically constant throughout such distances as are here considered.

THE DISTANCE PASSED OVER BY A BODY HAVING UNIFORMLY ACCELERATED MOTION

26. The velocity of a body has already been defined as the distance passed over divided by the time occupied by the body in passing over that distance. This relation may be written as follows:

$$v = \frac{d}{t} \tag{1}$$

in which v represents the velocity, d the distance, and t the time occupied by the body in traversing the distance d.

Transforming this equation,

$$d = vt$$

This expression gives the distance passed over by any moving body in terms of the velocity and the time. Evidently the velocity here contemplated is the **average velocity**.

Consider the case of the falling body. Let it be required to determine the distance it passes over in t seconds. We could make use of Equation (1) for this determination if we knew the average velocity of the falling body during the time interval under consideration.

The average velocity in this and similar cases is easily obtained. Let line v_0, Figure 13, represent to scale the velocity of the body at the beginning of the time interval t. Let the line v_1 represent the velocity of the body at the end of the first second, v_2 at the end of the second, v_3 at the end of the third, etc. If the body has uniformly accelerated motion, it follows that each of these lines differs in length from that on either side of it by the same amount, since **the gain in velocity per unit time is constant.** If the lines are equally spaced, the arrow-points will all lie on the same straight line as shown. Now the average velocity is the average of all the velocities, v_0, v_1,

v_2, etc., and might be obtained by adding together the velocities, represented by the lines v_0, v_1, v_2, etc., and dividing by 6, the number of velocities considered. From geometrical considerations, however, it is evident that we may use a short cut method in this case for finding the average of the six quantities, that is, we may add together the first and last of the six quantities and divide by 2, thus,

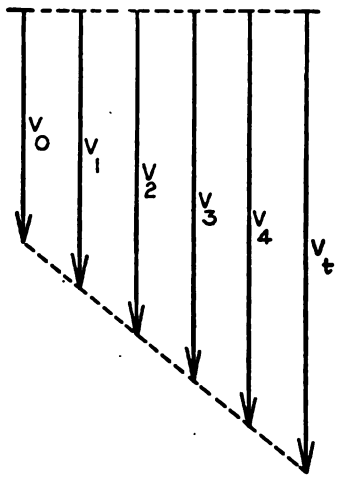

FIG. 13.—The Average Velocity in Uniformly Accelerated Motion equals the Half Sum of the Initial and Final Velocities.

$$\bar{v} = \frac{v_0 + v_t}{2} \tag{2}$$

in which $\bar{v}$ (read barred v) represents the average velocity.

For uniformly accelerated motion, therefore, the formula giving the distance passed over by the body in t seconds is as follows:

$$d = \frac{v_0 + v_t}{2} \cdot t \quad \text{(See Equation 1.)}$$

in which v_0 is the velocity of the body at the beginning of the time interval under consideration and v_t is the velocity at the end of that interval.

GENERAL EQUATIONS FOR UNIFORMLY ACCELERATED MOTION

27. It is desirable to have an expression for the final velocity in terms of the initial velocity. This is obtained as follows: Let a represent the acceleration, that is, the gain in velocity in each unit of time. We may therefore say that the

gain in velocity in 1 second $= a$
gain in velocity in 2 seconds $= 2a$
gain in velocity in 3 seconds $= 3a$
. . . .
. . . .
. . . .
Gain in velocity in t seconds $= ta$

Now the final velocity is equal to the initial velocity plus the gain in velocity. We have, therefore, at once:

$$v_t = v_0 + a \cdot t \tag{3}$$

Substituting in Equation (2) the value of v_t given in Equation (3) we have,

$$\bar{v} = v_0 + \tfrac{1}{2}\, at \tag{4}$$

And finally substituting this value for v in Equation (1), we obtain,

$$d = v_0 t + \tfrac{1}{2}\, at^2. \tag{5}$$

This equation is the general expression for the distance passed over by a body having uniformly accelerated motion in terms of the initial velocity v_0, the acceleration a, and the time t, during which the motion is considered.

We have assumed in the development of this formula that the initial velocity and the acceleration are in the same direction. In case the acceleration has a direction opposite to that of the initial velocity, one of the terms of the right-hand member of Equation (5) should be given a negative sign. If the direction of v_0 in this case is regarded as positive, then the second term becomes negative. If the acceleration is regarded as positive, v_0 in this case would be negative.

THE CASE OF A BODY STARTING FROM REST

28. In this case $v_0 = 0$, so that Equations (3), (4), and (5) become,

$$v_t = at \tag{6}$$

$$\bar{v} = \tfrac{1}{2}\, at \tag{7}$$

$$d = \tfrac{1}{2}\, at^2 \tag{8}$$

and

$$v_t = \sqrt{2\, ad} \text{ (from 6 and 8)} \tag{9}$$

THE CASE OF THE PROJECTILE

29. In this case it is assumed that the projectile is thrown in some direction other than the vertical, so that the case considered is really that in which the initial velocity makes an angle ϕ with the horizontal, ϕ being an angle which may have any

value, positive or negative, between 0° and 90°. Consider the case represented in Figure 14. Let the initial velocity with which the projectile is thrown be represented in magnitude and direction by the line v_0. Let it be required to find the velocity of the projectile after the lapse of t seconds. **The relations given by Equations (1) to (5) are applicable to this case, provided we interpret them as vector instead of algebraic equations, that is to say, provided we make the summations indicated in these equations by the method of vector analysis.** The final velocity of the projectile in the problem just stated is obtained as follows: The line AB having been drawn to scale to represent the initial velocity, draw from the arrowpoint downward a line BC representing in magnitude and direction the gain in velocity due to gravity, that is, at. Finally join A and C. This line represents in magnitude and direction the velocity of the projectile after the lapse of t seconds. This value may be obtained by measuring the length of the line AC and interpreting that length according to the scale number used in the drawing of AB and BC. In the same way the average velocity may be determined by joining A with the middle point of the line BC, since by Formula (4) the average velocity is equal to the initial velocity plus $\frac{1}{2}$ the gain in velocity. Therefore the line AD in Figure 14 represents in magnitude and direction the average velocity during the time t.

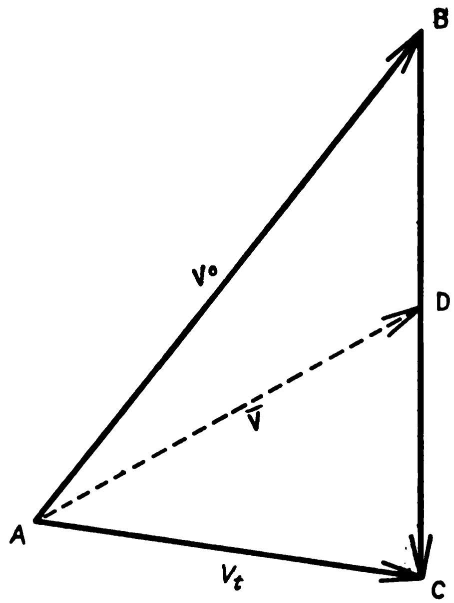

FIG. 14.—Relation between Initial and Final Velocities of a Projectile.

In a similar manner we can find the distance of the projectile

from the starting point after the lapse of t seconds. According to Equation (5) this distance $= v_0t + \frac{1}{2}at^2$, it being understood that the sum of these two quantities is to be taken according to the vector method. If, therefore, a line EF, Figure 15, is drawn to represent v_0t in magnitude and direction and FG to represent $\frac{1}{2}at^2$, then EG, the vector sum of EF and FG, **represents in magnitude and direction the distance of the projectile from the starting point** at the end of the time t.

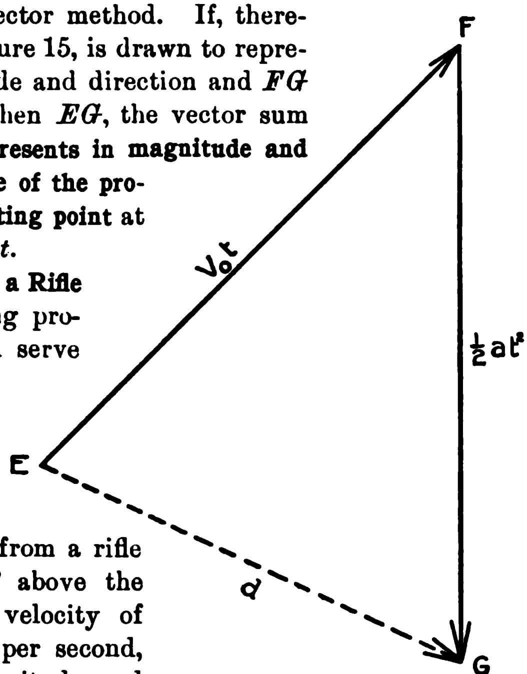

Fig. 15.—Distance of the Projectile from the Starting Point after t Seconds.

30. The Flight of a Rifle Ball. The following projectile problem will serve to emphasize the principle of vector analysis as applied to projectiles. Given that a rifle ball is fired from a rifle at an angle of 30° above the horizontal with a velocity of 25,000 centimeters per second, represented in magnitude and direction by the line AB, Figure 16. Let it be required to find the "range" and the time of flight, the "range" being defined as the horizontal distance passed over by the ball before it comes into the same horizontal plane as that from which it started. The solution of the problem is as follows: Draw a vertical line through B indefinite in extent. Draw a horizontal line through A until it intersects the vertical line through B. The point of intersection E determines the length of AE and BE. BE is $\frac{1}{2}$ the gain in velocity in the time interval under consideration. AE is the average velocity for that interval. This is evident from the fact that the average velocity must be directed from the starting point to the point reached by the ball at the end of the flight under consideration,

and from Equation (4). Measuring the line *BE* and interpreting its length according to the scale number to which *AB* was drawn, we find it represents a velocity of 12,500 centimeters per second. We have, therefore,

$$\tfrac{1}{2}\,at = 12{,}500\ \frac{\text{centimeters}}{\text{second.}}$$

Assume that the acceleration of bodies falling freely under gravity is 980 $\dfrac{\text{centimeters}}{(\text{second})^2}$.

$$\text{Therefore}\qquad t = \frac{12{,}500 \times 2}{980} = 25.5 \text{ seconds}$$
$$= \text{the time of flight}$$

that is to say, the rifle ball occupies 25.5 seconds in traversing the path represented by the curved line *ADC*, Figure 16.

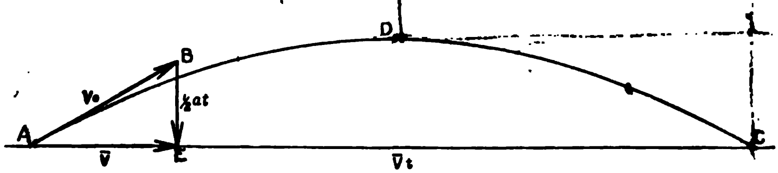

FIG. 16. — The Range of a Rifle Ball.

In order to determine the range we have only to determine the value of $\bar{v}$, the average velocity, and multiply this by the time of flight. $\bar{v}$ is found by measurement to have a value of 21,500 centimeters per second. Multiplying this value by 25.5, the time of flight, we obtain,

$$\begin{aligned} d = \bar{v}t &= 21{,}500 \times 25.5 \\ &= 548{,}250 \text{ centimeters} \\ &= 5.48 \text{ kilometers} \\ &= 3.3 \text{ miles (approximately)} \end{aligned}$$

In the above discussion of the flight of a rifle ball, the effect of air resistance is ignored. The effect of the resistance of the air is to retard the motion of the ball and hence to reduce its range. In practice it is found that the actual range of a projectile is materially less than the theoretical range. This is

especially true of projectiles of high velocity, since the retarding effect of air resistance increases rapidly with the speed of the moving body.

THE INDEPENDENCE OF FORCES

31. Any force acting upon a body produces its effect independent of all other forces which may be present in conjunction with it. The following simple experiment will serve to illustrate this principle.

Let it be assumed that a cannon ball B is fired from a cannon in a horizontal direction (see Figure 17). It will travel along a curved path AB, the curvature of which depends upon the

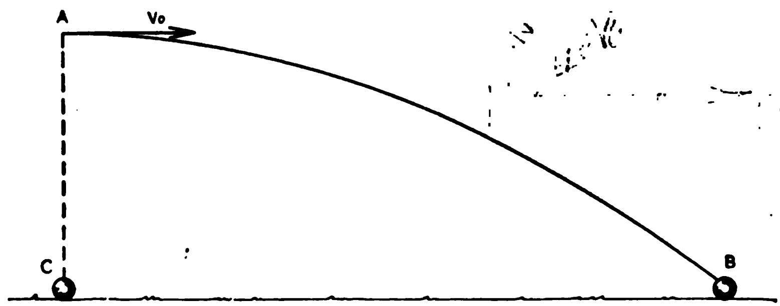

FIG. 17. — A Cannon Ball fired horizontally falls as rapidly as a Ball starting from Rest.

initial velocity v_0 with which the ball leaves the cannon. Let it be assumed that a second cannon ball C is released at the muzzle of the cannon at the instant the first ball B begins its flight and that the ball C falls freely under the action of gravity alone. C and B will reach the ground (*i.e.* the same horizontal plane) at the same instant. That is to say, according to the proposition above stated the ball B falls as rapidly as the ball C, whatever the value of its initial **horizontal** velocity. In other words, gravity produces its effect upon the ball B independent of the horizontal impulse given to it by the powder in the gun.

This proposition is conveniently demonstrated by the use of a spring gun which throws a ball in a horizontal direction, and which, by suitable adjustment, releases a second ball placed at the muzzle of the gun, or at any other convenient point in the same horizontal line, at the same instant. In carrying out an

experiment of this kind it will be found that the two balls *B* and *C* strike the floor at the same instant. A modification of this experiment is indicated in Figure 18. A ball *C* is supported in front of a spring gun *A*, as indicated, and starts to fall at the instant the ball *B* leaves the muzzle of the gun as in

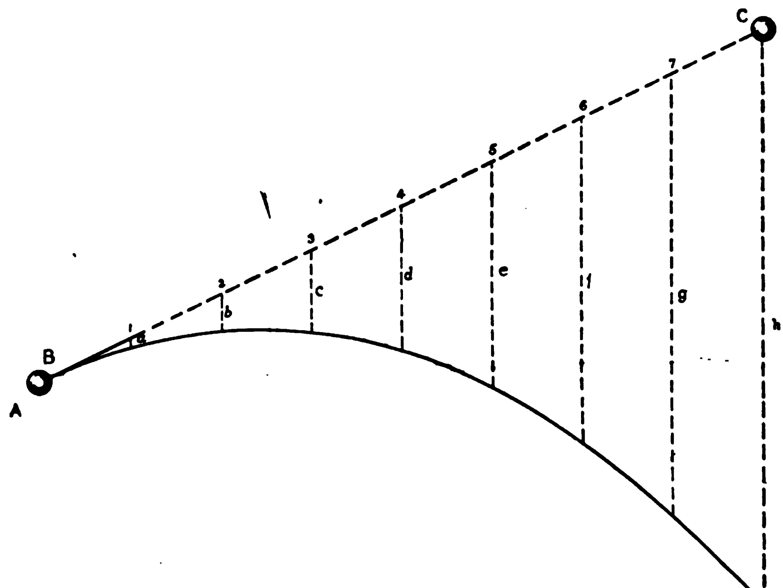

FIG. 18. — Ball fired at a Falling Target.

the experiment just described. The gun is aimed at the ball *C*. It will be found under these circumstances that the balls will meet in mid-air at some point such as *D*, demonstrating that the ball *B* has been caused to depart from its original course by exactly the same vertical distance that the ball *C* has fallen in the interval of time required for the ball *C* to reach the point *D*. It will be noted that this is exactly the same length of time that the ball *B* would require to travel from *A* to *C* under the impulse of the spring alone.

Problems

1. An elevator acquires a velocity of 9 ft./sec. in 3 sec. What is the acceleration if uniform?

2. What distance does the elevator of problem 1 pass over in the first 3 sec.?

16, 24, ...

3. A stone dropped into a well strikes the water 2 sec. after it starts to fall. How far below the surface of the ground is the water in the well?

4. A rifle ball is fired vertically downward from a balloon. Its velocity as it leaves the rifle is 25,000 cm./sec. What is its velocity 3 sec. later? (Neglect air friction.)

5. How far from the balloon will the rifle ball of problem 4 be at the end of the third second?

6. A body falls freely under gravity starting from rest. What is its velocity when it has fallen 100 ft.?

7. A baseball is thrown vertically upward. With what velocity must it leave the hand in order that it may rise to a height of 125 ft.?

8. A rifle ball was fired at an angle of 45° to the horizontal and rose to a height of 4000 feet. What was its range? (Neglect air friction.)

9. How far will a body having uniformly accelerated motion travel in 5 sec., if its velocity at the end of that interval is 600 ft./sec. and its acceleration is 30 ft./sec.^{2}?

10. A tower is 300 m. high. If a rifle ball is fired upward from the top of the tower at a velocity of 2000 ft./sec., what time will elapse before the rifle ball reaches the earth?

FORCE AND TORQUE

CHAPTER IV

FORCE

32. Force is that which changes or tends to change the motion of a body. It may be measured by observing its accelerating effect upon a given mass. It is found by experiment that the acceleration produced in any mass is strictly proportional to the force which causes that acceleration, that is to say,

$$F \propto a$$

in which F stands for the force and a for the acceleration. Experiment also shows that the acceleration produced by a given force is inversely proportional to the mass of the body upon which the force acts, *i.e.* $a \propto \frac{1}{M}$ and therefore $a \propto \frac{F}{M}$. This proportionality may be expressed in the form of an equation as follows:

$$F = Ma \tag{10}$$

providing the force is expressed in the units of force defined below.

Units of Force. Equation (10) expresses the general relation between a force and the acceleration which it produces in a given mass. Since mass and acceleration have already been defined, it will be evident that this relation may be used for defining the unit of force. Naturally we define unit force in terms of unit mass and unit acceleration. Therefore **the unit of force in the c. g. s. (centimeter-gram-second) system of units is that force which acting upon a mass of one gram will impart to it an acceleration of one centimeter per second per second.** This unit is called the **dyne.**

The unit of force in the f. p. s. (foot-pound-second) system is the **poundal,** and is defined as that force which will impart

to a mass of one pound an acceleration of one foot per second per second.

Weight. Since Equation (10) is general, it may be applied to the case of the force action between a body and the earth, *i.e.* the weight of the body. Let W stand for the force with which the earth attracts the body and g for the acceleration produced by this force in a freely falling body. We have, therefore:

$$W = Mg \tag{11}$$

in which M is the mass of the body as before, and g is the acceleration due to gravity.

Assume the value of g to be $\dfrac{980.6 \text{ centimeters}}{\text{second}^2} = \dfrac{32.2 \text{ feet}}{\text{second}^2}$.

Substituting these values for g and putting $M = 1$ in Equation (11), we find,

The weight of a gram, *i.e.* 1 gram weight = 980.6 dynes.

The weight of a pound, *i.e.* 1 pound weight = 32.2 poundals.

THE MEASUREMENT OF FORCE

33. It is evident that the relation given in Equation (10) may be made use of in the measurement of a force. If a force, the magnitude of which we wish to determine, is allowed to act upon a known mass and the acceleration produced is measured, we have the values of the quantities appearing on the right-hand side of the equation, and the equation may be solved for the value of F.

Another method of measuring force is by observing its effect in distorting an elastic body. Figure 19 represents an elastic body in a convenient form for the purpose of measuring forces. AB is a spiral of elastic wire the upper end of which is suitably supported, the force to be measured being applied to the

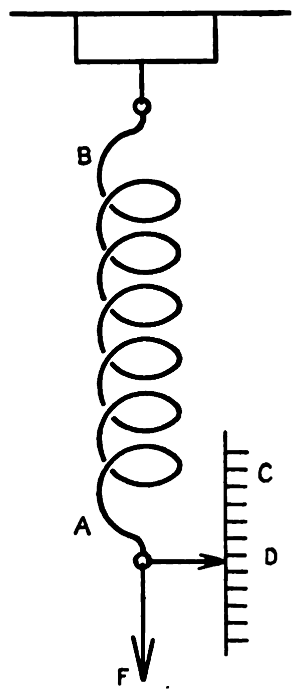

FIG. 19. — A Spring Dynamometer

lower end as indicated by the arrow F. Under the action of this force F the spiral is elongated by an amount which is proportional to, and therefore a measure of, the force F. Such a device is commonly known as a **dynamometer**, or spring balance.

THE PROPORTIONALITY FACTOR

34. Equation (10) in Section 32 was really developed from a proportionality by introducing the factor M and substituting the sign of equality for that of proportionality. A brief discussion here of what is known as the "proportionality factor" in an equation will be timely, inasmuch as we shall have occasion to make use of a proportionality factor in the development of many of the equations which follow. The meaning of the "proportionality factor" in an equation will be understood from the consideration of one or two examples. Let it be required to express the cost of a railway in terms of its length. Assuming that the cost per mile is uniform, we may write,

$$\text{cost} \propto \text{length in miles} \qquad (a)$$

It will be evident that the total cost is equal to the product of the cost per mile and the total length in miles. Let K represent the cost per mile; we have therefore,

$$\text{cost} = K \cdot \text{length in miles} \qquad (b)$$

(a) is an expression of **proportionality**, (b) is an equation. (a) is transformed into (b) by introducing the constant K and the sign of equality for the sign of proportionality. K is called the "proportionality factor."

In the same manner for the spring balance of the last section we may write,

$$\text{force} \propto \text{elongation}$$

or $F = K_1 \cdot e$, in which e is the elongation of the spring caused by the force F.

It will be evident from the example given that if one physical quantity is proportional to a second, the first may be expressed in terms of the second by means of an equation in which a suitable proportionality factor has been introduced. Furthermore, by substituting known values for the two quantities concerned, the equation may be solved for the value of

the proportionality factor. **The equation used in this way becomes a defining equation for the proportionality factor.**

EXAMPLE. — Let it be assumed that in the case of the dynamometer cited above, the spring is elongated 3 inches by a force of 15 pounds weight. We have,

$$\text{force} = K_1 \times \text{elongation}$$

or $$15 \text{ pounds wt.} = K_1 \times 3 \text{ inches}$$

Solving for K_1, $$K_1 = \tfrac{15}{3} = 5 \, \frac{\text{pounds weight}}{\text{inch}}$$

which in ordinary language means that 5 pounds weight are required to elongate the spring one inch.

TORQUE

35. **When a force acts upon a body in such way that it tends to produce rotatory motion, it is said to have torque action. This torque action of a force depends upon the magnitude of the force and its distance from the axis about which the body tends to turn.** The **torque** is given by the product of the force and the distance from the axis to the force measured perpendicular to the latter.

$$T = FL \tag{12}$$

in which T is the torque, F the force, and L the perpendicular distance of the axis from the force.

Experiment shows that the **angular acceleration** produced in a given body by a torque is proportional to the torque, that is,

$$\alpha \propto T$$

Experiment also shows that if a given torque is caused to act upon different bodies, the **angular acceleration** produced in each case is inversely proportional to the **moment of inertia** of the body acted upon. Or,

$$\alpha \propto \frac{1}{I}$$

in which I is the moment of inertia of the body. These expressions for α may be combined as follows:

$$\alpha \propto \frac{T}{I}$$

and written in the form of an equation provided the units of torque, angular acceleration, and moment of inertia are properly chosen, or provided the equation is used for defining I. Thus, —

$$T = I \cdot \alpha. \tag{13}$$

This relation may be used for determining the value of an unknown torque. The torque is applied to a body of known moment of inertia and the angular acceleration is observed. I and α being known, Equation (13) may be solved for T. In similar manner Equation (13) may be used in the measurement of moment of inertia of a body by applying a known torque to it and observing the angular acceleration.

MEASUREMENT OF ANGLE

36. The unit of angle employed in science is the **radian. A radian is an angle whose arc is equal to its radius.** Referring to Figure 20, the angle ϕ is one radian when the arc CB, measured in centimeters or inches, is equal to the radius AB. Hence the value of any angle in radians is given by the following relation:

$$\text{angle in radians} = \frac{\text{arc}}{\text{radius}}$$

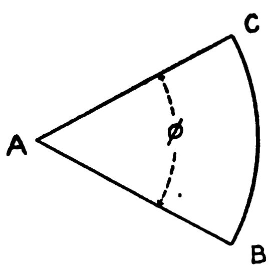

FIG. 20. — Unit Angle, the Radian.

The **degree** is used for certain kinds of angular measurement. The relation between the radian and the degree is obtained as follows. The circumference of a circle is equal to its radius $\times\ 2\pi$. Therefore the total angle in a plane about a point is 2π radians, whence it follows that,

$$360^\circ = 2\pi \text{ radians}$$

and

$$1 \text{ radian} = 57.3^\circ \text{ (approximately)}$$

Angular velocity is measured in radians per second. Angular acceleration is measured in radians per second per second.

BALANCED FORCES. FIRST CONDITION OF EQUILIBRIUM

37. **In order that a number of forces may be in equilibrium, their vector sum must be zero.** (See Section 15.) When this condition is fulfilled, the body acted upon by these forces will have no linear acceleration. If the vector sum of the forces is not zero, they have an unbalanced resultant which will tend to accelerate the motion of the body acted upon.

BALANCED TORQUES. SECOND CONDITION OF EQUILIBRIUM

38. **In order that a number of torques may be in equilibrium, their sum must be zero.** Another way of stating this condition is to say that the sum of the positive torques must equal the sum of the negative torques. When this condition is fulfilled, the body acted upon will have no angular acceleration.

(Those torques which act **against** the hands of a clock, or **counterclockwise**, are called positive torques; those acting clockwise are called negative.)

CONSTANT TORQUE ACTION OF BALANCED FORCES

39. **The torque action of two or more forces in the same plane, satisfying the first condition of equilibrium, is the same about all points in that plane.** Consider the three forces shown in Figure 21. Their values are 3, 4, and 5 and their vector sum is zero. Assume that the lines in the figure represent the actual directions of the forces. Now if the point of intersection of two of the forces is chosen as the **pivot**, only the third force will have torque action. Thus,

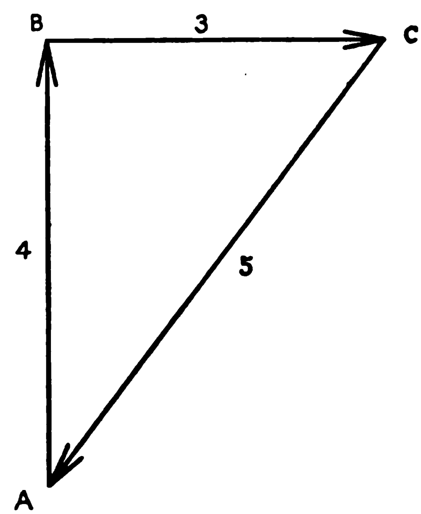

FIG. 21.—The Torque Action of Forces in Equilibrium is the same about all Points in the Plane of the Forces.

$$\text{the torque about } A = 3 \times 4 \quad = 12 \text{ units}$$
$$\text{the torque about } C = 4 \times 3 \quad = 12 \text{ units}$$
$$\text{the torque about } B = 5 \times 2.4 = 12 \text{ units}$$

since 2.4 is the **perpendicular** distance from CA to the pivot B. If now any other point inside or outside the triangle is chosen as pivot, the total torque action about it will be found to be 12 units. In other words, the torque action of these three forces is the same about all points in their plane.

THREE FORCES IN EQUILIBRIUM INTERSECT IN ONE POINT

40. This is evident from the fact that if any three forces do not intersect in a common point, then any one of them will have

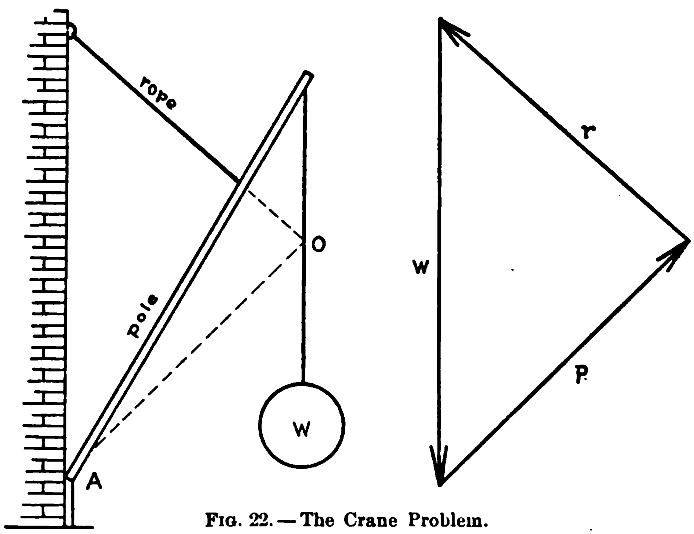

Fig. 22. — The Crane Problem.

torque action about the point of intersection of the other two, and the second condition of equilibrium will not be satisfied. Therefore three forces can only be in equilibrium when they intersect in a common point.

This principle may be used to great advantage in solving problems of a certain class. As an example, consider the crane problem represented in Figure 22.

The pole is attached to the wall at its lower end, and sup-

ported by a rope near its upper end. The weight W hangs from the end of the pole. Required the tension in the rope and **the force with which the wall pushes against the pole.**

SOLUTION. The three forces acting upon the pole are in equilibrium, therefore they intersect in a common point. This common point of intersection must be O, since this is the point of intersection of the forces in the two ropes. It follows therefore that the third force acting through A must have the direction AO. The directions of the several forces are thus determined: Draw a triangle wpr, the side w to scale to represent the weight. Then the lines p and r, parallel respectively to AO and the upper rope, represent to scale the corresponding forces.

NEWTON'S LAWS OF MOTION

41. There are three fundamental physical laws which state the effect of force upon matter, the dependence of matter upon force for change in motion, and the reaction of matter to force action. In honor of the celebrated Englishman who first formulated them these laws are called **Newton's laws of motion.**

1. **A body at rest tends to remain at rest and a body in motion tends to continue in uniform motion except in so far as it is acted upon by disturbing force.**

2. **The acceleration produced in a given mass by the action of a given force is proportional to the force and in the direction of that force.**

3. **Action is equal to reaction.**

The first law is a statement of the inertia of matter. A body cannot of itself in any way alter its state of motion. If a bullet were fired into the air, or a ball tossed from the hand, and shielded from the action of all other forces, it would continue in motion indefinitely in the same direction and with the same velocity that it had at the outset. Of course, such an experiment cannot be tried since it is impossible to eliminate the effects of air friction and of gravity, and these two effects gradually change the velocity of the moving body, bringing it, eventually, to the ground.

The second law is simply a statement of the equation $F = M \cdot a$ (Equation 10) in common language. It has already

been pointed out (Section 25) that acceleration is to be regarded as evidence of the presence of an unbalanced force. A stationary body acted upon by a system of balanced forces has no tendency to start into motion. Neither can a system of balanced forces in any way change the motion of a moving body. An unbalanced force is the only agency which can alter the state of motion of a body; that is, it is the only agency which can produce acceleration, and the acceleration produced by such a force is proportional to, and in the direction of, that force, as stated in the second law.

The third law calls attention to the double aspect of a force. In every case of force action two bodies are concerned, the body acting and the body acted upon. Call these bodies *A* and *B*. This law states that if *A* pushes *B* with a certain force, *B* reacts with an equal push in the opposite direction.

DIMENSION FORMULÆ

42. The dimension formula for a physical quantity is an expression which shows how **mass**, **length**, and **time** (the chosen fundamental quantities) are involved in its defining equation. For example, the defining equation of velocity is,

$$\text{velocity} = \frac{\text{distance}}{\text{time}}$$

Since distance is measured in units of length, we may write,

$$[v] = \frac{L}{T}$$

or,

$$[v] = LT^{-1}$$

which is called the dimension formula for velocity. The brackets are placed about v to indicate that the equation is not an algebraic relation in the ordinary sense and is only to be used for the purpose of determining the dimensions of v. In dimension formulæ M is written for mass, L for length, and T for time.

The dimensions of the quantities thus far considered are given below:

Quantity	Dimensions
Area	L^2
Volume	L^3
Velocity	LT^{-1}
Acceleration	LT^{-2}
Angular Velocity	T^{-1}
Angular Acceleration	T^{-2}
Force	MLT^{-2}
Torque	ML^2T^{-2}
Moment of Inertia	ML^2

Several important uses are made of dimension formulæ. One of these is to substitute the dimensions of a quantity for a name for the unit of that quantity. For example the unit of acceleration has been given no name. We therefore write for an acceleration of 5 units in the f. p. s. system:

$$5\,\frac{\text{feet}}{\text{second}^2}$$

since the foot and the second are the units of length and time in that system of units. In the same way instead of 1000 c. g. s. units of moment of inertia we write:

$$1000 \text{ gram-centimeters}^2$$

Again, if it is desired to change the measure of a quantity from one system to another, we find the dimension formula of great assistance. To illustrate, let it be required to express a velocity of 40 miles per hour in meters per second. Call the velocity in meters per second x. We then have:

$$x\,\frac{\text{meters}}{\text{second}} = 40\,\frac{\text{miles}}{\text{hour}}$$

$$\therefore\; x = 40\,\frac{\text{mile}}{\text{meter}}\cdot\frac{\text{second}}{\text{hour}}$$

$$\doteq 18$$

that is, $40\,\frac{\text{miles}}{\text{hour}}$ is equivalent to $18\,\frac{\text{meters}}{\text{second}}$. In making this determination we have simply substituted the names of the units of length and time in the dimension formula for velocity. The ratio of these units we know. The reduction is therefore a simple matter.

Still another use of the dimension formula is in testing the accuracy of complicated formulæ. The terms of an equation

involving physical quantities must all be of the same dimensions. Take as an illustration the expression for the distance passed over by a body having uniformly accelerated motion,

$$d = v_0 t + \tfrac{1}{2} at^2 \qquad (5 \text{ bis})$$

Writing the dimensions of each term, we have:

$$[L] = [LT^{-1} \cdot T] + [LT^{-2} \cdot T^2]$$

that is, $$[L] = [L] + [L]$$

Problems

1. An elevator having a mass of 1000 lb. acquires a velocity of 9 ft./sec. in 3 sec. What force in pounds weight is required to accelerate its motion?

2. Let it be assumed that a man weighing 175 lb. is standing in the elevator of problem 1. With what force in pounds weight do his feet press against the floor as the elevator is being started?

3. A force of 10 lb. weight acts upon a 10 lb. mass. Find the velocity of the body at the end of 20 sec.

4. How long would it take a body of 1000 g. mass to acquire a velocity of 100 cm./sec. if acted upon continuously by a force of 10,000 dynes?

5. A mass of 1 T. is moving at a rate of 20 ft./sec. A force of 100 lb. weight opposes it. In what length of time will the opposing force bring the mass to rest?

6. A mass of 2 kg. has a velocity of 50 m./sec. What force will bring it to rest in 5 sec?

7. Two weights of 1000 and 1100 g. mass are connected together by a flexible cord which passes over a pulley. What is the acceleration of the bodies and the tension in the cord? Neglect friction effects.

8. The hammer of a pile driver weighs 1000 lb. It falls 30 ft. and drives a pile 5 in. Assuming the resistance to be uniform, with what force does the hammer push on the pile as it is brought to rest?

9. The flywheel of an engine makes 250 R. P. M. What is its angular velocity in rad./sec.?

10. A torque of 5,000,000 dyne-cm. acts upon a rotating body having a moment of inertia of 2.5×10^6 c. g. s. units. What acceleration does it produce?

11. Under the action of a torque of 400 pound-feet a flywheel acquires an angular velocity of 2 rad./sec. in 10 sec. What is its moment of inertia?

12. What torque would be required to impart an angular velocity of 25 rad./sec to a rotating body, having a moment of inertia of 8×10^6 c. g. s. units, in 100 sec.?

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CIRCULAR AND SIMPLE HARMONIC MOTION

CHAPTER V

UNIFORM CIRCULAR MOTION

43. A body is said to move uniformly in a circle when it moves over equal arcs in equal times. The velocity of a body having uniform circular motion is constant in magnitude but continually changing in direction. Such a body is accelerated just as truly as a body whose velocity increases or decreases in magnitude, and, as will be shown further on, is necessarily acted upon by an unbalanced force in order that it may have such motion. That such a body is accelerated and therefore has an unbalanced force acting upon it is evident at once from the following considerations. Let the circle, Figure 23, represent the path of a body having uniform circular motion. Let it be assumed that at a given instant it is in the position A, and that its velocity is represented in magnitude and direction by the line AC. After the lapse of a certain interval of time the body will be in the position B, its velocity being indicated by the arrow drawn through this point. Calling angle AOB ϕ, then ϕ is the angle between the velocities at A and at B. Since a body is unable

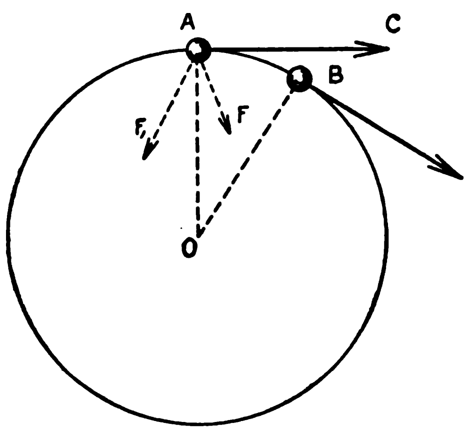

FIG. 23.—A Body in Uniform Circular Motion.

to change its own state of motion, it is evident that a force must have been acting upon the body as it passed from A to B in order that the given change in velocity, that is to say in direction of velocity, may have taken place. The simplest consideration will indicate that the general direction of this force must have been toward the concave side of the curved path AB. Further consideration will show that the direction of the force must have been at all times toward the center, for if it be assumed that the direction of the force acting upon the body in the A position has a direction which is not toward the center but in some such direction as AF, then such force has a component in the direction of the velocity v, and this component would of course tend to increase the value of v. Inasmuch as the assumption has been made that v does not change in magnitude, it is evident that no such component of F can exist. In the same way, if the assumption is made that the direction of the force is AF_1, it is evident that a component of the force will be present which would tend to decrease the value of the velocity v; and upon the assumption that v does not change in magnitude, it is evident that such component cannot exist. Therefore **the force which acts upon the body causing it to move in the curved path is at all times directed toward the center of the circle.** It is further evident that the value of this central force is constant, inasmuch as its effect in changing the direction of the velocity of the body is constant; that is to say, as the body passes over, let us say, 30° of arc in the region A, the direction, of the velocity changes through an angle of 30° (the angle between tangents to a circle is the same as the angle between the radii of the circle drawn to the points of tangency), and as the body passes over 30° of arc in the region C or at any other part of the circle the change in the direction of velocity will be 30° as before.

A convenient way of finding the value of this central force is to determine the acceleration which it produces. Knowing the acceleration, the force may be found by the use of Equation (10). Evidently the acceleration produced by the central force is **radial.** Its value is determined in the following section.

RADIAL ACCELERATION IN UNIFORM CIRCULAR MOTION

44. Let AB, Figure 24, represent a circular path along which a body is moving with a uniform velocity v. Consider the motion of the body from A to B. Call the velocity of the body at A, v_0, and the velocity of the body at B, v_t. Let t be the time required for the body to move from A to B. From a common point O' draw v_0 and v_t to scale, as shown in the upper part of the figure. Join the arrowpoints E and F as indicated. It will be evident that EF represents the gain in velocity in the time interval t, since it is the quantity which added (vector addition) to v_0 gives v_t. Hence EF represents $a \cdot t$ (see Equation 3) to the same scale that v_0 is represented by $O'E$. Now $O'EF$ and OAB are similar triangles, since $O'E$ is perpendicular to OA and $O'F$ is perpendicular to OB; and if A and B are taken very close together, arc AB may be regarded as a straight line. We may therefore write:

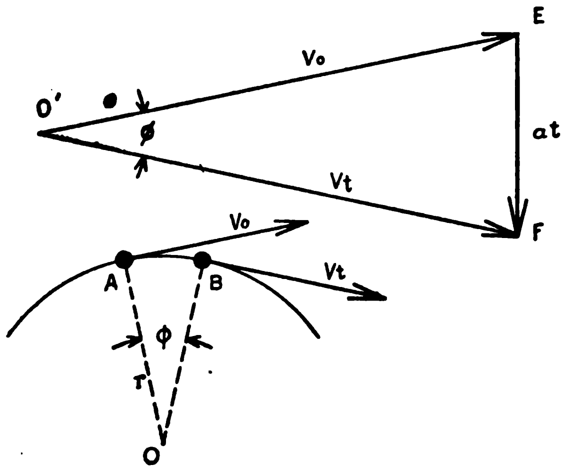

FIG. 24. — Illustrating Effect of Central Force in Uniform Circular Motion.

$$\frac{AB}{r} = \frac{EF}{v_0}$$

But $$AB = v_0 t \qquad \text{(See Equation 1.)}$$

and $$EF = at$$

$$\therefore \frac{v_0 t}{r} = \frac{at}{v_0}$$

or $$a = \frac{v_0^2}{r}$$

or, writing v in place of v_0, since **numerically** the velocity is the same at all points in the circular path, we have

$$a = \frac{v^2}{r} \tag{14}$$

It is convenient to express the value of a in terms of the **revolutions per second**, *i.e.* the number of times which the body passes around the circle in one second. If the body makes one revolution per second, then (Section 26),

$$v = 2\pi r$$

If the body makes n revolutions per second,

$$v = 2\pi rn$$

Placing this value of v in Equation (14), we have

$$a = 4\pi^2 n^2 r \tag{15}$$

CENTRAL FORCE REQUIRED TO MAINTAIN UNIFORM CIRCULAR MOTION

45. From Equation (10), Section 33, we see that the force acting on any body having accelerated motion is given by the product of its mass and its acceleration. Having therefore determined the acceleration a body experiences when in uniform circular motion, namely,

$$\text{acceleration} = \frac{v^2}{r}$$

we may write the expression for the force which must act upon such body to keep it moving uniformly in a circle.

Thus, $$F_c = M \cdot \frac{v^2}{r}$$

or (Equation 15), $$F_c = 4\pi^2 n^2 r \cdot M \tag{16}$$

In order, therefore, that a body may move uniformly in a circle, it is necessary to supply a force action at all times directed toward the center of the circle having a value which varies directly as the mass of the moving body, as the square of its velocity, and inversely as the radius of the circle in which the

body is moving. If the force, the value of which is determined by Equation (16), is not supplied, the body will not move in the circle of radius r and at a velocity v as contemplated in the expression.

THE "BURSTING" FLYWHEEL

46. Consider the outermost portions of a flywheel. Let it be assumed that the flywheel is turning at a given instant at a speed such that the velocity of these outer portions is v. Let the radius of the flywheel be r. If now the flywheel is speeded up so that n is made twice as great, then from Equation (16) it is evident that F_c, the central force required to maintain any given particle in its circular path, will be four times as great. If n is made three times as great, F_c will be nine times as great. The central forces acting upon these outer portions are in this case supplied by the cohesion of the various parts of the flywheel. Now if v is made larger and larger, there will come a time when these cohesive force actions will no longer be adequate to supply the requisite central forces. The result will be that these outer portions will no longer move in the curved paths in which they were traveling before the speed became excessive, but leaving these paths they will tend, according to the first law of motion, to move off along tangents to the circumference of the wheel drawn to those points in which the portions were traveling when the cohesive forces failed. These portions of the flywheel travel off in straight lines with high velocities, and may be very destructive in their effects upon their surroundings. A flywheel going to pieces in this manner is said to burst. **It is evident that the flywheel does not burst in the sense that a bomb bursts; it goes to pieces, not because of any outward acting force, but because of a failure of the cohesion in the iron to supply the necessary inward acting central force.** The term "centrifugal force" is oftentimes made use of to indicate the tendency of the outer portions of revolving bodies to move outward (tangentially). The term is really misleading in that it implies the existence of an outward acting force.

THE CREAM SEPARATOR AND CENTRIFUGAL DRIER

47. Consider a chain of particles *a*, *b*, *c*, *d*, etc., Figure 25. Let it be assumed that they all have uniform circular motion about the center. Let it be assumed that the particles are of the same volume, but of unequal mass. *a*, *c*, *e* may be thought of as having greater mass than *b*, *d*, *f*. Let it be further assumed that the central forces necessary to maintain *b*, *d*, *f* in uniform circular motion are supplied, but that the central forces acting upon *a*, *c*, and *e* are not sufficient to maintain these particles of larger mass in the uniform circular motion assumed at the outset. It is evident, under these assumptions, particles *a*, *c*, and *e* will move away from the dotted circle shown in the diagram while *b*, *d*, and *f* will continue in the positions shown. That is to say, *a*, *c*, and *e* will be separated from *b*, *d*, and *f*.

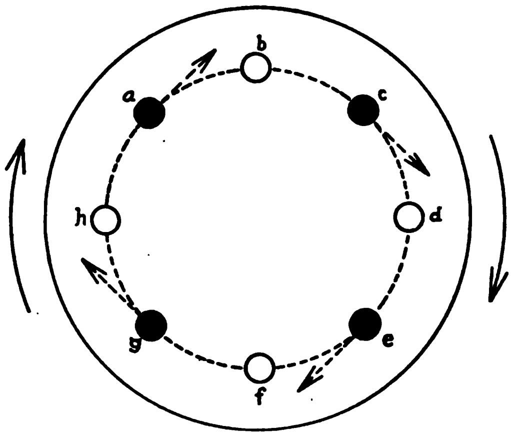

FIG. 25. — The Action in a Cream Separator.

We may imagine that *a*, *c*, and *e* are milk particles, while *b*, *d*, and *f* are cream particles, all situated side by side in a cream separator. It will be evident that, as the vessel in which these particles are placed is set in rapid rotation, a tendency for the separation of the milk particles from the cream particles will exist, the milk particles tending to pass to the circumference of the rotating vessel, the cream particles remaining nearer the center.

The **centrifugal drier** affords another illustration of the application of this principle. The wet clothing which is to be dried is placed in a cylindrical iron vessel with perforated sides, which is caused to rotate at a high velocity. The water becomes separated from the clothing from the fact that its cohesion for the

material of the clothing is not sufficient to keep the water particles moving uniformly in the circle in which the clothing is constrained to move.

ELEVATION OF THE OUTER RAIL IN RAILWAY CURVES

48. When a car passes around a curve, it must be supplied with a force equal to $\frac{mv^2}{r}$, directed toward the center of the curve, in order that it may not tend to leave the track. This central force must of course be supplied by the track. It is

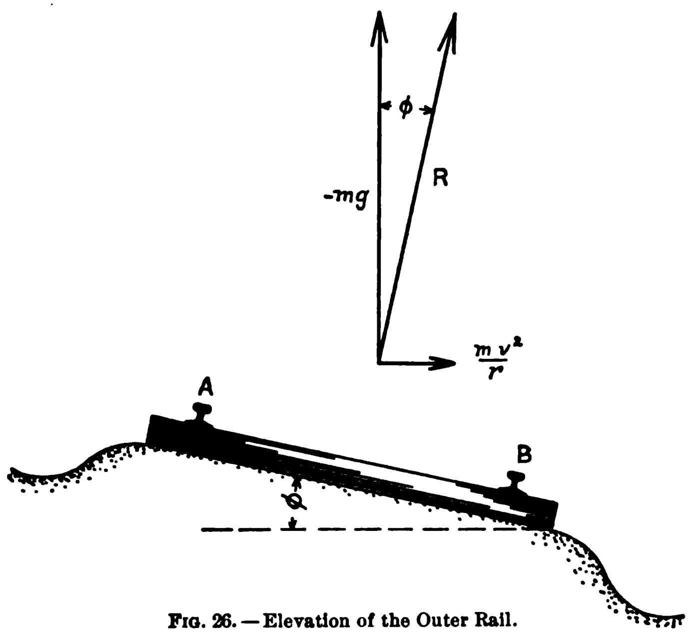

FIG. 26.—Elevation of the Outer Rail.

found that by elevating the outer rail the reaction of the track may be caused to furnish the necessary central force and at the same time the force actions on the two rails may be made equal. Let A and B, Figure 26, represent the rails on a railroad curve. Call the angle which AB makes with the horizontal ϕ. Let it be assumed that A is elevated to such an extent that the force action between the car and each of the rails is the same. Therefore R, the reaction of the track upon the car, is perpendicular

to AB. Evidently the vertical component of R balances the weight of the car and is, therefore, equal to Mg (Equation 11). The horizontal component of R is the required central force equal to $\frac{mv^2}{r}$. Since the angle between R and the vertical is the same as that between AB and the horizontal, we have at once,

$$\tan\phi = \frac{\frac{mv^2}{r}}{mg} = \frac{v^2}{rg}$$

which determines the value which ϕ must have in order that the car may press equally upon the two rails and have no tendency to leave the curve tangentially.

SIMPLE HARMONIC MOTION

49. Many phenomena of physics are characterized by a certain kind of vibration or oscillation known as simple harmonic motion. In linear simple harmonic motion the vibrating body moves to and fro in a straight line. In simple harmonic motion of rotation the vibrating body moves to and fro in a circular arc. In the following paragraphs inquiry is made as to the nature of the force which acts to maintain a body in simple harmonic motion.

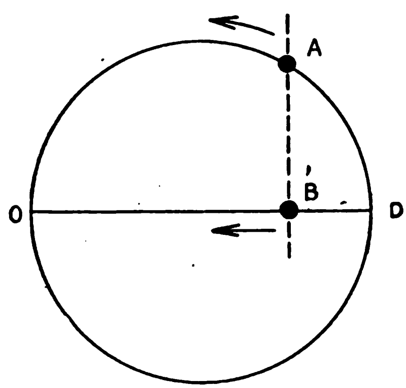

FIG. 27. — Simple Harmonic Motion and Uniform Circular Motion Compared.

For the purposes of the following discussion **simple harmonic motion is defined as a component of uniform circular motion parallel to a diameter of the circle.** Referring to Figure 27, let the circle represent the path of a body A which has uniform circular motion. Let the body B move in the horizontal diameter of the circle OD in such manner that B is always in the vertical line which passes through A; that is to say, let it be always directly beneath A while A is in the upper half of

the circle, and always directly above A when A is in the lower half of the circle. Under these conditions B is said to have simple harmonic motion. To determine the acceleration the body B must have in order that it may be at all times in the vertical line through A, we proceed in the following manner: Draw a line AE, Figure 28, to represent in magnitude and direction the acceleration of the body A. Let this acceleration be resolved into its vertical and horizontal components V and H. Then EF is the horizontal component of the acceleration of the body A at the moment it arrives at the point A. It will be evident that the acceleration of B must equal the horizontal component of the acceleration of A, since it is plain that **if B is to remain at all times directly below or above A, it must possess at every instant the same horizontal velocity that B has, and at each and every instant the same horizontal acceleration that B has.** Thus the acceleration of the body B at the instant under consideration is EF. To find the value of EF, produce the line AE to O, the center of the circle. We have then at once from similar triangles:

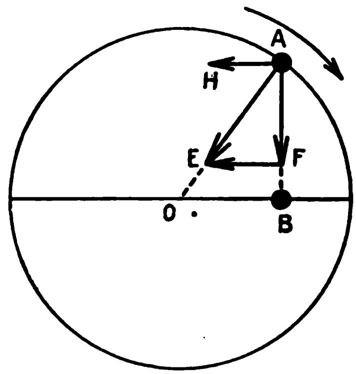

FIG. 28.—The Acceleration of a Body in Simple Harmonic Motion is at all Times toward the Position of Rest.

$$\frac{EF}{AE}=\frac{OB}{AO}$$

or,
$$EF=\frac{OB}{AO}\cdot AE$$
$$=\frac{x}{r}\cdot AE$$

in which x is written for OB, which is called the displacement, that is to say, the distance of the body B from its mean position O. r is the radius of the circle. But AE is the acceleration of the body A having uniform circular motion. We

have therefore $$AE = 4\pi^2n^2 \cdot r$$

and therefore, $$EF = 4\pi^2n^2r \cdot \frac{x}{r}$$
$$= 4\pi^2n^2x$$

Knowing the value of the acceleration, we may write the expression for the force which must be acting upon the body B at the given instant, since

$$F = M \cdot a$$

Therefore, $$F_h = 4\pi^2n^2M \cdot x$$

It will be noticed that the horizontal component EF of the acceleration of the body A is directed toward the left when the body A is on the right-hand side of the circle, and is directed toward the right when the body A is on the left-hand side of the circle. Hence **the acceleration of B is always toward the left when B is on the right of its mean position and always toward the right when B is on the left of its mean position.** It is customary to indicate this fact by introducing a negative sign before the right-hand member of the expression for F_h. We have therefore

$$F_h = -4\pi^2n^2M \cdot x \tag{17}$$

Comparing this expression for F_h, the force which acts upon a body when it is in simple harmonic motion, with that for F_c given in Equation (16), which is the force which acts upon a body in uniform circular motion, it will be seen that they are identical in form, the difference being that in place of the radius of the circle which enters the expression for F_c we have the displacement of the particle x in the expression for F_h. Further, the expression F_h is given the negative sign for the reason above indicated. Thus, while F_c is constant in value, it is evident that F_h is all the time changing and has a different value for each position of the body in its path, that is to say, for each displacement. Equation (17) is sometimes written as follows:

$$F_h = -K' \cdot x$$

in which K' stands for the group of constants $4\pi^2n^2M$. The conditions which must be met in order that a body may have simple harmonic motion may be stated in ordinary language as

follows: **First, the body must at each moment be acted upon by a force proportional to its displacement from its mean position. Second, the direction of the force must be such as to tend to decrease the displacement.**

The converse of the proposition may be stated as follows: Any body which is so conditioned that when it is displaced from its position of equilibrium it is acted upon by a force tending to draw it back into that position, and having a value proportional to the displacement, will execute simple harmonic motion, if displaced and then left free to move.

EXAMPLES OF SIMPLE HARMONIC MOTION

50. Consider a mass *M* suspended by a spiral spring, Figure 29. The weight of the body stretches the spring until the upward pull due to the stretched spring is equal to the downward pull due to the weight. Under these conditions the body *M* will remain at rest. If the body is drawn down to some lower position, the stretch in the spring is increased and the upward pull of the spring becomes greater than the downward pull, *i.e.* the weight of the mass *M*. Furthermore, the farther *M* is drawn down the greater is the unbalanced part of the upward force. On the other hand, if the mass *M* is lifted above its position of equilibrium, it is evident that the upward force due to the spring will be less than the weight of the mass *M*. There will thus be an unbalanced part of the weight tending to pull *M* back to the position of rest. Evidently as *M* is lifted to a higher position, the unbalanced part of the weight becomes greater. In general, we may say that if *M* is displaced in either direction from the position of rest, the unbalanced force tending to draw it back into that position is proportional to the displacement: the conditions for simple harmonic motion are therefore satisfied; and

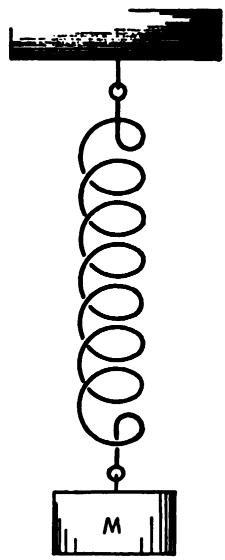

FIG. 29. — Illustrating Simple Harmonic Motion.

if the body M is displaced in an upward or downward direction and left free to move, it will execute simple harmonic motion.

The following may be cited as further examples of simple harmonic motion: the prongs of a tuning fork when displaced vibrate to and fro through their mean positions in simple harmonic motion; a particle of air which is transmitting a simple sound wave vibrates to and fro in simple harmonic motion; etc.

THE SIMPLE PENDULUM

51. As an example of the application of the law of harmonic motion we may consider the simple pendulum and the determination of what is known as the law of its vibration.

The simple pendulum is defined as a body of large but concentrated mass suspended by a very light thread.

In Figure 30 let OB represent such a pendulum. In order to determine the law of its vibration we may proceed as follows: When the pendulum is at the position indicated in the diagram, it is acted upon by a force tending to draw it back into the vertical position. This force is evidently a component of its weight. Let the line W represent in magnitude and direction the weight of the pendulum bob B. Let this weight be resolved into components, the one f, in the direction in which the bob is moving at the instant under consideration, the other T in the direction OB, that is to say, in the direction of the length of the pendulum. Call the length of the pendulum

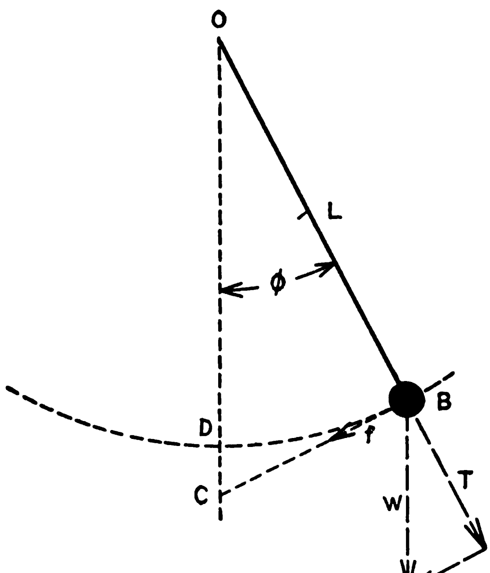

FIG. 30.—The Simple Pendulum.

L, and call the distance BD, of the bob from its mean position, x. It is evident that when the angle ϕ is small, that is to say, when x is small, the points C and D will coincide. We may therefore write from similar triangles:

$$\frac{f}{W}=\frac{x}{L}$$

or,
$$f=\frac{x}{L}\cdot W$$

or,
$$f=\frac{W}{L}\cdot x \tag{18}$$

From this last expression it is evident that f, the force which tends to draw the bob back to its position of equilibrium, is proportional to x, since it is equal to a constant times the displacement. Furthermore, it is evident that when the pendulum bob is on the right, f is always directed toward the left; and when the pendulum bob is on the left, f is directed toward the right. It will therefore be evident that the conditions for simple harmonic motion are in this case completely satisfied. Therefore, Equation (17) is applicable to the case of the simple pendulum, and F_h and f being one and the same thing for the simple pendulum, we have,

$$f=F_h=\frac{W}{L}\cdot x=4\,\pi^2n^2M\cdot x$$

i.e.
$$\frac{W}{L}=4\,\pi^2n^2M \quad \text{(Compare Equations 17 and 18.)}$$

or,
$$\frac{Mg}{L}=4\,\pi^2n^2M \quad \text{(Since } W=Mg\text{, Equation 11.)}$$

Solving for n, we have $n=\dfrac{1}{2\,\pi}\sqrt{\dfrac{g}{L}}$

This is known as the law of the simple pendulum. n is the number of complete vibrations which the pendulum will make in unit time, g is the acceleration of gravity, and L the length of the pendulum. It is evident from this expression that the number of vibrations is independent of the mass of the bob and of the displacement x (it should not be forgotten that we have made the assumption that **the displacement x is in all cases small**).

It is convenient to have the law of the simple pendulum in such form that it gives the time required for one complete vibration of the pendulum instead of the number of vibrations which the pendulum makes in unit time. This may be obtained by inverting the expression for n given above. This is evident from the following considerations. If a body makes n vibrations per second, it requires $\frac{1}{n}$ of a second to make one vibration. The time T of one complete vibration is called the period. We have therefore

$$T = 2\pi\sqrt{\frac{L}{g}} \tag{19}$$

SIMPLE HARMONIC MOTION OF ROTATION

52. Let AB, Figure 31, represent an elastic wire clamped at the upper end and fastened at the lower end to the body B. If the body B is turned about the wire as an axis, the wire will be twisted, and the elastic forces in the wire will tend to restore B to its original position. We know also from common experience that the greater the twist given to the lower end of the wire, the greater is the resistance to the twisting; or what amounts to the same thing, the greater is the torque in the wire which tends to bring the body B back to its original position. It is evident, therefore, that the body B is conditioned for simple harmonic motion as required by Equation (17), except that the different parts of the body B have been displaced along circular arcs instead of in straight lines. If B is turned through a small angle and then released, it will vibrate to and fro about AB as an axis, in **simple harmonic motion of rotation.** Evidently, the equation for simple harmonic motion of rotation is an expression involving torque T and moment of inertia I, instead of the force F and mass M, Equation (17). The torque acting on a body in simple harmonic motion of rotation when displaced through an angle ϕ is

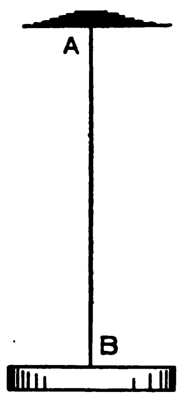

FIG. 31.—The Torsion Pendulum.

$$T = -4\pi^2 n^2 I \cdot \phi \tag{17 a}$$

THE ADDITION OF ANGULAR VELOCITIES

53. Angular velocity is a vector quantity. This is evident when it is considered that to describe the angular velocity of a body, the direction of the axis about which it is rotating must be given as well as the rate at which it turns through angular space. It follows, therefore, that **the angular velocity of a rotating body may be represented by a straight line, and that two or more such velocities may be combined by the usual method of dealing with vectors.**

To represent an angular velocity by a straight line, proceed as follows: Draw a line parallel to the axis about which the body is rotating. Make its length such as to represent to scale the angular velocity in radians per second or revolutions per second, as desired, and place an arrowhead pointing in the direction in which one must look along the axis in order that the rotation may be clockwise.

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THE GYROSCOPE

54. The gyroscope affords an interesting illustration of the addition of angular velocities. **This instrument consists of a wheel of large moment of inertia so mounted as to be free to turn about three axes, each perpendicular to the plane of the other two.** Figure 32 represents a form of gyroscope. *AB* is the primary axis. *E* is a pivot which supports one end of the axle *AB*.

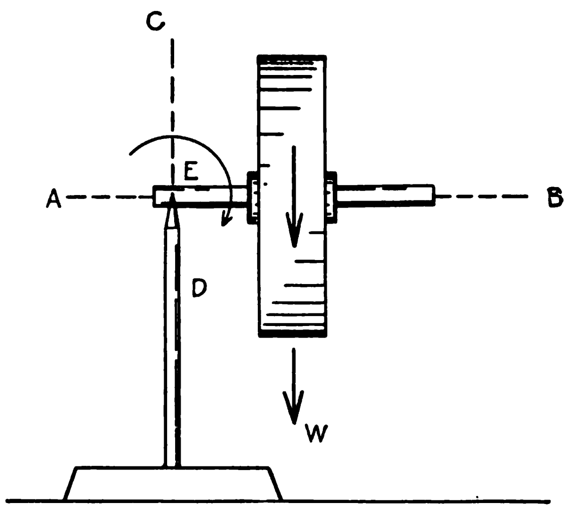

Fig. 32. — A Simple Gyroscope.

The hub of the wheel is supposed to be fitted with ball bearings so that the axle *AB* does not rotate. The wheel is free to rotate about an axis through the pivot *E*, perpendicular to the plane of the figure. Call this axis *EF*. Evidently the wheel can also rotate about the axis *CD*. Imagine the wheel to be revolving rapidly about the primary axis, the rotation being **clockwise as seen from *A*.** If now it is supported as shown in the figure, and left to itself, it will tend to fall as shown by the curved arrow at *E*; that is, it will **tend to rotate about the axis *EF*** because of its weight. **Under these circumstances the wheel will revolve about the third axis *CD*.** This resultant rotation about the third axis is called **precession.**

The vector analysis of the wheel's motion is as follows: Imagine the apparatus to be viewed from above. Let *AB*, Figure 33, represent the angular velocity about the primary axis and *EF* the angular velocity about the axis *EF*. Combining these by the vector polygon method, we obtain *R* as the resultant, which means that the effect of introducing the velocity *EF* is to shift the primary axis from *AB* to *R*. **The analysis shows that if the wheel is rotating about its primary axis, a torque about the second axis will produce rotation about the third axis.**

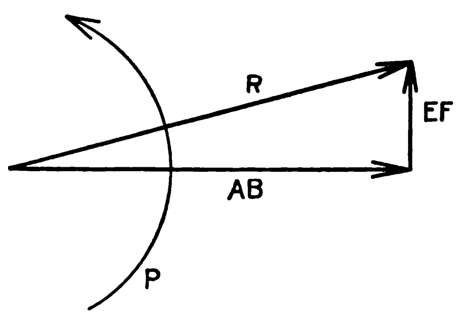

Fig. 33. — Illustrating Precession.

It is convenient to call the primary axis the **axis of spin,** the second axis the **axis of torque,** and the third axis the **axis of precession.** A convenient rule for determining the precession in a given case is the following: Holding the thumb and first and second fingers of the **right hand** perpendicular to each **other,** point with the **forefinger along the axis of spin** and with the **second finger along the axis of torque; the thumb will then extend along the axis of precession.** The clockwise aspect of each rotation must be taken in applying this rule.

VELOCITY OF PRECESSION

55. The relation between torque, velocity of spin, and velocity of precession is a simple one, and may be derived as follows:

Let OA, Figure 34, drawn to scale represent ω, the velocity of spin. Call the velocity of precession Ω. This velocity is indicated by the curved arrow. Now if OA represents to scale the velocity of spin at a given instant and OB the velocity of spin a certain number of seconds later, then AB represents to the same scale αt, **the change in the velocity of spin** during that interval. (Compare Figure 24.) But the angular velocity of OA (the angle AOB swept by OA in one second) is Ω. Therefore (arc) AB described in unit time is $\omega\Omega$. Also when AOB is small, $AB =$ arc AB, and for unit time AB represents the angular acceleration, α, of the moving body about the axis of precession through the point O.

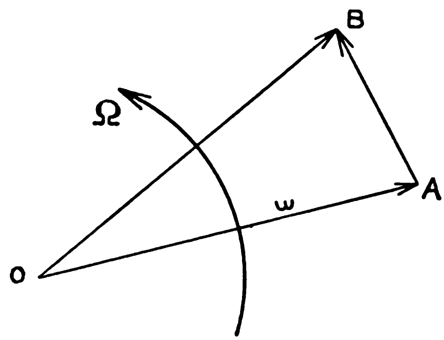

FIG. 34.—Showing the Relation between Ω and ω.

We have therefore $$\alpha = \omega\Omega \tag{20}$$

But the torque acting upon a body is equal to the product of the moment of inertia of the body and the angular acceleration produced by the torque $$T = I\alpha$$

hence, $$T = I\omega\Omega \tag{21}$$

AN EXPLANATION OF PRECESSION

56. The following explanation, based upon Newton's laws of motion, will help one to a better understanding of precession. Let the large circle, Figure 35, represent a rapidly rotating disk. The axis of the disk (the axis of spin) is perpendicular to the paper, and the rotation of the disk is clockwise. Let it be imagined that a torque is acting upon this rotating disk tending

to tip the upper edge of the disk towards the reader, and the lower edge away from the reader. Consider any small particle of the rotating disk, for example, the particle *B* in the upper left-hand quadrant. The linear velocity of this particle at a given instant is represented in magnitude and direction by the arrow.

Now the spin of the disk is carrying *B* farther from the torque axis. Hence, on the assumption that the angular acceleration

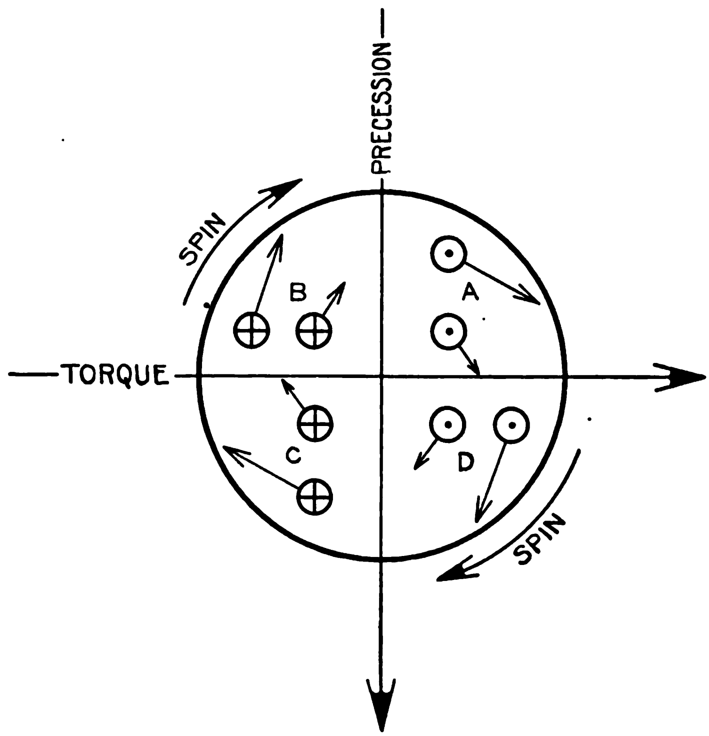

Fig. 35. — Illustrating the Reactions of a Precessing Body.

about the torque axis is constant, **the force (torque) which must act upon *B* to maintain its acceleration about the torque axis becomes greater and greater as *B* moves away from that axis.** This discussion applies to any particle, and therefore to all particles, in the *B* quadrant. But action and reaction are equal and oppositely directed. Therefore it is evident that **the *B* quadrant, as a whole, reacts in opposition to the torque, that is, away from**

the reader. This may be indicated by placing a cross in the small circle representing the particle B.

Consider next the particle C. **This particle by the spin of the disk is brought closer to the torque axis.** Its linear speed in the direction of the torque is decreasing. Its acceleration is, therefore, negative, and its reaction is in the direction of its motion about the torque axis, that is, **in the direction of the torque.** This discussion applies to all particles in the C quadrant. Hence the C quadrant, as a whole, reacts in the direction of the torque, that is, **away from the reader.** This is indicated as before by placing a cross in the small circle representing the C particle.

The analysis shows that all particles in the B and C quadrants react away from the reader. By similar reasoning it may be shown that **all particles in the A and D quadrants react towards the reader.**

The reactions of quadrants B and C away from the reader, together with those of A and D towards the reader, constitute a torque about a vertical axis. If the disk has freedom of motion in this direction, it will therefore revolve about the vertical axis in a clockwise direction, as seen from above. This is precession.

EXAMPLES OF GYROSCOPIC ACTION

57. A bicycle rider can turn his machine by tilting it. If, for example, he wishes to turn to the right, he leans to the right. In Figure 36 let AB represent the front wheel of a bicycle seen from above and

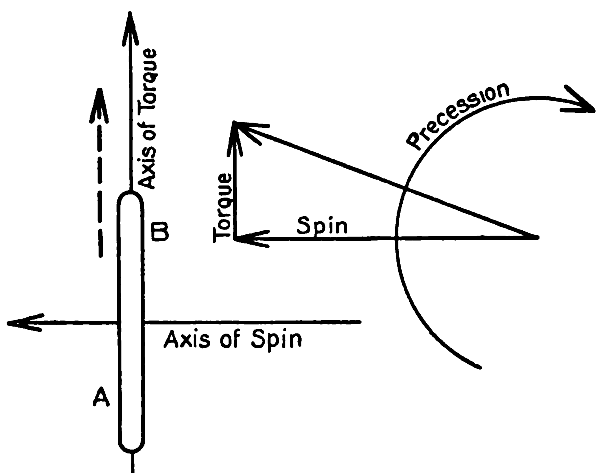

FIG. 36. — Precession in a Bicycle Wheel.

moving in the direction of the dotted arrow. The angular velocity of the wheel is clockwise as seen from the right. When the rider leans to the right he introduces a torque in the direction shown. The vector triangle shows how the spin and torque combine to produce clockwise precession.

When a rapidly moving automobile turns a corner, the engine flywheel tends to precess in a direction depending upon the direction of its axis and its rotation. Let F, Figure 37, represent the flywheel of an automobile. A and B are the wheels. The flywheel rotates clockwise as seen from the front. Assume that the car is moving in the direction of the dotted arrow and turns to the right. The torque is then given by the vertical arrow. The flywheel **tends** to precess about a horizontal axis, as shown in the vector diagram. Of course, precession does not in this case actually take place, being prevented by forces within the frame of the machine. The effect in this case is a tendency to

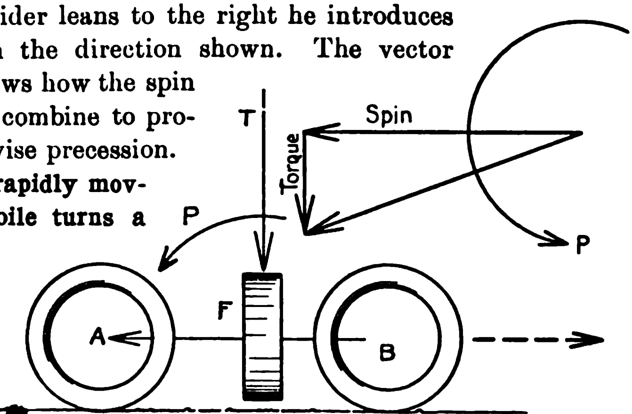

FIG. 37. — Precessional Tendency in the Flywheel of an Automobile.

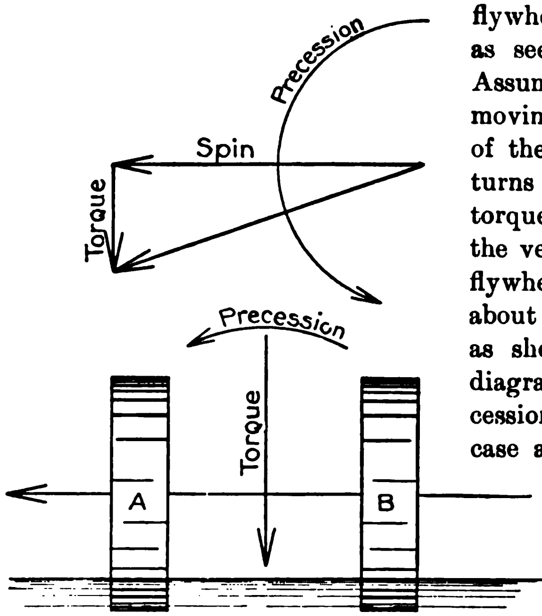

FIG. 38. — Precession in a Side-wheel Steamboat.

lift the front wheels of the automobile and increase the pressure upon the rear wheels.

When a railway engine or a side-wheel steamer moves in a curve, the precession of its wheels acts with the centrifugal effect, tending to overturn the engine or boat. In Figure 38 let AB represent the wheels of a side-wheel steamboat, moving away from the observer. Let it be assumed that the boat is turning to the right. Then the torque axis is vertical, as shown. Under these circumstances the wheels will precess in a counter-clockwise direction, looking forward.

LINEAR AND ANGULAR MOTION

58. The **linear velocity** of a body which moves over a distance d in t seconds is

$$v = \frac{d}{t} \quad \text{(Section 26, Equation 1)}$$

Similarly the **angular velocity** of a body which moves over an angular distance ϕ in t seconds is

$$\omega = \frac{\phi}{t}$$

The **linear acceleration** of a body is given by its change in linear velocity divided by the time in which that change takes place. That is,

$$a = \frac{v}{t} \quad \text{(Section 24)}$$

Similarly the **angular acceleration** of a body is

$$\alpha = \frac{\omega}{t}$$

in which ω is the change in angular velocity occurring in the time t.

In general, for each equation in **linear motion** there is a corresponding equation in **angular motion**. In each case the equation for angular motion is **similar** to that for linear motion, **angular distance, velocity, and acceleration** being substituted for **linear distance, velocity, and acceleration, torque** for **force**, and **moment of inertia** for **mass**.

Below is given a table of these relations. The equations used are those derived above, together with that for **momentum**, which it is convenient to include.

The **linear momentum** of a body is the product of its mass and linear velocity. The **angular momentum** of a body is the product of its **moment of inertia** and **angular velocity**.

LINEAR		ANGULAR
	Fundamental Quantities	
Length, mass, and time		Angle, moment of inertia, and time
d, m, t		ϕ, I, t
	$I = r^2 \cdot m$	
	Velocity	
$v = \frac{d}{t}$		$\omega = \frac{\phi}{t}$
	$v = r\omega$	
	Acceleration	
$a = \frac{v}{t}$		$\alpha = \frac{\omega}{t}$
	$a = r \cdot \alpha$	
	Force and Torque	
$F = ma$		$T = I\alpha$
	$T = rF$	
	Momentum	
mv		$I\omega$
	$I\omega = r \cdot mv$	
	Central Force and Precessional Torque	
$F = mr\omega$		$T = I\Omega\omega$
	Uniformly Accelerated Motion	
$v_t = v_0 + at$ $\bar{v} = v_0 + \frac{1}{2} at$ $d = v_0 t + \frac{1}{2} at^2$		$\omega_t = \omega_0 + \alpha t$ $\bar{\omega} = \omega_0 + \frac{1}{2} \alpha t$ $\phi = \omega_0 t + \frac{1}{2} \alpha t^2$
	Simple Harmonic Motion	
$F = -4\pi^2 n^2 M \cdot x$		$T = -4\pi^2 n^2 I \cdot \phi$

Problems

1. A stone having a mass of 1 lb. is whirled, at the end of a string 3 ft. long, in a circular path at a velocity of 30 ft./sec. What is the tension in the string in pounds weight?

2. The flywheel of an engine is 5 ft. in diameter and makes 300 revolutions per minute. What is the radial acceleration of those portions of the wheel which lie farthest from the axis? What radial force in pounds weight is acting upon each pound mass of the outermost portions of the wheel?

3. A mass of 50 g. is whirled at the end of a string 50 cm. long. Find the tension in the string when the mass makes 2 revolutions per second.

4. How will the tension in the string of problem 3 change if the velocity of the body is doubled? if the length of the string is halved?

5. A certain railway curve has a radius of 400 ft. If trains are to pass this curve in safety at 25 mi./hr., what should be the angular elevation of the outer rail?

6. A body is moving uniformly in a circle of 5 ft. radius. At what velocity will the radial accelerating force just equal the weight of the body?

7. A body having a mass of 5 lb. is vibrating in simple harmonic motion. It makes one vibration per second. What is the force acting upon it when it is 3 in. from the center of its path?

8. What must be the length of a simple pendulum in order that it will make one swing per second at a place where $g = 981$ cm./sec.^{2}?

9. A small sphere of lead is suspended by a thread 25 ft. long. What is its time of vibration? Assume $g = 981$ cm./sec.^{2}.

10. Two simple pendulums are so adjusted that their periods are as 2 to 3. What is the ratio of their lengths?

WORK AND ENERGY. FRICTION

CHAPTER VI

WORK

59. In the scientific sense work is defined as the product of force by distance. A force is said to do work when it moves in its own direction or when the body to which the force is applied moves in the direction of the force. This conception of work is somewhat different from that commonly accepted. As the term is usually applied, a man, for example, is said to be doing work, or working, when he does that which causes fatigue. In the sense in which we are now to use the term, motion is an essential part of work; thus, the pillars which support a roof are exerting large force actions upon the roof, but they are doing no work since there is no motion. In the same way a man tugging at a heavy stone which he is unable to move is doing no work because the force applied does not move the body upon which it is acting. The measure of the **work done by a force is the product of the force and the distance through which it moves in the direction of the force**, that is,

$$W = F \cdot d \tag{22}$$

UNITS OF WORK

60. The **erg** is the unit of work in the c. g. s. system. It is defined as the work done by a force of one dyne in moving through a distance of one centimeter. The erg is too small to serve as a practical unit. It has therefore been found convenient to adopt, as a practical unit of work, the **joule**, which is ten million times as large, that is,

$$1 \text{ joule} = 10{,}000{,}000 = 10^7 \text{ ergs}$$

The **foot-poundal** is the unit of work in the f. p. s. system, and is defined as the work done by a force of one poundal in moving through a distance of one foot.

Gravitational Units of Work. The kilogram-meter and the foot-pound are sometimes used as units of work. A **kilogram-meter** is the work done by a force of one **kilogram weight** in moving a distance of one meter. A **foot-pound** is the work done by a force of **one pound weight** in moving a distance of one foot.

The relation between the units of work in the two systems is given by the following relation:

$$\begin{aligned} 1 \text{ foot-pound} &\doteq .138 \text{ kilogram-meter} \\ &\doteq 1.356 \text{ joules } (g = 980.6) \end{aligned}$$

THE WORK DONE BY A FORCE

61. The distance d of Equation (22) is the distance through which the force moves **in its own direction.** If a body upon which a force is acting moves in a direction which is not parallel to the force, then in determining the work done we must resolve the actual motion of the body into its components parallel and perpendicular to the force. The work done is then given by the product of the force and that component of the motion which is parallel to the force. Consider, for example, the case represented in Figure 39.

A rope attached to a sled S makes an angle ϕ with the surface over which the sled is moving. A force F acting in the direction of the rope causes the sled to move forward so that

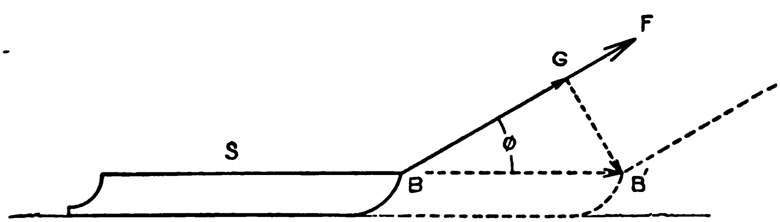

FIG. 39. — Work done $= F \times BB' \cos \phi$.

the point B comes to the position B'. Let it be required to find the work done by the force F. Applying the principle given above, the distance BB' through which the sled has moved must be resolved into components parallel and perpen-

dicular to F, namely, BG and GB'. We have, therefore, work done by the force F is

$$W = F \cdot BG$$

It is evident that this gives the value of the work done by the force F as the sled moves from B to B' from the following consideration: The sled might, for example, be supposed to be free to move in any direction and to pass from B to B' by first traveling over the distance BG and then over the distance GB'. As it moves from B to G it is moving parallel to the force F and the force F is doing work. As it moves from G to B' it is moving at right angles to the force F. The force F is therefore not moving in its own direction at all and hence is doing no work. Calling the angle $B'BG$, ϕ, it is evident that

$$BG = BB' \cos \phi$$

$$\therefore\ W = F \cdot BB' \cos \phi$$

This expression for the work done may be reached in a different way. Instead of resolving the **motion** into two components, we may resolve the **force** F into two components, the one of which is parallel to the motion, and is therefore effective in doing work as the sled moves the other perpendicular to the motion, and therefore doing no work. Thus in Figure 40, f is that component of F which is parallel to the motion, and

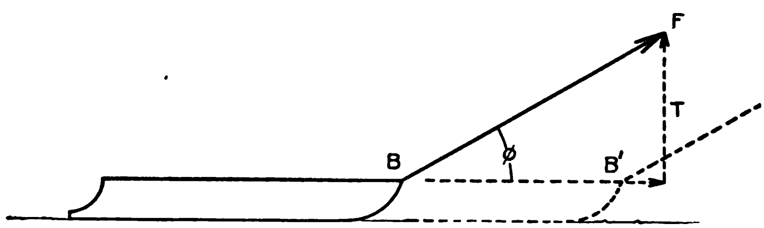

FIG. 40. — Work done = $BB' \times F \cos \phi$.

which might therefore be called the working component, while T is the component perpendicular to the motion. We have therefore for the work done as the sled moves from B to B',

$$W = f \cdot BB'$$

but evidently

$$f = F \cdot \cos \phi$$

Therefore, $W = F \cdot BB' \cos \phi$

The general rule for finding the work done by a force is as follows: Multiply the total distance through which the body moves by that component of the force which is parallel to the motion, or multiply the total value of the force by that component of the motion which is parallel to the force.

When a body is moved in opposition to a force which is acting upon it, work is said to be done against the force. Thus in lifting a weight work is done against the force of gravity.

THE WORK DONE BY A TORQUE

62. The work done by a torque T in turning a body through an angle ϕ is given by the product of the torque and the angle. Thus,

$$W = T\phi \qquad (23)$$

This relation may be demonstrated as follows. Referring to Figure 41, let it be assumed that a force F acts upon a body OB which is pivoted at O, for example, a wrench applied to the nut of a bolt. Call the perpendicular distance of the point O from F, r. Then the torque action of the force F upon the body OB is

$$T = Fr$$

Let it be assumed that under the action of the force F (which is supposed to remain at all times perpendicular to r) the point B moves to B' so that the line OB has turned about O through the angle ϕ. The work done, according to Section 59, is

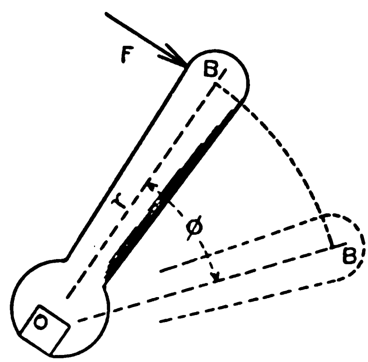

FIG. 41.—The Work done in tightening a Nut.

$$W = F \cdot BB'$$

But, $\dfrac{BB'}{r} = \phi$

Or, $BB' = r\phi$

Therefore, $W = Fr\phi$

i.e. $W = T\phi$

ENERGY

63. **A body is said to possess energy when by reason of its position or condition it is capable of doing work,** and the energy of a body is defined as its capacity for doing work. The unit of energy is evidently the same as the unit of work.

Energy is of two forms, potential energy and kinetic energy. **The potential energy of a body is the energy which the body possesses in virtue of its position or configuration.** For example, a lifted weight has potential energy in virtue of the higher position into which it has been lifted, since it can do work as it sinks back, under the action of gravity, to the position from which it was lifted. A clock spring when wound possesses potential energy (of configuration), since in unwinding it is capable of doing work.

The kinetic energy of a body is the energy which it possesses in virtue of its motion. Thus a moving rifle ball possesses kinetic energy since it can do work in stopping. The rotating flywheel of an engine possesses kinetic energy since it can do work in coming to rest.

THE TRANSFORMATION OF ENERGY

64. Energy may be transformed from potential to kinetic or *vice versa:* For example, a lifted weight, which possesses potential energy in its highest position, if allowed to fall, will gradually lose its potential energy. At the same time it will acquire velocity, and therefore kinetic energy. A ball thrown vertically into the air possesses at the beginning of its flight kinetic energy, but as it rises it gradually comes to rest, thus losing its energy in that form. However, as it rises it is acquiring potential energy. In other words, as it rises its kinetic energy is transformed into potential energy.

THE CONSERVATION OF ENERGY

65. It is found in every case of energy transformation that the energy possessed by the body or system of bodies under consideration before and after the transformation takes place is the same. Thus in the first example given in the last section

the potential energy of the weight in its highest position is the same as its kinetic energy after it has fallen to its lowest position. In the second example of the last section the potential energy of the ball when it finally comes to rest in its upward flight is (neglecting air friction) equal to the kinetic energy with which it began its flight. We may say in general that the energy possessed by an isolated body or system of bodies is constant. This fact is sometimes expressed by saying that **energy can neither be created nor destroyed** and is known as the principle of the conservation of energy.

In a device for transforming mechanical energy a certain part of the energy disappears as mechanical energy and reappears in some other form, for example, heat, noise, etc.; but it is found to be an invariable rule that the total energy delivered by the transforming device, after the transformation occurs, in whatever form it may exist, is equal to the original energy supplied to the transforming device.

THE POTENTIAL ENERGY OF A LIFTED WEIGHT

66. From the foregoing statements it is evident that the energy of a body may be measured in either of two ways: **First, by the work which it is capable of doing; second, by the work which has been done upon it to bring it into the condition in which it possesses energy.**

The potential energy of a lifted body is given by the following equation,

$$E = Mgh \tag{24}$$

where E stands for the work done in lifting the body through a vertical height h against the force of gravity Mg. This expression is obtained by multiplying the weight of the body Mg by the height h through which it has been lifted.

THE KINETIC ENERGY OF A BODY IN LINEAR MOTION

67. The kinetic energy of a moving body may be expressed in terms of its mass M, and its velocity v, as follows: Let it be assumed that an unbalanced force F is applied to a body of mass M at rest. Under the influence of the force F the body

will start into uniformly accelerated motion. The acceleration is given by

$$F = Ma \qquad (10 \text{ bis})$$

The velocity acquired by the body in t seconds is

$$v = at \qquad (6 \text{ bis})$$

and the distance passed over the body while acquiring the velocity v is

$$d = \tfrac{1}{2}\, at^2 \qquad (8 \text{ bis})$$

Multiplying Equation (10) by Equation (8), member for member, we have

$$Fd = \tfrac{1}{2}\, M \cdot a^2t^2$$

or since

$$a^2t^2 = v^2 \qquad \text{(from Equation 6)}$$

$$Fd = \tfrac{1}{2}\, Mv^2$$

But Fd is the work done by the force F while the body acquires the velocity v. **This work is stored in the body as kinetic energy**. Therefore, the kinetic energy of a body having a mass M and a velocity v is

$$E = \tfrac{1}{2}\, Mv^2 \qquad (25)$$

If M is expressed in grams and v in $\frac{\text{centimeters}}{\text{second}}$, Equation (25) gives kinetic energy in ergs. If M is expressed in pounds and v in $\frac{\text{feet}}{\text{second}}$, the kinetic energy is given in foot-poundals.

THE KINETIC ENERGY OF A BODY IN ROTATORY MOTION

68. Under the influence of an unbalanced torque T a body having a moment of inertia I will start into uniformly accelerated rotatory motion, the acceleration being such that

$$T = I \cdot \alpha \qquad (13 \text{ bis})$$

The angular velocity acquired in t seconds is

$$\omega = \alpha \cdot t \qquad (26)$$

since, as in linear motion, the velocity acquired by a body having uniformly accelerated motion and starting from rest is given by the product of the acceleration and the time.

The angular distance passed over by the body while acquiring this velocity is

$$\phi = \tfrac{1}{2}\, \alpha \cdot t^2 \qquad (27)$$

since, as in linear motion, the distance passed over by a body having uniformly accelerated motion and starting from rest is given by one half the product of the acceleration and the square of the time.

Multiplying Equation (13) by Equation (27), we have

$$T\phi = \tfrac{1}{2} Ia^2t^2$$

or since $$a^2t^2 = \omega^2 \qquad \text{(from Equation 26)}$$

$$T\phi = \tfrac{1}{2} I\omega^2$$

But $T\phi$ is the work done by the torque T while the body acquires angular velocity ω (Section 62). **This work is stored in the rotating body as kinetic energy.** Therefore, the kinetic energy of a body having a moment of inertia I, and an angular velocity ω, is,

$$E = \tfrac{1}{2} I\omega^2 \qquad (28)$$

Comparing this equation with Equation (25), the fact is again emphasized that the **moment of inertia of a body in rotatory motion is that which corresponds to the mass of the body in linear motion.**

THE RELATION BETWEEN MOMENT OF INERTIA AND MASS

69. Consider a particle P, Figure 42, moving uniformly in a circle of radius r. Call the **linear velocity** of the particle v. The kinetic energy of this particle is

$$E = \tfrac{1}{2} mv^2$$

in which m is the mass of the particle. But

$$v = r \cdot \omega \qquad (29)$$

in which ω is the **angular velocity** of the radius vector r. Hence $v^2 = r^2\omega^2$. Substituting this value of v^2 in the equation above, we have

$$E = \tfrac{1}{2} mr^2 \cdot \omega^2$$

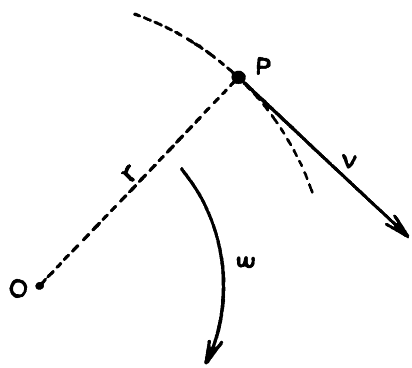

FIG. 42.—Moment of Inertia and Mass.

Comparing this expression with Equation (28), evidently

$$I = m \cdot r^2 \qquad (30)$$

That is, **the moment of inertia of a particle about a given axis is equal to the product of the mass of the particle and the square of its distance from the axis.**

Now consider a body made up of any number of particles. Let m_1 and r_1 be respectively the mass and the distance from the axis of one particle, m_2 and r_2 the corresponding values for a second particle, m_3 and r_3 the values for a third particle, etc. Then evidently the moment of inertia of the body as a whole is,

$$I = m_1r_1^2 + m_2r_2^2 + m_3r_3^2 + \text{etc.}$$

or

$$I = \Sigma mr^2 \tag{31}$$

The **radius of gyration** of a rotating body is the distance R from the axis of rotation at which the entire mass of the body might be concentrated without altering the moment of inertia of the body. If the entire mass M were concentrated at a distance R from the axis, the moment of inertia would be equal to MR^2 according to the above discussion. That is,

$$I = MR^2$$

therefore,

$$R = \sqrt{\frac{I}{M}} \tag{32}$$

The moments of inertia of some of the regular solids are given in the table below.

Moments of Inertia

Axis passing through center of mass . . .	I	R
Sphere, radius r, mass M	$\frac{2}{5} Mr^2$	$r\sqrt{\frac{2}{5}}$
Cylinder, radius r, mass M, axis of cylinder is the axis of rotation	$\frac{1}{2} Mr^2$	$r\sqrt{\frac{1}{2}}$
Slender rod, length L, mass M, axis of rotation at right angles to rod	$\frac{1}{12} ML^2$	$L\sqrt{\frac{1}{12}}$
Rectangular parallelopiped, length L, width B, mass M, axis at right angles to L and B	$\frac{1}{12} M(L^2+B^2)$	$\sqrt{\frac{L^2+B^2}{12}}$

TRANSFORMATION OF ENERGY IN A SLED ON A HILLSIDE

70. When the sled is at the top of the hill, its energy is all potential. This energy was stored in it as it was drawn up the hill (lifted through the vertical height of the hill). As it slides down the hill its energy changes to the kinetic form.

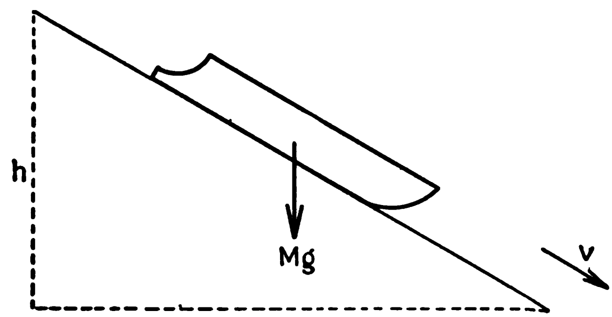

FIG. 43. — The Energy Transformations in a Coasting Sled.

Assume that it moves without friction, then all of its potential energy will be transformed into kinetic energy when it reaches the bottom of the hill.

Call the vertical height of the hill h and the mass of the sled M. Its potential energy at the top is therefore

$$E_1 = Mg \cdot h \qquad \text{(See Figure 43.)}$$

If V is its velocity when it reaches the bottom of the hill, then its kinetic energy there is

$$E_2 = \tfrac{1}{2} MV^2$$

and since we have assumed that friction effects are absent, therefore

$$E_2 = E_1$$

that is, all of its potential energy has been transformed into kinetic.

$$\therefore Mgh = \tfrac{1}{2} MV^2$$

whence

$$V = \sqrt{2gh} \qquad \text{(9 bis)}$$

where V is the velocity with which the sled reaches the foot of the hill.

It will be noted that the horizontal component of the distance traveled by the sled does not appear in this expression.

The significance of this fact is that **it makes no difference whether the hill be long or short, steep or of gentle slope; providing the vertical height is the same, the velocity of the sled at the bottom of the hill will be the same.** This is true even in the limiting case in which the hillside is vertical. That is, if the sled falls freely through the air, its velocity after falling a distance h will be as before, $V = \sqrt{2\,gh}$ (see Equation 9).

TRANSFORMATION OF ENERGY IN A BALL ON A HILLSIDE

71. Assume that the ball rolls down the hill without slipping. Evidently it acquires kinetic energy in two forms, that due to the linear motion of the ball, and that due to its rotatory

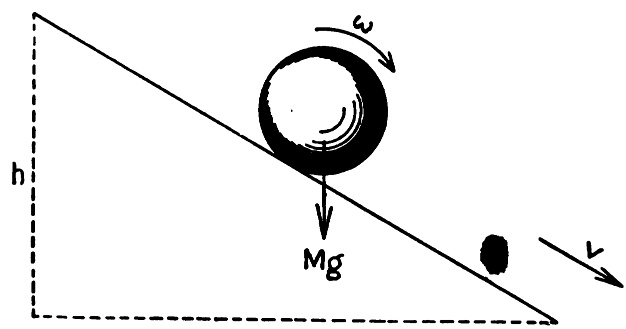

Fig. 44. — Energy Transformations in a Ball on a Hillside.

motion. If when it reaches the foot of the hill its linear velocity is v and its angular velocity is ω, we have,

$$Mgh = \tfrac{1}{2}\,Mv^2 + \tfrac{1}{2}\,I\omega^2 \qquad \text{(See Figure 44.)}$$

If it be assumed that the mass of the ball is the same as that of the sled in the preceding section, then their kinetic energies at the foot of the hill will be the same. It follows that **the ball will have a linear velocity v less than that of the sled,** since the velocity of the sled must be large enough to store all of its energy as $\frac{1}{2}\,MV^2$, while the ball stores a part of its energy as $\frac{1}{2}\,I\omega^2$.

KINETIC ENERGY AND MOMENT OF INERTIA

72. Experiment shows that the **moment of inertia** of a body depends upon its mass and the distance of its mass from the

axis about which it is rotating. The moment of inertia of any small portion of the body is proportional to its mass and to the **square of its distance** from the axis of rotation. Hence **the moment of inertia of a body may be changed without altering its mass, by placing the mass farther from the axis of rotation.** Thus the flywheels of engines are given large moments of inertia by building them with massive rims. The following experiment shows how the moment of inertia of a body depends upon the distribution of its mass: *A* and *B*, Figure 45, represent the ends of two cylinders of the same diameter and length. The center of *B* is of iron and the outside of wood. *A* has a wooden center surrounded by iron. **The mass of wood and iron in each is the same. Hence the masses and weights of the two cylinders are equal. Their moments of inertia, however, are very different.** This is because the larger part of *B*'s mass is near its center, while the larger part of *A*'s mass is far from its center.

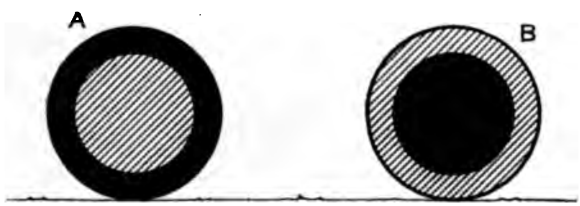

FIG. 45.—Cylinders having Equal Masses and Unequal Moments of Inertia.

If these cylinders are placed side by side and allowed to roll down an inclined plane (hillside) *B* will roll faster than *A*, since its lost potential energy will be stored largely as $\frac{1}{2}MV^2$, while *A*, having relatively large moment of inertia, will store its energy largely as $\frac{1}{2}I\omega^2$.

FRICTION

73. A moving body which slides or rolls upon another body is acted upon by forces which tend to bring it to rest. This effect is called **friction**. When a box is dragged along the floor, the **friction** between the floor and the box tends to stop the box; when water flows through a pipe, **friction** between the water and the walls of the pipe opposes the flow; when a rifle ball is fired into the air, its motion is opposed by air **friction**.

It is evident that **in all such cases work is done against friction, and as long as the body moves energy is expended in overcoming the drag of friction.**

THE COEFFICIENT OF FRICTION

74. Consider a block of wood lying upon a table, Figure 46. Assuming that the top of the table is horizontal, the block is evidently stationary under the balanced forces *W*, the weight of the block, and *R*, the reaction of the table upon the block. *R* is equal and opposite to *W* and both forces are perpendicular to the surface *CD*.

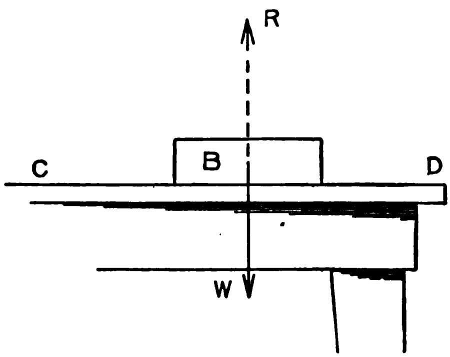

FIG. 46. — Balanced Forces on a Block lying on a Table.

Let it be assumed that a force *F*, parallel to the surface *CD*, is now applied to the block, Figure 47, *a*. Let *F* be such that it is just sufficient to keep the block in **uniform motion. The three forces *F*, *W*, and *R* will, under these circumstances, be in equilibrium.** If *F* and *W* are known, the magnitude and direction of *R* may be determined by applying the principle of Sec-

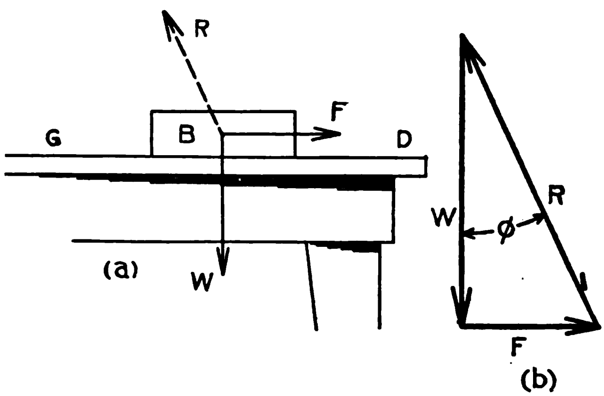

FIG. 47. — Balanced Forces on a Block in Uniform Motion on a Table.

tion 37. See Figure 47, *b*. The angle between R and W is called the **angle of friction.** It is found that for given conditions, if W is increased, F must be increased in the same ratio, in other words, $F \propto W$, or, $\frac{F}{W} =$ a constant (for a given pair of surfaces, in given condition as to smoothness, etc.).

The ratio $\frac{F}{W}$ for a given pair of surfaces is called the coefficient of friction for those surfaces.

The above relation may be written

$$F = \mu W \tag{33}$$

In which the proportionality factor μ is the **coefficient of friction.** Evidently from Figure 47, *b*,

$$\mu = \frac{F}{W} = \tan \phi$$

The coefficient of friction between two surfaces depends upon the nature of the materials and upon their roughness or smoothness. It is found to be nearly independent of the area of contact and the velocity with which the one body moves over the other.

ROLLING FRICTION

75. When one body rolls upon another, the friction is less than would be the case if sliding took place. The resistance to rolling motion between two bodies is called rolling friction.

The friction effects in machinery are diminished by the use of ball bearings. Rolling friction is in this manner substituted for sliding friction. Figure 48 represents a **ball bearing.** The shaft A rolls upon the balls which in turn roll upon the bearing B. Thus the sliding friction between A and B is avoided. In order to secure the best results in a ball bearing, the balls must be slightly separated. Otherwise there will be sliding friction between adjacent balls.

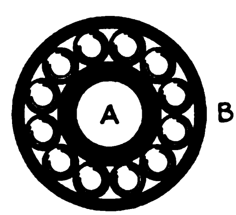

FIG. 48.—A Ball Bearing.

Problems

1. How much work in joules is required to lift a mass of 10,000 gr. (10 kilos) from the floor and place it upon a table 120 cm. high?

2. A man whose mass is 175 lb. climbs a ladder 20 ft. long. The ladder is leaning against a wall, its lower end being 10 ft. from the wall. How much work is done against gravity?

3. A man draws a box along a sidewalk for a distance of 100 ft. He draws it by means of a rope which makes an angle of 30° to the horizontal. How much work is done in moving the box if the pull in the rope is 25 lb. weight?

4. What is the potential energy of a ton (2000 lb.) of water on the brink of a fall of 150 ft.?

5. What is the potential energy of a tankful of water 100 ft. high, the base of the tank being at the surface of the ground, if the capacity of the tank is 200 tons (approximately 50,000 gal.)?

6. What is the kinetic energy of a rifle ball having a mass of 0.04 lb. and a velocity of 1000 ft./sec.?

7. What is the kinetic energy of a 40-ton car at a velocity of 60 mi./hr.?

8. A box of 250 lb. weight lies upon a level sidewalk. The coefficient of friction between the box and the walk is 0.37. What horizontal force is required to move the box?

9. A man presses an ax against a grindstone with a force of 25 lb. weight. The radius of the stone is 15 in. The torque required to turn the grindstone is 12.5 pound-feet. What is the coefficient of friction between the ax and the stone?

10. A block having a mass of 1000 gr. slides down an inclined plane. The height of the plane h is 50 cm. The velocity acquired by the sliding block is 250 cm./sec. How much work is done against friction?

11. It requires a force of 100 lb. weight to stretch a certain spiral spring 6 in. What is the potential energy of the spring when stretched in this manner?

12. A constant torque of 50 pound-feet is acting upon a rotating body. If the body makes 20 revolutions, how much work is done by the torque?

THE SIMPLE MACHINES

CHAPTER VII

DEFINITION OF A MACHINE

76. A machine is a device which facilitates the doing of work. It is to be regarded as a **transmitting** device, since a machine is able to do work upon other bodies only when work is done upon it. The force which operates a machine is called the **working force**; and the force against which the machine operates, the **resisting force.**

The **simple machines** are the lever, the wheel and axle, the pulley, the inclined plane, the wedge, and the screw.

The mechanical advantage of a machine is the ratio of the resisting force to the working force.

The efficiency of a machine is the ratio of the work done by the machine to the work done upon the machine. The efficiency of a machine is always less than 100%, since in every machine a certain amount of work is done against friction.

THE LEVER

77. The lever is a rigid bar, straight or curved, which when in use rotates about a fixed point called the fulcrum. There are three classes of levers (see Figure 49): (*a*) that in which the fulcrum is between the working force and the resisting force; (*b*) that in which the resisting force is between the working force and the fulcrum; (*c*) that in which the working force is between the fulcrum and the resisting force.

The pump handle is a lever of the (*a*) class. The oar of a boat is a lever of the (*b*) class. A man's forearm is a lever of the (*c*) class.

The mechanical advantage of the lever is given by the ratio

of the length of the lever arm of the working force to the length of the arm of the resistance force. This is evident from the following: Consider that the lever is without weight and the torques are in equilibrium, then (Section 38),

$$f \cdot AP = F \cdot BP$$

(See Figure 49.)

$$\therefore \frac{F}{f} = \frac{PA}{PB}$$

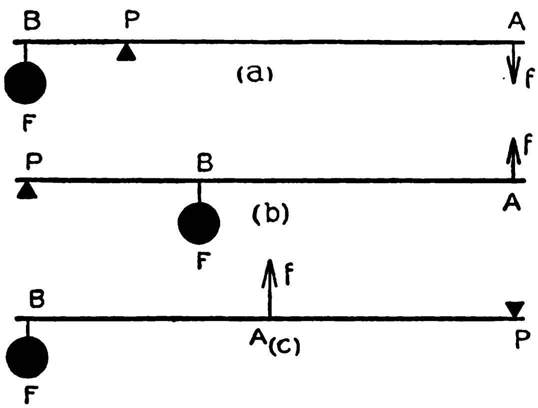

Fig. 49. — The Three Kinds of Lever.

It is evident that with the lever of the class (*a*) the mechanical advantage may have any value. In a lever of the class (*b*) the mechanical advantage is always greater than unity. In class (*c*) it is always less than unity.

THE WHEEL AND AXLE

78. The mechanical advantage of the wheel and axle is given by the ratio of the radius of the wheel to the radius of the axle. This may be shown as follows: In Figure 50 let the small circle represent the axle upon which the cord *c* is wound as the wheel is turned. It is assumed that the working force is applied at the circumference of the wheel and it may be represented by a small weight *f*. Assuming the torques to be in equilibrium, we have (Section 38),

$$f \cdot R = F \cdot r$$

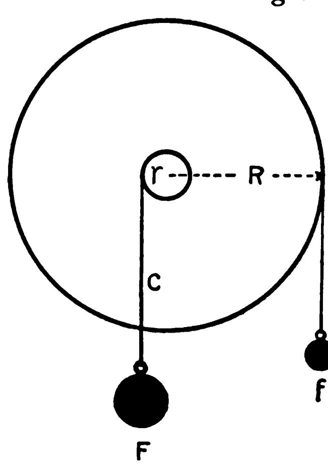

Fig. 50. — Wheel and Axle.

in which R represents the radius of the wheel and r the radius of the axle.

$$\therefore \frac{F}{f} = \frac{R}{r}$$

THE PULLEY

79. The mechanical advantage of the pulley when the "block," that is, the frame which supports the pulley wheel, is fixed, is unity. This is evident from the following considerations: In Figure 51, f represents the working force applied at one end of a rope passing over a fixed pulley. The weight which is being lifted hangs from the other end of the rope. Let it be assumed that the torques are balanced (Section 38),

then, $\quad f \cdot r = F \cdot r$

or $\quad f = F$

and $\quad \therefore \frac{F}{f} = 1$

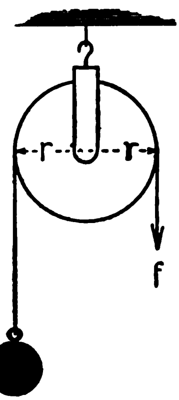

FIG. 51. — Fixed Pulley.

A pulley used in this manner is termed a "**fixed pulley.**"

A pulley when used as shown in Figure 52 is called a "loose pulley."

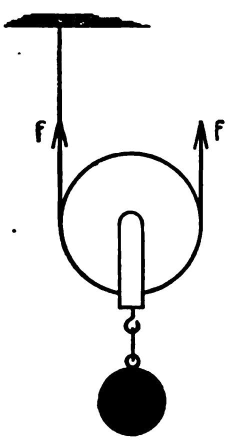

FIG. 52. — Loose Pulley.

The mechanical advantage of the "**loose pulley**" is 2, providing the two ropes which support the loose pulley are parallel. This is evident from the following consideration: If the forces are in equilibrium, then (Section 37),

$$f + f = F$$

$$\therefore \frac{F}{f} = 2$$

The "**block and tackle**" is a combination of fixed and loose pulleys with one continuous rope running between them. In effect the arrangement of pulleys and ropes in the block and tackle is like that

shown in Figure 53, in which *a*, *b*, and *c* are fixed pulleys all attached to the same block, and *d*, *e*, and *f* are loose pulleys attached to one block. The rope passes around them in the manner indicated in the diagram. *W* represents the weight lifted or the resisting force to be overcome. *f* represents the working force. The mechanical advantage of the arrangement is obtained as follows: Let it be assumed that in the use of the apparatus *W* is lifted through a distance *h*. Then evidently with the arrangement shown *f* moves in its own direction a distance of $6h$. We have, therefore, neglecting friction,

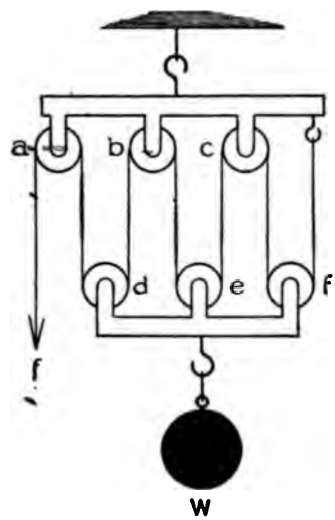

FIG. 53. — Combination of Pulleys illustrating the Block and Tackle.

$$W \cdot h = f \cdot 6h$$

or

$$\frac{W}{f} = 6$$

It will be observed that the **mechanical advantage** is given by **the number of ropes leading to the system of loose pulleys.** In the block and tackle the three pulleys, *a*, *b*, *c*, are placed side by side on the same axle, and *d*, *e*, and *f* also side by side on one axle.

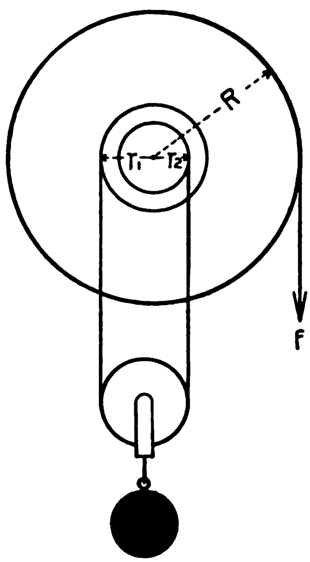

FIG. 54. — Differential Block and Tackle.

The "**differential block and tackle**" is shown diagrammatically in Figure 54. It consists essentially of two fixed pulleys of unequal diameters attached to the same shaft, and one loose pulley. A chain passes over the fixed pulleys in such way that as it winds up on one, it unwinds from the other. Let it be assumed that the fixed pulleys are turned in

THE SIMPLE MACHINES

CHAPTER VII

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THE LEVER

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The pump handle is a lever of the (*a*) class. The oar of a boat is a lever of the (*b*) class. A man's forearm is a lever of the (*c*) class.

The mechanical advantage of the lever is given by the ratio

of the length of the lever arm of the working force to the length of the arm of the resistance force. This is evident from the following: Consider that the lever is without weight and the torques are in equilibrium, then (Section 38),

$$f \cdot AP = F \cdot BP$$

(See Figure 49.)

$$\therefore \frac{F}{f} = \frac{PA}{PB}$$

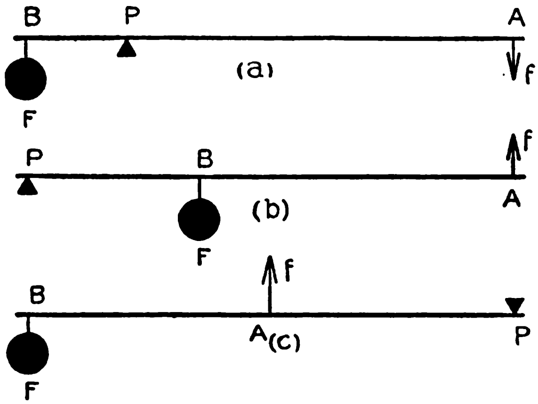

FIG. 49. — The Three Kinds of Lever.

It is evident that with the lever of the class (*a*) the mechanical advantage may have any value. In a lever of the class (*b*) the mechanical advantage is always greater than unity. In class (*c*) it is always less than unity.

THE WHEEL AND AXLE

78. The mechanical advantage of the wheel and axle is given by the ratio of the radius of the wheel to the radius of the axle. This may be shown as follows: In Figure 50 let the small circle represent the axle upon which the cord *c* is wound as the wheel is turned. It is assumed that the working force is applied at the circumference of the wheel and it may be represented by a small weight *f*. Assuming the torques to be in equilibrium, we have (Section 38),

$$f \cdot R = F \cdot r$$

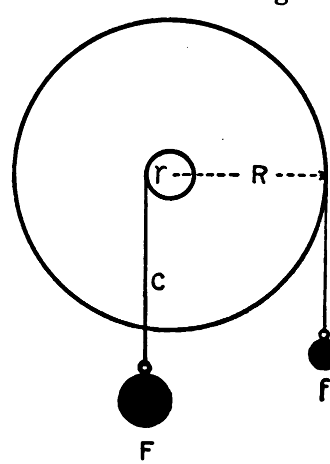

FIG. 50. — Wheel and Axle.

in which R represents the radius of the wheel and r the radius of the axle.

$$\therefore \frac{F}{f} = \frac{R}{r}$$

THE PULLEY

79. The mechanical advantage of the pulley when the "block," that is, the frame which supports the pulley wheel, is fixed, is unity. This is evident from the following considerations: In Figure 51, f represents the working force applied at one end of a rope passing over a fixed pulley. The weight which is being lifted hangs from the other end of the rope. Let it be assumed that the torques are balanced (Section 38),

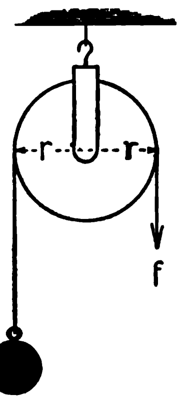

FIG. 51. — Fixed Pulley.

then, $$f \cdot r = F \cdot r$$

or $$f = F$$

and $$\therefore \frac{F}{f} = 1$$

A pulley used in this manner is termed a "**fixed pulley.**"

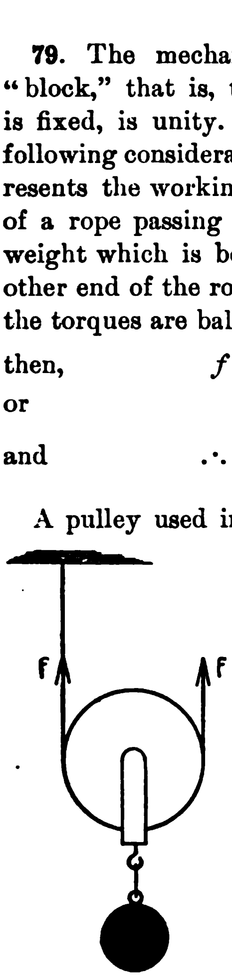

FIG. 52. — Loose Pulley.

A pulley when used as shown in Figure 52 is called a "loose pulley."

The mechanical advantage of the "**loose pulley**" is 2, providing the two ropes which support the loose pulley are parallel. This is evident from the following consideration: If the forces are in equilibrium, then (Section 37),

$$f + f = F$$

$$\therefore \frac{F}{f} = 2$$

The "**block and tackle**" is a combination of fixed and loose pulleys with one continuous rope running between them. In effect the arrangement of pulleys and ropes in the block and tackle is like that

shown in Figure 53, in which *a*, *b*, and *c* are fixed pulleys all attached to the same block, and *d*, *e*, and *f* are loose pulleys attached to one block. The rope passes around them in the manner indicated in the diagram. *W* represents the weight lifted or the resisting force to be overcome. *f* represents the working force. The mechanical advantage of the arrangement is obtained as follows: Let it be assumed that in the use of the apparatus *W* is lifted through a distance *h*. Then evidently with the arrangement shown *f* moves in its own direction a distance of 6 *h*. We have, therefore, neglecting friction,

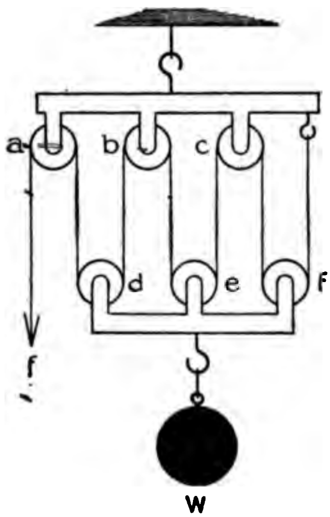

FIG. 53. — Combination of Pulleys illustrating the Block and Tackle.

$$W \cdot h = f \cdot 6h$$

or

$$\frac{W}{f} = 6$$

It will be observed that the **mechanical advantage** is given by **the number of ropes leading to the system of loose pulleys.** In the block and tackle the three pulleys, *a*, *b*, *c*, are placed side by side on the same axle, and *d*, *e*, and *f* also side by side on one axle.

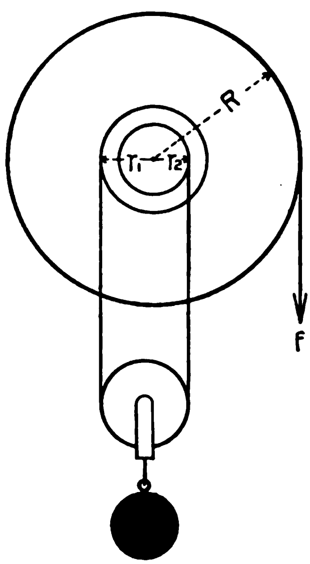

FIG. 54. — Differential Block and Tackle.

The "**differential block and tackle**" is shown diagrammatically in Figure 54. It consists essentially of two fixed pulleys of unequal diameters attached to the same shaft, and one loose pulley. A chain passes over the fixed pulleys in such way that as it winds up on one, it unwinds from the other. Let it be assumed that the fixed pulleys are turned in

such direction that the chain is wound up on the larger one. In one revolution the amount of chain taken up by the apparatus is, therefore,

$$2\,\pi(r_1 - r_2)$$

Evidently the weight is lifted through one half this distance, that is,

$$\pi(r_1 - r_2)$$

If the working force f is applied at a distance R from the center, it moves in one revolution through a distance $2\,\pi R$. Neglecting friction, we have,

$$W \cdot \pi(r_1 - r_2) = f \cdot 2\,\pi R$$

Whence,

$$\frac{W}{f} = \frac{2R}{r_1 - r_2}$$

Evidently by means of this device a high mechanical advantage may be secured, other things being equal, by making $r_1 - r_2$ small, that is to say, having the two fixed pulleys of nearly the same diameter.

THE INCLINED PLANE

80. The mechanical advantage of the **inclined plane** may be expressed in terms of the length and the height of the plane as follows: Referring to Figure 55, if f causes the body B to move from a to b, the work done is $f \cdot ab$, while the work done against gravity is $W \cdot bc$. Neglecting friction, then on the theory of the conservation of energy, we have

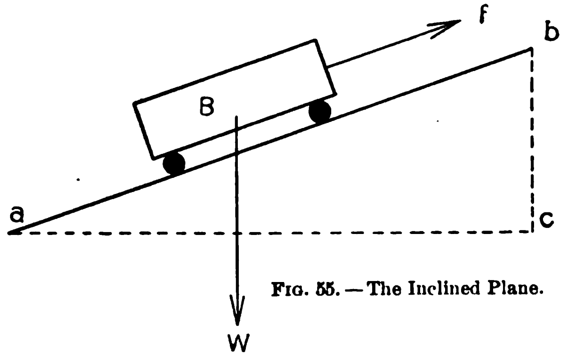

FIG. 55.—The Inclined Plane.

$$W \cdot bc = f \cdot ab$$

Whence

$$\frac{W}{f} = \frac{ab}{bc}$$

That is to say, the mechanical advantage of the inclined plane is given by the ratio of its length to its height.

THE WEDGE

81. The **wedge** is a modification of the inclined plane. Its mechanical advantage is found as follows: Referring to Figure 56, let f represent the force with which the wedge is pushed into the log. Let F represent the force with which the log pushes against the face of the wedge on one side, friction neglected. Evidently the three forces F, f, and F_1 are in equilibrium. Therefore, they form a closed triangle, as shown in Figure 56, the sides of which are parallel to the three forces concerned, and are of such length that they represent these forces in magnitude. The angle between F and F_1 is evidently the same as that formed by the sides of the wedge. Then, calling the side of the wedge L and its greatest thickness D, from similar triangles, we have

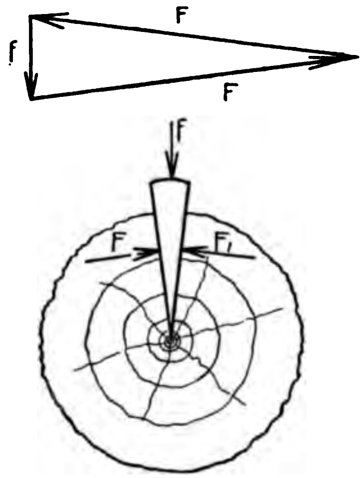

Fig. 56. — The Wedge.

$$\frac{F}{f} = \frac{L}{D}$$

THE SCREW

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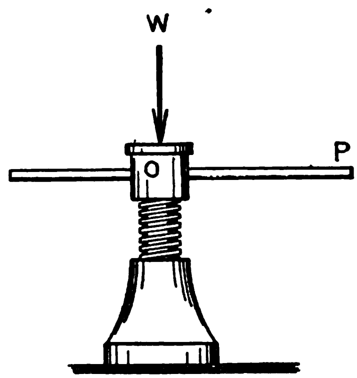

Fig. 57. — Jack Screw.

Let Figure 57 represent a

"jack screw," and OP the lever used for turning it. Call the pitch of the thread, that is to say, the vertical distance from one thread to the next, p, and let it be assumed that the weight of the body resting upon the screw head is W. Then the work done against the resisting force in one revolution of the screw is $W \cdot p$. Let the working force be attached at the point P, and be so directed as to always be perpendicular to the lever arm OP. Call the distance OP, R. Then the work done by the working force f in one revolution of the screw is $f \cdot 2\pi R$. Therefore, in the absence of friction,

$$W \cdot p = f \cdot 2\pi R$$

$$\therefore \frac{W}{f} = \frac{2\pi R}{p}$$

EFFECTS OF FRICTION ON THE SIMPLE MACHINES

83. In the above discussion of the simple machines, friction has been neglected; that is, the efficiency in each case has been assumed to be 100 %. As a matter of fact, **friction is always present**, and a part of the work put into the machine is used in overcoming friction effects. It follows, therefore, that **the efficiency of a machine is always less than** 100 %, and the actual mechanical advantage is less than that obtained in the discussion above.

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The center of mass of a body is the point at which the entire mass of the body might be concentrated without changing the effect of outside forces upon the body. For example, if a body of irregular form is suspended from any point, it will come to rest with its center of mass directly below the point of sup-

port. It thus behaves as if its weight were concentrated at its center of mass. For this reason the center of mass is sometimes called the **center of gravity.**

If it is desired to determine the torque action on a lever due to its own weight, it is necessary to first determine its center of mass. Then the product of the weight of the lever into the horizontal distance of its center of mass from the point of support gives the torque action due to the weight of the lever. Evidently if the lever is supported at its center of mass the torque action due to its own weight is 0.

EXAMPLE. Let AB, Figure 58, represent a lever of uniform density and cross section having a mass of 10 pounds.

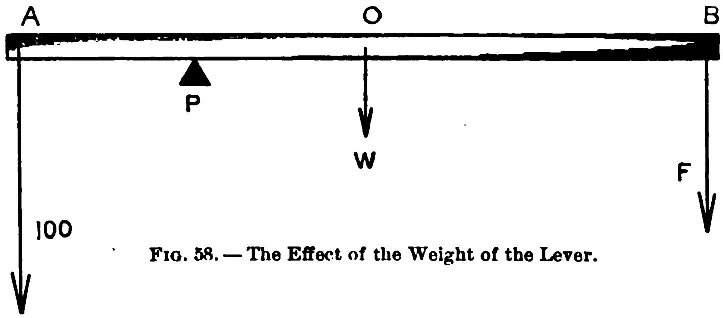

FIG. 58. — The Effect of the Weight of the Lever.

Its center of mass is therefore at its center of figure O. Let it be assumed that a force of 100 pounds weight is acting downward at A. It is required to find the value of F acting downward at B, which will hold the lever in equilibrium. Let it be assumed that the fulcrum P is at a point $\frac{1}{4}$ the distance from A to B as indicated, *i.e.* $AP = \frac{L}{4}$, in which $L = AB$.

The condition for no turning (Section 38) is that the tendency to turn counterclockwise is just balanced by the tendency to turn clockwise. We have, therefore,

$$100 \cdot AP = F \cdot BP + W \cdot OP$$

But, $AP = OP = \frac{L}{4}$, $BP = \frac{3}{4} L$, and $W = 10$ pounds weight.

$$\therefore 100 \cdot \frac{L}{4} = 10 \cdot \frac{L}{4} + F \cdot \frac{3}{4} L$$

Whence, $\qquad F = 30$ pounds weight

Had the lever been without weight, evidently the balancing force would have been $33\frac{1}{3}$ pounds weight. It is evident, therefore, that it is necessary to take account of the weight of the lever itself in problems of this nature.

WEIGHING MACHINES

85. The lever is used in weighing machines, the simplest form of which is the steelyard. This is represented in Figure 59. A heavy mass of iron CA is attached to one end of a long slender steel rod AB and so proportioned that the center of

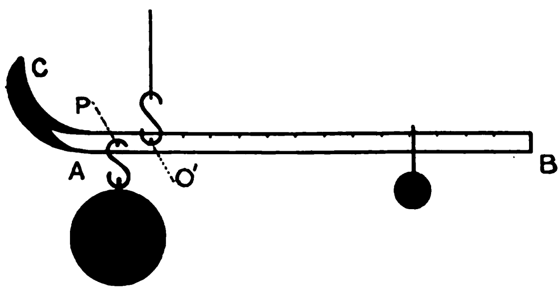

FIG. 59. — The Steelyard.

mass of the apparatus is at O', the point of support. The body W to be weighed is attached at P. A small weight is made to slide along the arm AB until the steelyard is balanced. The weight of the body W is indicated by the position of the sliding weight.

In the platform scales, which is a form of weighing device designed for weighing heavy bodies, combinations of levers are used in such manner as to make it possible to balance the weight of a very heavy body by that of a small one, without the use of a lever of excessive length.

Problems

1. AC is a straight bar. B is a point between A and C, the ends of the bar, the distance AB being $\frac{9}{10}$ of the length of the bar. What is the mechanical advantage of the bar as a lever of the first class, the fulcrum being at B?

2. What is the mechanical advantage of the bar in the last problem as a lever of the second class, fulcrum at C?

shown in Figure 53, in which a, b, and c are fixed pulleys all attached to the same block, and d, e, and f are loose pulleys attached to one block. The rope passes around them in the manner indicated in the diagram. W represents the weight lifted or the resisting force to be overcome. f represents the working force. The mechanical advantage of the arrangement is obtained as follows: Let it be assumed that in the use of the apparatus W is lifted through a distance h. Then evidently with the arrangement shown f moves in its own direction a distance of $6\,h$. We have, therefore, neglecting friction,

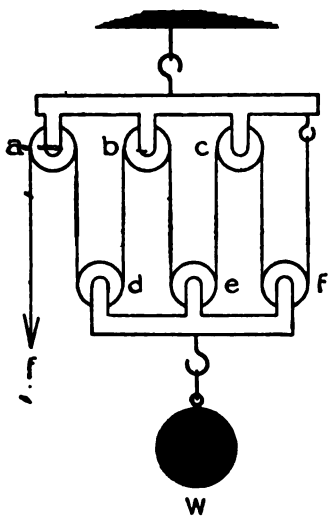

FIG. 53. — Combination of Pulleys illustrating the Block and Tackle.

$$W \cdot h = f \cdot 6\,h$$

or

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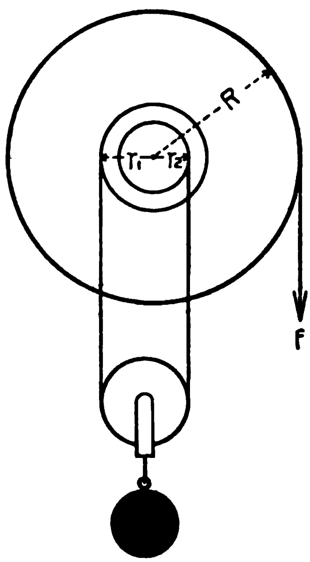
It will be observed that the **mechanical advantage** is given by **the number of ropes leading to the system of loose pulleys.** In the block and tackle the three pulleys, a, b, c, are placed side by side on the same axle, and d, e, and f also side by side on one axle.



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$$2\pi(r_1 - r_2)$$

Evidently the weight is lifted through one half this distance, that is,

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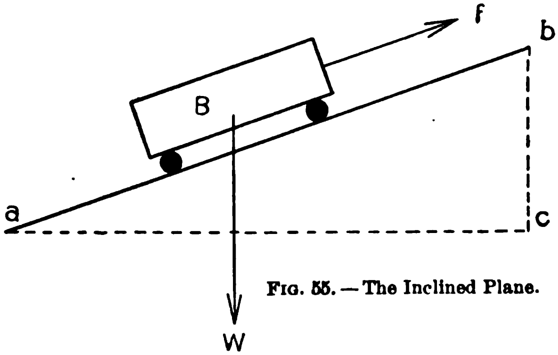

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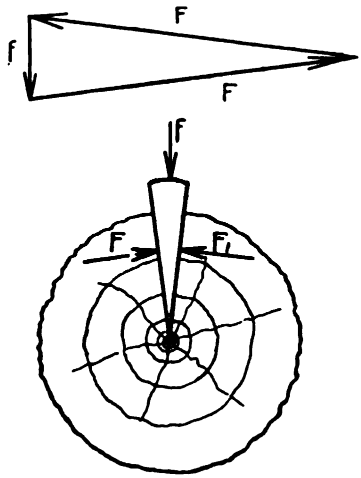

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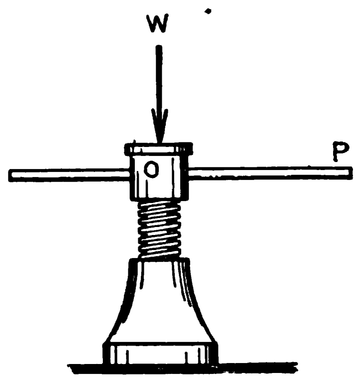

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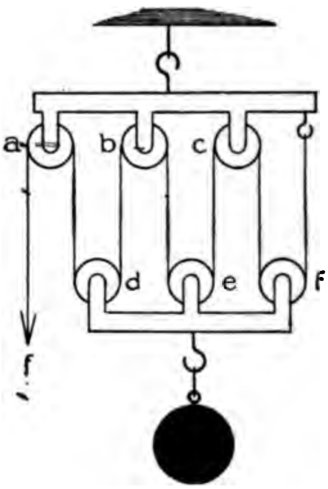

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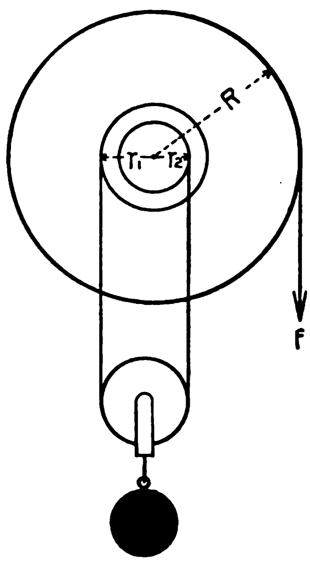

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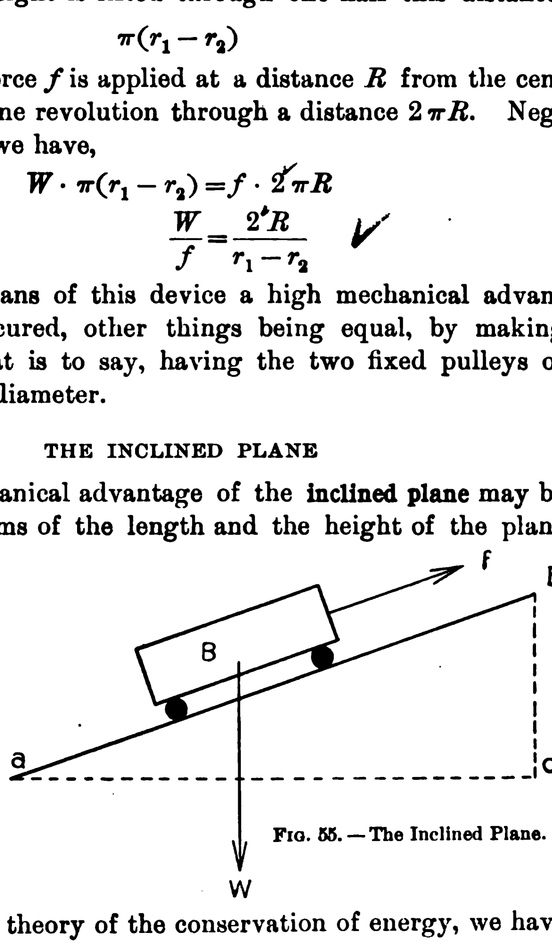

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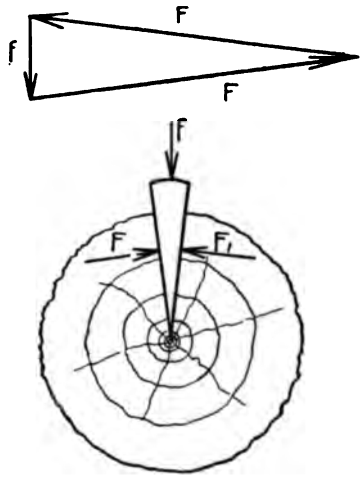

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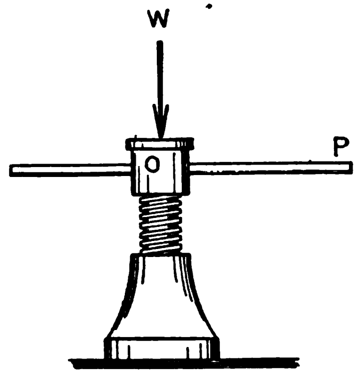

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83. In the above discussion of the simple machines, friction has been neglected; that is, the efficiency in each case has been assumed to be 100 %. As a matter of fact, **friction is always present**, and a part of the work put into the machine is used in overcoming friction effects. It follows, therefore, that **the efficiency of a machine is always less than** 100 %, and the actual mechanical advantage is less than that obtained in the discussion above.

GENERAL DISCUSSION OF THE LEVER

84. The lever is so widely used in mechanical appliances that a more complete discussion of it is warranted. The lever as commonly employed has appreciable weight, and this must be taken account of in determining the conditions under which the forces acting upon it are in equilibrium. In order to determine this influence of weight, we make use of the conception of the center of mass of the lever.

The center of mass of a body is the point at which the entire mass of the body might be concentrated without changing the effect of outside forces upon the body. For example, if a body of irregular form is suspended from any point, it will come to rest with its center of mass directly below the point of sup-

3. What is the mechanical advantage of a system consisting of one fixed and one loose pulley, (*a*) when the working force moves upward? (*b*) when it moves downward?

4. What would be the mechanical advantage of the system of pulleys shown in Figure 53 if inverted? (Neglect friction.)

5. With a system like that of problem 4 it is found that a working force of 100 lb. weight is required to lift a weight of 500 lb. What is the efficiency of the system?

6. The diameter of a large cylinder of a differential block and tackle is 12 in. and the diameter of the small cylinder 11 in. R is 2 ft. What is the mechanical advantage? (Neglect friction.)

7. A plank 12 ft. long has one end resting upon the ground, the other upon a box 3 ft. high. What force parallel to the plank would suffice to maintain a block of stone weighing 200 lb. in uniform motion up the plank, assuming the block to be mounted on frictionless rollers? Would the same force maintain the block in uniform motion down the plank?

8. Conditions the same as in problem 7. What horizontal force would maintain uniform motion?

9. If the stone in problem 7 rests upon the plank, coefficient of friction .75, what force parallel to the plank will maintain uniform motion?

10. What is the efficiency of the inclined plane in problem 9?

11. A jack screw has a pitch of $\frac{1}{4}$ in. The lever used for turning it is 3 ft. long. If the efficiency of the screw is 50 %, what force at the end of the lever will raise the screw when it supports a weight of 1 T.?

POWER

CHAPTER VIII

THE RATE OF DOING WORK

86. In the various discussions which have thus far been given upon the subject of work, no reference has been made to the time occupied by a given force in accomplishing its work. This item, however, is usually of great importance. We are generally interested in knowing, not only how much work a force can do, but in what length of time the work can be accomplished.

Power is the rate of doing work.—The power expended by an agency is determined by dividing the work done by the time occupied in doing the work. We have, therefore,

$$P = \frac{W}{t} \tag{34}$$

in which P represents the power expended, W the work performed, and t the time in which the work is accomplished.

The unit of power in any system is the unit of work per unit time. Thus in the c.g.s. system the unit of power is the erg per second. The practical unit in the c.g.s. system is the joule per second. This is called the **watt**; that is to say,

1 watt = 1 joule per second.

In the English gravitational system the unit of power is the **foot-pound per second**. The capacity of engines and electric motors is measured in horse power.

1 horse power = 550 foot-pounds per second

In the measurement of the capacity of dynamos the kilowatt (kw.) is employed:

1 kilowatt = 1000 watts

The relative values of these two units of power are given in the following relation:

$$1 \text{ horse power} = 746 \text{ watts (very approximately)}$$

THE MEASUREMENT OF POWER

87. The power developed by any agency may be measured by determining the work done in a measured time and dividing by the time.

EXAMPLE. Let it be required to find the power developed by a man climbing a mountain. It has been observed that a man weighing 180 lb. can climb up an average slope at such rate that he lifts himself 2000 ft. vertically in 1 hr. The power expended in this case is therefore

$$\begin{aligned} P &= 180 \text{ lb.} \times \frac{2000}{3600}\frac{\text{feet}}{\text{seconds}} \\ &= 100\frac{\text{foot-pounds}}{\text{second}} \\ &= 0.18 \text{ h.p.} \end{aligned}$$

Since the work done by a moving force is given by the product of the force and the distance through which it moves, Equation (34) may be written in the following form:

$$P = \frac{W}{t} = \frac{f \cdot d}{t}$$

or

$$P = f \cdot v \tag{35}$$

since $\frac{d}{t}$ is the velocity of the working force. That is to say, the power developed by a moving force is given by the product of the force and the velocity with which it moves in the direction of the force.

EXAMPLE. Let it be required to find the power developed by a man drawing water from a well. Let it be assumed that he is making use of a bucket which, when full of water, weighs 44 lb. Let it be assumed further that he can lift this at the rate of 5 feet per second. We have, therefore,

$$
\begin{aligned}
P &= f \cdot v \\
&= 44 \text{ lb.} \times 5 \frac{\text{feet}}{\text{second}} \\
&= 220 \frac{\text{foot-pounds}}{\text{second}} \\
&= \tfrac{2}{5} \text{ h.p.}
\end{aligned}
$$

In a similar manner it may be shown that the power developed by a moving torque is given by

$$P = T \cdot \omega \qquad (36)$$

That is to say, the power developed by a moving torque is given by the product of the torque and its angular velocity.

BRAKES

88. The capacity of an electric motor or a steam engine is sometimes determined by applying to the pulley or flywheel what is known as a **brake**. The power developed by the machine is used in overcoming the frictional resistance of the brake. If, therefore, the frictional resistance of the brake is known and the distance through which such resistance is overcome is determined, we have at once the data for determining the capacity of the machine. In Figure 60 is shown a simple form of brake used for this purpose. A represents the pulley or flywheel of the machine. CD is a strap which is held against the periphery of the wheel. To the ends of this strap spring balances are attached. Let F_1 represent the force indicated by the spring balance on the right and F the force indicated by the spring balance on the left.

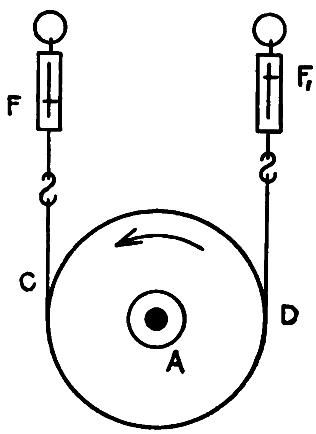


Fig. 60. — The Strap Brake.

Let it be assumed that the wheel A is moving in the direction of the arrow. It will be evident that F is larger than F_1. The difference between them is the frictional resistance between the surface of the wheel and the strap. It is this fric-

tional resistance against which the machine is working. Therefore, if the wheel A makes n revolutions per second, so that its angular velocity in radians per second is $2\pi n$, the velocity of a point on its circumference is $2\pi rn$, where r is the radius of the wheel. Therefore, the work done per second (*i.e.* the power developed) by the machine, against the frictional resistance at the circumference of the wheel, is

$$P = 2\pi rn\,(F - F_1)$$

In Figure 61 is shown another simple device which is used for the purpose of determining the capacity of a motor. It is known as the Prony brake. Let A represent the pulley of the motor to be tested, and B and C represent two pieces of wood which are clamped on the pulley A as indicated. If A is rotating in the direction of the arrow, the frictional resistance between A and B and C tends to rotate the brake in the same direction. This tendency to rotate, that is, the torque acting upon the brake, is balanced by means of the spring balance E attached to the end of the arm B. Let it be assumed that the indication of the spring balance is F. Then the torque which prevents the rotation of B is

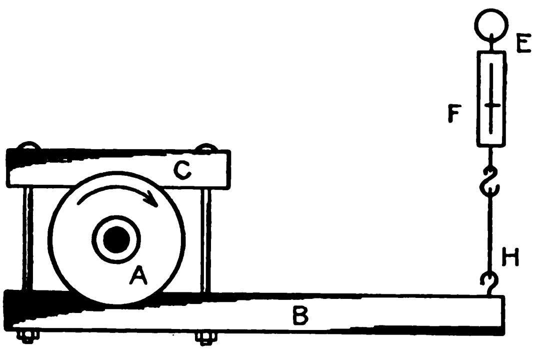


Fig. 61. — The Prony Brake.

$$T = F \cdot AH$$

but this torque is equal to the torque action of the pulley on the brake, since it just balances it. Therefore, assuming that A makes n revolutions per second, so that its angular velocity in radians per second is $2\pi n$, we have

$$P = T \cdot \omega \qquad \text{(Equation 36.)}$$
$$= 2\pi n \times F \cdot AH.$$

Hence, if the value of F is read from the spring balance, AH is measured, and n is determined by means of a speed counter, it is a simple matter to calculate the power developed by the machine.

Problems

1. What pull must a horse traveling 4 mi. per hour exert upon a wagon in order to develop one horse power?

2. A force of 5×10^7 dynes is acting upon a body. If the body moves 40 m. in the direction of the force in 20 sec., what horse power does the force develop?

3. What power would be required to raise water to a height of 50 ft. at the rate of 500 T. per hour?

4. What horse power would be required to raise a loaded elevator having an unbalanced weight of 3000 lb. at the rate of 6 ft. per second?

5. What horse power will a man weighing 165 lb. develop in running upstairs, if he climbs a vertical distance of 10 ft. in 3 sec?

6. A strap brake (Figure 60) is applied to the pulley of an electric motor. The difference in the indications of the spring balances is 5 lb. weight. The pulley is 12 in. in diameter and makes 2000 R. P. M. What is the brake horse power?

7. A Prony brake (Figure 61) is applied to an engine flywheel. The brake arm $AH = 4$ ft. and the spring balance reads 40 lb. weight. The engine makes 300 R. P. M. What horse power does it develop?

8. An elevator having an unbalanced weight of 2000 lb. starts from rest and in 3 sec. has acquired a velocity of 9 ft./sec. What is the average horse power developed during this interval?

9. What is the work done and the average horse power developed in the first second in problem 8? in the second? in the third?

10. A shaft making 200 R. P. M. is transmitting 200 horse power. What is the torque?

11. A torque of 500 pound-feet has an angular velocity of 10 rad./sec. What power does it develop?

ELASTICITY

CHAPTER IX

ELASTIC AND INELASTIC BODIES

89. Experience teaches that solid bodies offer resistance to a change in form or size. Certain substances when forcibly distorted exhibit the property of recovery; that is to say, when the distorting force is removed, they return more or less completely to the original form or size which they had before the distortion took place. **Bodies which exhibit this property of recovery in a large degree are said to be elastic.** Those in which this property is not strongly marked are referred to as **inelastic bodies.** Thus elasticity may be thought of as that property of a body which enables it to recover from distortion.

STRESS AND STRAIN

90. When a force acts upon an elastic body in such manner as to cause distortion, it is opposed by force actions within the body. These internal force actions are larger for large distortions than they are for small ones; and when a distorting force is applied to a body, the distortion increases until the internal force actions balance the distorting force.

The internal force action per unit of area, across which the forces are acting, **is called the stress.** Thus if a column AB, Figure 62, 5 inches square, supports a weight of 5000 pounds, the stress is,

$$\text{Stress} = \frac{\text{total force}}{\text{area}} = \frac{5000}{25}\left(\frac{\text{pounds weight}}{\text{square inches}}\right)$$

$$= 200\left(\frac{\text{pounds weight}}{\text{square inch}}\right)$$

For the "total force," we have taken the **external force,** since, as stated above, when the various parts of the column are in

equilibrium, the internal force action balances (*i.e.* is numerically equal to) the external force.

The distortion of a body, per unit length, or unit volume, as the case may be, is called the **strain**. Thus, if the column represented in Figure 62 is 20 inches high and is shortened $\frac{1}{10}$ inch by the weight supported, the strain is given by

$$\text{Strain} = \frac{\text{change in length}}{\text{total length}}$$

$$= \frac{0.1}{20}\left(\frac{\text{inch}}{\text{inches}}\right) = 0.005$$

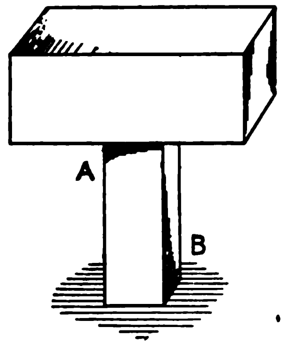


FIG. 62.—Column under Stress.

THREE KINDS OF STRESS

91. There are three kinds of stress, viz.:

1. Tensile Stress.
2. Hydrostatic Pressure.
3. Shearing Stress.

A body is said to experience a **tensile stress** when the forces acting upon the body tend to change its length. The strain which accompanies this kind of stress is a change in length and is called **a stretch**. For example, a vertical wire or string supporting a weight is subjected to a tensile stress.

A body is said to be subjected to a **hydrostatic pressure** when the pressure upon it from all sides is the same; for example, a small object submerged in a body of water is under hydrostatic pressure. The strain corresponding to hydrostatic pressure is a **change in volume**.

A body is said to be subjected to a **shearing stress** when the system of forces acting upon it tends to cause one layer of particles in the body to slide over an adjacent layer. For example, in the shearing or punching of plates of metal one part of the plate, the part sheared off, is made to slide past another part, that is to say, one layer of particles is made to slide upon an adjacent layer. The strain which accompanies a shearing stress is called a **shearing strain**.

ELASTIC LIMIT

92. If the strain in a body exceeds a certain value, the body will not recover completely when the distorting force is removed. Such a body is said to be strained beyond its elastic limit.

HOOKE'S LAW

93. In 1676 Robert Hooke discovered that, for elastic bodies under any kind of stress, stress is proportional to strain, that is,

$$\frac{\text{stress}}{\text{strain}} = \text{a constant}$$

This is known as Hooke's law.

MODULUS OF ELASTICITY

94. The ratio of tensile stress to tensile strain is called **Young's modulus**, the **stretch modulus**, or the **modulus of elasticity**.

The ratio of hydrostatic pressure to the corresponding strain is known as the **bulk modulus**.

The ratio of shearing stress to shearing strain is called the **coefficient of simple rigidity**.

The stretch modulus of a few common materials is given in the following table:

	Dynes per Cm².	Lb. Wt. per Sq. In.
Copper	11×10^{11}	16×10^6
Glass	6×10^{11}	9×10^6
Wrought Iron	19×10^{11}	27×10^6
Lead	1×10^{11}	1.5×10^6
Steel	23×10^{11}	33×10^6

HOW THE MODULUS IS USED

95. The stretch modulus of any building material is one of its most important physical properties. Before an engineer can design a structure, whether it be a bridge, a building, or a machine, he must have knowledge of the elastic properties of the material to be used. Having such knowledge, he can determine how large a rod, beam, or column should be to sustain a given weight or load.

An example will serve to illustrate how the stretch modulus is used in such calculations. A steel rod in a certain structure

is 20 feet (=240 in.) long. Let it be required to determine how large this rod should be to support a weight of 5 tons with an elongation of **not more** than $\frac{1}{10}$ inch. Assume that the stretch modulus of steel is 33,000,000 pounds weight per square inch.

Now, $$E = \frac{\text{stress}}{\text{strain}}$$

where E is the stretch modulus of the steel. But stress $= \frac{F}{a}$, that is, the total force acting upon the rod divided by the cross-sectional area of the rod. Also strain $= \frac{e}{L}$ in which e is the elongation and L the length of the rod, therefore,

$$E = \frac{F/a}{e/L}$$

or, $$E = \frac{FL}{ea} \qquad (37)$$

$$\therefore\ a = \frac{F \cdot L}{E \cdot e}$$

Substituting the assumed values for F, L, E, and e,

$$a = \frac{10{,}000 \times 240}{33{,}000{,}000 \times \frac{1}{10}} = 0.727 \text{ square inch}$$

FLEXURE

96. When a rod or beam is bent, the convex side is **stretched** and the concave side **compressed.** Let AB, Figure 63, represent a beam resting on supports SS. Imagine a heavy weight W to be placed upon the beam at its center. Under the action of W the beam will be bent into the form shown (exaggerated) in the lower part of the figure. Since the lower horizontal lay-

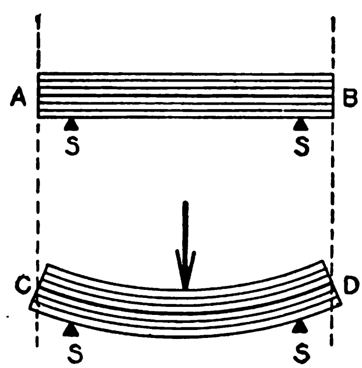


FIG. 63. — Beam under Flexure.

since the pendulum was stationary ($V_1 = 0$) before the impact and v_2 is negligible in comparison with v_1, that is, the change in the velocity of the bullet is practically v_1.

Hence
$$v_1 = \frac{M}{m} \cdot V_2$$

In the actual use of the apparatus V is determined by observing the horizontal distance through which the pendulum moves under the impulse of the bullet. Knowing this distance and the length of the pendulum, it is an easy matter to calculate the height through which the pendulum bob is lifted. But the height through which the pendulum bob is lifted multiplied by the weight of the pendulum bob is the potential energy of the pendulum at the extremity of its swing. Under the theory of the conservation of energy this must be just equal to the kinetic energy of the pendulum as it started to swing toward the left. This kinetic energy is equal to

$$\tfrac{1}{2}(M + m) V_2^2$$

Hence V_2 may be calculated.

Problems

1. A rod 1 m. long and 0.2 sq. cm. cross section sustains a weight of 100 Kg. and is stretched so that its length is 100.04 cm. Find the stress, the strain, and the stretch modulus.

2. A vertical rod of wrought iron 10 ft. in length and 1 in. in diameter supports a weight of 5 T. What is the increase in length of the rod?

3. A vertical rod 10 ft. long and 1 in. in diameter is stretched 0.05 in. by a certain weight. What stretch will be produced in a rod of the same material 5 ft. long and $\frac{1}{2}$ in. in diameter by the same weight?

4. A vertical iron wire 3 m. long and 1 mm. in diameter is attached to a copper wire of the same diameter 5 m. long. A weight of 3 Kg. is attached to the lower end. What is the elongation of each wire?

5. The stretch modulus of a certain metal is 2×10^{12} dynes/cm^2. What force would be required to stretch a rod of this metal 1 sq. cm. in cross section until its length is doubled, assuming that the elastic limit is not passed in the operation?

6. Two vertical wires of steel and copper of the same diameter carry the same load. What must be their relative lengths in order that their elongations may be equal?

thus led to the conclusion that **in the case of impacting bodies the change in momentum experienced by each body is the same.**

ELASTIC AND INELASTIC IMPACT

98. In elastic impact work is done in distorting the impacting bodies and energy is stored momentarily as potential energy. If the bodies are perfectly elastic, they will recover completely from the distortion and will therefore return all of the energy which was expended in distorting them. Therefore,

$$\tfrac{1}{2}MV_1^2+\tfrac{1}{2}mv_1^2=\tfrac{1}{2}MV_2^2+\tfrac{1}{2}mv_2^2 \qquad (39)$$

If the impacting bodies are inelastic, the energy expended in distorting them is not returned, hence the kinetic energy of the bodies after impact is less than before.

THE BALLISTIC PENDULUM

99. The ballistic pendulum is a device used for measuring the velocity of a bullet. The principle upon which its use is based will be understood from the following discussion: Let M, Figure 64, represent a large block of wood suspended pendulumwise as indicated. Let it be imagined that a rifle bullet is fired into this pendulum. It is required to find the velocity of the bullet v at the moment it comes in contact with the pendulum. Call the mass of the pendulum M and the mass of the bullet m. Evidently the pendulum and the bullet swing as one body after the impact, the mass being $M+m$. If m is very small in comparison with M, it may be neglected after impacting with M. Therefore from the principle above enunciated, we have

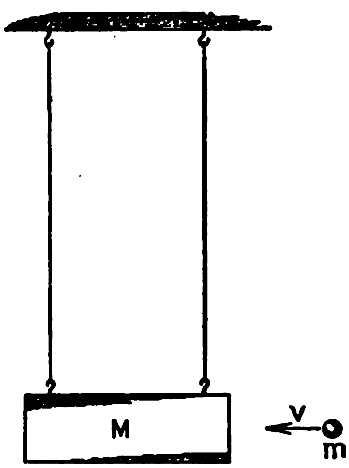


FIG. 64. — Ballistic Pendulum.

$$M(V_2-V_1)=m(v_2-v_1)$$

or

$$MV_2=mv_1$$

since the pendulum was stationary ($V_1 = 0$) before the impact and v_2 is negligible in comparison with v_1, that is, the change in the velocity of the bullet is practically v_1.

Hence
$$v_1 = \frac{M}{m} \cdot V_2$$

In the actual use of the apparatus V is determined by observing the horizontal distance through which the pendulum moves under the impulse of the bullet. Knowing this distance and the length of the pendulum, it is an easy matter to calculate the height through which the pendulum bob is lifted. But the height through which the pendulum bob is lifted multiplied by the weight of the pendulum bob is the potential energy of the pendulum at the extremity of its swing. Under the theory of the conservation of energy this must be just equal to the kinetic energy of the pendulum as it started to swing toward the left. This kinetic energy is equal to

$$\tfrac{1}{2}(M + m) V_2^2$$

Hence V_2 may be calculated.

Problems

1. A rod 1 m. long and 0.2 sq. cm. cross section sustains a weight of 100 Kg. and is stretched so that its length is 100.04 cm. Find the stress, the strain, and the stretch modulus.

2. A vertical rod of wrought iron 10 ft. in length and 1 in. in diameter supports a weight of 5 T. What is the increase in length of the rod?

3. A vertical rod 10 ft. long and 1 in. in diameter is stretched 0.05 in. by a certain weight. What stretch will be produced in a rod of the same material 5 ft. long and ½ in. in diameter by the same weight?

4. A vertical iron wire 3 m. long and 1 mm. in diameter is attached to a copper wire of the same diameter 5 m. long. A weight of 3 Kg. is attached to the lower end. What is the elongation of each wire?

5. The stretch modulus of a certain metal is 2×10^{12} dynes/cm². What force would be required to stretch a rod of this metal 1 sq. cm. in cross section until its length is doubled, assuming that the elastic limit is not passed in the operation?

6. Two vertical wires of steel and copper of the same diameter carry the same load. What must be their relative lengths in order that their elongations may be equal?

7. An elastic ball, M_1, moving with a velocity v_1, strikes a stationary elastic ball, M_2, of equal mass. What are the velocities of the balls after impact?

8. If the mass of M_1, problem 7, is twice that of M_2, what will be the velocity of the balls after impact?

9. A ball of putty having a mass of 700 g. and moving with a velocity of 10 m./sec. strikes a stationary ball of putty having a mass of 250 g. What is the velocity of the balls after impact?

10. A rifle bullet is fired into a ballistic pendulum having a mass of 2000 g. The mass of the bullet is 1 g. Under the impulse of the bullet the pendulum swings so that its center of mass rises 1 cm. Required the velocity of the bullet.

FLUIDS AT REST

CHAPTER X

THE THREE FORMS OF MATTER

100. We are familiar with matter in three forms; namely, solids, liquids, and gases. Liquids and gases are commonly considered together as fluids.

A solid may be defined as a portion of matter which offers resistance to any force action which tends to change either its form or its size.

A fluid is a portion of matter which offers resistance to a change of size but none to change of form. A **liquid** is distinguished from a gas in that when placed in an open vessel it will present a free surface, and also by the property of forming itself into drops.

A **gas** is distinguished by the property of indefinite extension. A gaseous body tends to expand until it fills all available space.

Generally speaking, any of the simple substances may exist in either the solid, liquid, or gaseous state. The most common example is that of water, which in the form of ice is a solid, in the form of water is a liquid, and in the form of very hot steam is a gas. Any other substance, for example, iron, may be made to pass from one of these states to another; a piece of solid iron, if placed in a furnace and strongly heated, melts and assumes the liquid form. If still more strongly heated, it vaporizes and becomes a gas.

THE GENERAL PROPERTIES OF THE THREE STATES OF MATTER

101. As general properties of a solid we may mention density, elasticity, hardness, ductility. We think of the ultimate particles (molecules) of a solid as being bound together by some sort of intermolecular force action which causes them to cohere and resist any force which tends to separate them. They do not possess any great degree of freedom of motion, that is to

say, we imagine that a molecule in one part of a given solid continues in that neighborhood. In the solid the molecules are to be thought of as sliding over one another with great difficulty.

In a liquid, while the ties which bind one molecule to its neighbors are present as in the solid body, the molecules are to be thought of as sliding upon one another with great freedom; and furthermore, it is to be considered possible for a molecule which is in one given portion of a liquid to wander to an entirely different part of the liquid. In matter in the liquid state, as in the solid state, the molecules are to be thought of as being very close together.

In matter which is in the gaseous state the molecules may be more or less widely separated. The bond of union between the adjacent molecules is not nearly so strong as in the other forms of matter. The molecules in the body in this state move freely about from place to place, the spaces between molecules being large as compared with the size of the molecules.

DENSITY

102. The density of a body is its mass per unit volume. In the c. g. s. system the density of a substance is given in grams per cubic centimeter. In this system the density of distilled water is for practical purposes unity, since the mass of one cubic centimeter of water (at 4° C.) is almost exactly one gram. The density of water in the f. p. s. system is about 62.3 (pounds per cubic foot).

The following table gives the densities of some of the common substances.

DENSITIES

Solids

Aluminum	2.58	Iron, Wrought	7.86
Copper	8.92	Lead	11.3
Cork	0.24	Platinum	21.5
Glass (Crown)	2.6	Silver	10.53
Ice	0.91	Tin	7.29

Liquids

Alcohol	0.789	Mercury	13.596
Glycerine	1.26	Olive Oil	0.91

Gases

At Freezing Temperature and Standard Atmospheric Pressure

Air	0.00129	Hydrogen	0.000089
Carbon Dioxide	0.00197	Oxygen	0.00143

THE PRESSURE IN A LIQUID DUE TO ITS WEIGHT

103. The freedom of motion possessed by the molecules of a fluid give rise to certain phenomena which are characteristic of fluid bodies and which distinguish them from solids. One of these phenomena is that of distributed pressure on the walls of a containing vessel.

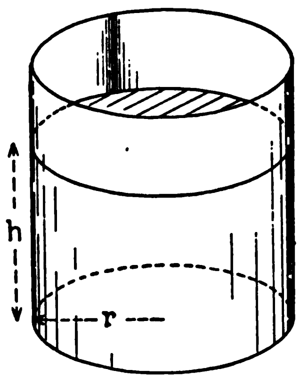


FIG. 65. — Pressure due to the Weight of a Liquid.

Consider a cylindrical vessel standing on end and partly filled with liquid; see Figure 65. Let it be required to find the pressure p on the bottom of the vessel due to the liquid contained. If h is the height of the liquid and r the radius of the base, evidently the volume of the liquid is

$$V = \pi r^2 \cdot h$$

Let the mass per cubic centimeter of the liquid, that is to say, the density of the liquid, be d. Then the total mass of the water contained in the vessel is

$$M = \pi r^2 hd$$

and the weight of the liquid, that is $M \cdot g$, is

$$W = \pi r^2 hdg$$

This weight is supported by the bottom of the vessel. The force action per unit area of the bottom is found by dividing the total weight supported by the total area supporting that weight. **This force action per unit area is called the *pressure*** on the bottom of the vessel. We have, therefore,

$$p = \frac{\text{weight}}{\text{area}}$$

$$= \frac{\pi r^2 hdg}{\pi r^2}$$

$$\therefore p = hdg \tag{40}$$

That is to say, **the pressure at a point in a liquid due to the weight of that liquid is proportional to the vertical distance of the point from the free surface of the liquid, to the density of the liquid, and to the acceleration of gravity.**

One of the important consequences of this law is that the pressure is independent of the lateral extent of the body of liquid. Thus the pressure at the bottom of a well is the same as the pressure at the bottom of a lake, providing the depth of water in the lake is the same as that in the well.

THE HYDROSTATIC PARADOX

104. Let *AB*, Figure 66, represent a wide vessel communicating with a narrow vessel *CD* by means of the tube *BC*. A liquid poured into the vessel *AB* will rise to the same height in the two vessels, that is to say, the narrow column of liquid in *CD* balances the wide column of liquid in *AB*. Inasmuch as these columns of liquid represent entirely different weights, the condition of equilibrium seems paradoxical. However, when we remember that the pressure at *C* due to the column *DC* is determined by the height of that column and the pressure at *B* due to the column *AB* is determined by the height of the column *AB*, it will be evident that the pressure at *C* acting toward the left is equal to the pressure at *B* acting toward the right if the heights of the columns *AB* and *CD* are the same.

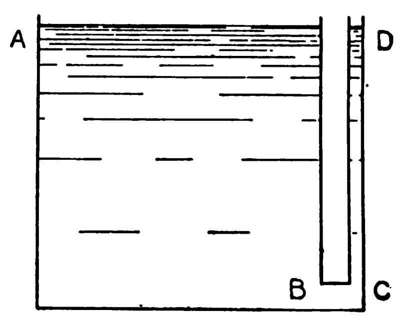


Fig. 66. — Hydrostatic Paradox.

THE PRESSURE PERPENDICULAR TO THE WALLS

105. The pressure on the walls of a vessel due to a contained liquid at rest is at every point perpendicular to the wall. This is evident from the following consideration. Consider the pressure at any point in the bottle represented in Figure 67.

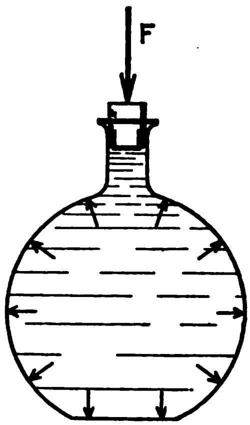


FIG. 67. — Fluid Pressure is Perpendicular to the Walls.

Let it be assumed that the pressure at a certain point is not perpendicular to the wall of the vessel. Then this pressure may be resolved into two components, one of which is perpendicular and the other parallel to the wall. That component which is parallel to the wall will tend to move those portions of the liquid which lie at this point along the wall. It is assumed, however, that the liquid is at rest. Therefore there can be no component of the pressure parallel to the wall; that is to say, the pressure must act perpendicular thereto.

THE PRESSURE THE SAME IN ALL DIRECTIONS

106. Consider the vessel represented in Figure 68. The points A, B, C, and D are all at the same vertical distance from the free surface of the liquid. Therefore, according to Equation (40), the pressure at each of these several points is the same.

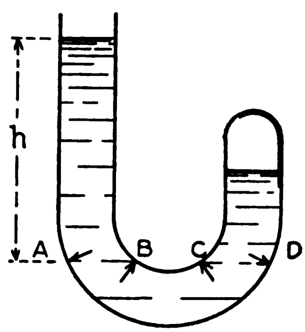


FIG. 68. — The Pressure at all points in a Horizontal Plane is the same.

THE PRINCIPLE OF ARCHIMEDES

107. A body submerged in a fluid is acted upon at each and every part of its surface by a pressure the value of which is given by Equation (40). It will be evident that for every small area on the right-hand side of the body there is a corresponding small area of the same size on the opposite side of the body upon which the pressure is the same. In a general way, therefore, it can be seen that the pressures right and left will neutralize each other. If we consider, however, points on the upper and lower surfaces of the submerged body, it will be evident that the pressures acting from above are less than those acting from below, since the height of

liquid in the first case is less than that in the second. Therefore the resultant of all of the force actions of a fluid upon a body submerged in it is an upward force action. This upward force action can be shown to be equal to the weight of the fluid displaced. Let *A*, Figure 69, represent a small portion of the liquid which fills the vessel *BC*. Assuming that the liquid is at rest, the forces acting upon *A* are in equilibrium. Therefore, since the weight *W* of the portion *A* is urging it downward, **the resultant of all the pressures upon the *A* portion due to the remaining parts of the liquid must be an upward force action *F* exactly equal to the weight of *A*.** Imagine the *A* portion to be removed and in its place some other body, for example, a stone having exactly the same form and size as the *A* portion of the liquid removed. Evidently the remaining portions of the liquid will act upon this stone in exactly the same manner that they acted upon the *A* portion of the liquid which it replaces. The stone will, therefore, be acted upon by two forces: First, the weight of the stone acting downward. **Second, an upward force action *F* equal to the weight of the *A* portion of the liquid which the stone displaces.** Thus the stone in this position will apparently weigh less than it does outside of the liquid by exactly the weight of the liquid which it displaces. This apparent loss of weight is found in all submerged bodies. The following is a general statement of the fact, and is known as the **Principle of Archimedes. A body submerged in a fluid loses a portion of its weight equal to the weight of the fluid displaced.**

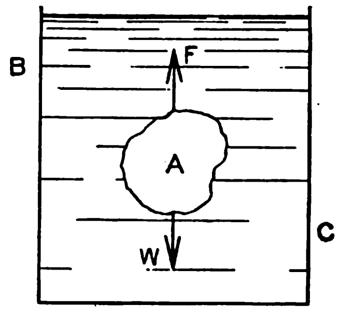
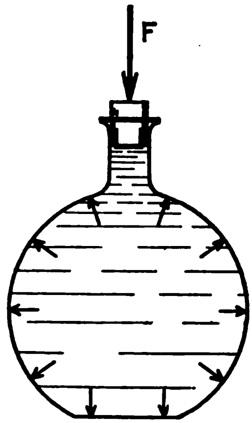


FIG. 69. — Archimedes' Principle.

SUBMERGED FLOATING BODIES

108. Let it be imagined that in place of the stone referred to in the last section the *A* portion of the liquid is replaced by a body having not only the same size and shape as *A*, but having also the same weight as *A*. **Under these circumstances the body will float in the liquid *BC*.** The condition, therefore, for the



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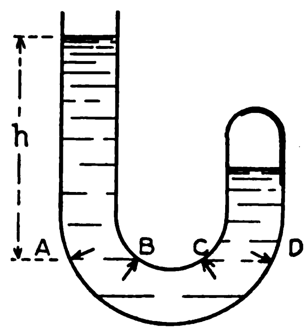

FIG. 68. — The Pressure at all points in a Horizontal Plane is the same.

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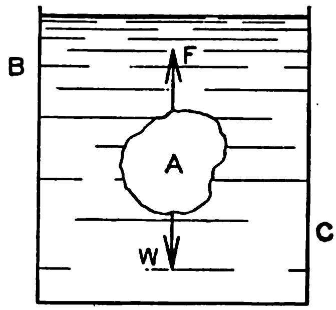

Fig. 69. — Archimedes' Principle.

SUBMERGED FLOATING BODIES

108. Let it be imagined that in place of the stone referred to in the last section the *A* portion of the liquid is replaced by a body having not only the same size and shape as *A*, but having also the same weight as *A*. **Under these circumstances the body will float in the liquid *BC*.** The condition, therefore, for the

floating of a submerged body is that **it must displace a weight of fluid equal to its own weight.** If its weight is greater than the weight of the fluid displaced, it will sink. If its weight is less than the weight of the fluid displaced, it will rise.

THE CARTESIAN DIVER

109. The apparatus shown in Figure 70 is used to illustrate the effect of a change in density upon a submerged floating body.

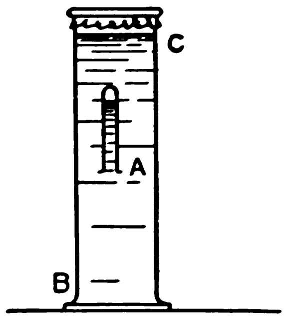

FIG. 70. — Cartesian Diver.

A is a small hollow vessel, for example, a bottle or test tube, with its opening at the bottom. It is nearly filled with the liquid in which it is floating, but contains enough air at the top to give it an average density equal to that of the liquid in which it floats. Under these conditions it will tend neither to rise nor sink. If the pressure on the liquid is increased, the air bubble in the top of the diver will be compressed, the average density (and therefore the total weight) of the floating body will be increased, and the diver will sink. If the pressure is decreased, the diver will rise.

THE BALLOON

110. A floating balloon affords a good example of the application of the principle of Archimedes. The combined weight

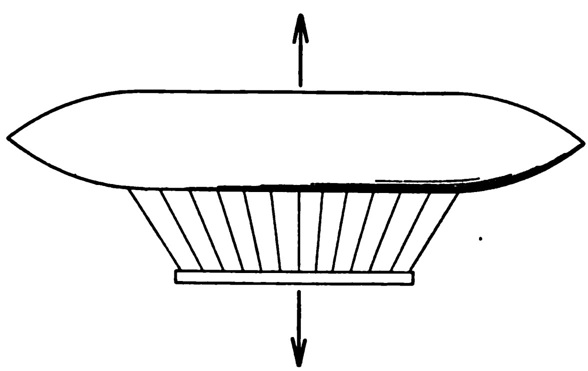

FIG. 71. — The Balanced Forces acting on a Balloon.

of the gas bag, gas, car, engine, passengers, etc., tends to drag the balloon towards the earth. This tendency to fall is opposed by the buoyant force of the air, which, according to the principle above stated, is equal to **the weight of the air displaced.** See Figure 71. In other words, **in order that the balloon may float it must displace its own weight of air.** Since the density of air is small, therefore, the balloon **must be large if heavy**, in order that it may displace a sufficient weight of air.

In the dirigible balloon the weight is counterbalanced by the air displacement as above described, and the balloon is moved through the air by a propeller driven by an engine.

THE PRINCIPLE OF ARCHIMEDES AS APPLIED TO BODIES FLOATING ON THE SURFACE OF A LIQUID

111. A body submerged in a liquid of the same density will float; in a denser liquid it will rise to the surface and a certain portion will project. As it begins to project above the surface, the liquid which it displaces becomes less and less. Evidently there will come a time when, having projected itself a certain distance above the surface, the weight of the liquid displaced will be equal to the weight of the body. The body will, under these conditions, be in equilibrium under the action of its own weight and the buoyant force of the liquid.

The statement of Archimedes' principle may be modified to fit the case of a body floating on the surface of a liquid as follows: **a floating body sinks in a liquid to such depth that the weight of the liquid displaced is equal to the weight of the floating body.**

THE MEASUREMENT OF DENSITY

112. The density of a body as above defined is the mass of the body per unit volume, that is,

$$D = \frac{M}{V} \qquad (41)$$

To determine the density of a body therefore we may find its mass, and its volume, and the quotient of mass by volume gives at once the density.

DENSITY OF SOLIDS

(*a*) In the case of a regular body the volume may be determined by measuring the linear dimensions of the body. From these measurements the volume of the body may be calculated. The mass of the body may be determined by weighing.

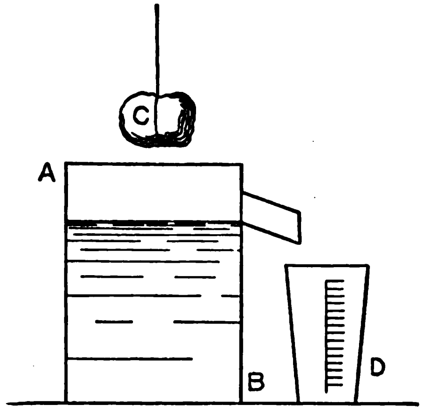

FIG. 72. — Determining the Volume of an Irregular Body.

(*b*) In the case of an irregular solid the volume may be determined by "displacement." This method of determining volume is as follows: A vessel *AB*, Figure 72, is filled with some convenient liquid, for example, water, up to the level of the spout as shown. The irregular body *C*, after being weighed to determine its mass, is lowered into the liquid. The volume of liquid which overflows is evidently equal to the volume of the body *C*. This volume is measured by the measuring vessel *D*.

DENSITIES OF LIQUIDS

(*c*) The density of a liquid may be determined by weighing a known volume. For this purpose it is customary to make use of a "specific gravity" bottle, a common form of which is represented in Figure 73. The capacity of the bottle in cubic centimeters being known, it is filled with the liquid in question and weighed. Subtracting the known weight of the bottle from this weight, we have at once the weight of the contained liquid in grams weight, which of course is numerically equal to its mass in grams, and, knowing the volume, the density of the liquid may be calculated in the usual way.

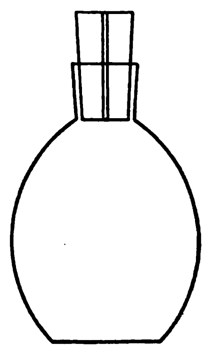
FIG. 73. — Specific Gravity Bottle.

(*d*) Another method of determining the density of a liquid is by means of comparing its density with that of some other standard liquid in the following manner. Referring to Figure 74, let A, B, D, C represent a U-tube the lower part of which, BD, is filled with mercury. The liquid whose density is to be determined is placed in one branch of the tube. The liquid to be used as a standard, for example water, is placed in the other branch of the tube. Assume that the ends of the mercury column B and D stand at the same level, then so far as the mercury is concerned, there is a balance of pressures right and left. Furthermore, the pressure upon the surface B due to the height h_1 of the liquid in the tube AB is equal to the pressure upon the surface D due to the height h_2 of the standard liquid in the other branch of the tube. Calling the density of the liquid in AB, d_1, and the density of the standard liquid in DC, d_2, we have at once:

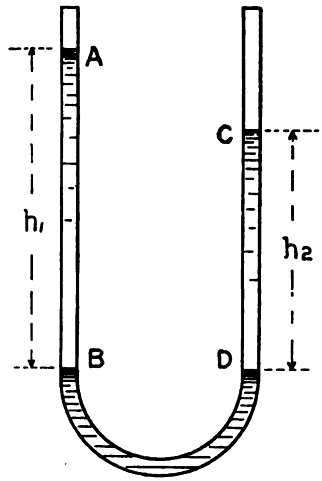

FIG. 74.—Balanced Columns.

$$h_1 d_1 g = h_2 d_2 g$$

$$\therefore \frac{d_1}{d_2} = \frac{h_2}{h_1}$$

In other words the heights of the two columns are inversely as the densities of the liquids.

Finally, $$d_1 = d_2 \cdot \frac{h_2}{h_1}$$

which gives the density of the liquid tested in terms of the density of the standard liquid.

SPECIFIC GRAVITY

113. **The specific gravity of a body is the ratio of its density to the density of water at the same temperature,** that is,

$$\text{specific gravity} = \frac{\text{density of the substance}}{\text{density of water at same temperature}}$$

or,
$$S = \frac{D}{d}$$
in which S is the specific gravity of the body.

Measurement of Specific Gravity.

Since the density of a body is equal to its mass divided by its volume, we may write
$$S = \frac{D}{d} = \frac{\frac{M}{V}}{\frac{M_1}{V_1}}$$
and if
$$V = V_1$$
$$\therefore S = \frac{M}{M_1}$$
or, multiplying the numerator and denominator by g, we have:
$$S = \frac{Mg}{M_1 g_1} = \frac{W}{W_1} \qquad (42)$$
in which W is the weight of the substance in question and W_1 is the weight of an equal volume of water. In determining the specific gravity of a substance, we therefore **find the weight of any convenient volume of the substance and the weight of an equal volume of water at the same temperature.** The ratio of these weights gives the specific gravity of the substance.

Specific Gravity of Solids.

(*a*) The specific gravity of a solid body heavier than water and insoluble may be conveniently determined in the following manner: First, weigh the body in air. Second, weigh the body in water (suspended, for example, by a thread from the arm of the balance so as to be completely submerged in a vessel of water). Call this weight w and the weight in air W. Then the **loss of weight in water is evidently $W - w$.** By the principle of Archimedes **this is precisely the weight of the water displaced,** that is to say, the weight of an equal volume of water. We have, therefore,
$$S = \frac{W}{W_1}$$
or,
$$S = \frac{W}{W - w} \qquad (43)$$

Liquids.

(*f*) Let a heavy body be weighed: First, in air — call this weight W. Second, in a liquid whose specific gravity we wish to determine — call this weight W_1. Third, in water — call this weight W_2. We have, therefore, the specific gravity of the liquid:

$$S = \frac{W - W_1}{W - W_2} \tag{44}$$

The Hydrometer.

(*g*) The hydrometer is an instrument which is designed to indicate the specific gravity of a liquid by the depth to which it sinks in the liquid. The usual form of the hydrometer is that shown in Figure 75. It is usually made of a thin glass tube weighted at the bottom so as to be in stable equilibrium in an upright position. Placed in a liquid it will float, sinking deeper into a liquid of low density and less deeply in a liquid of a higher density. The depth to which it sinks is therefore an indication of the density of the liquid in which it is floating. If the instrument is graduated by reference to the depth to which it sinks in water, it indicates specific gravities.

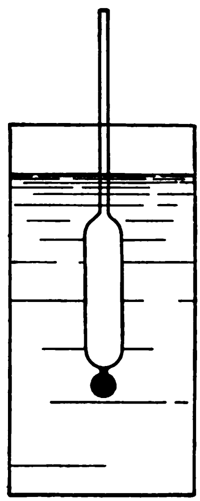

Fig. 75. — The Hydrometer.

The hydrometer of *Beaumé* has an arbitrary scale determined as follows: The instrument is first placed in distilled water at the desired temperature, and the point on the scale to which the instrument sinks is marked 0°. It is then placed in a solution of 15 parts common salt to 85 parts water, at the same temperature, and the point to which it sinks is marked 15°. The distance between these two points is divided into 15 equal parts, and this scale of equal divisions is extended down the stem of the instrument. The hydrometer graduated in this way is used for heavy liquids only; that is, for liquids of a specific gravity greater than unity.

(*h*) The specific gravity bottle as used for the determination of specific gravities is employed as follows. A bottle like that

represented in Figure 73 is first weighed empty and dry. It is then filled with distilled water and weighed again. Finally, after emptying and drying the bottle, it is filled with the liquid the specific gravity of which it is desired to determine, and again weighed. Subtracting the weight of the bottle from each of these two weights, we have at once the weight of a bottleful of water and the weight of a bottleful of liquid; that is to say, we have the weights of *equal volumes* of the unknown liquid and water, from which we may determine the specific gravity of the liquid.

ARCHIMEDES' PRINCIPLE APPLIED TO A GAS

114. In the general statement of the principle of Archimedes given in Section 107, we considered the loss of weight of a body when submerged in a **fluid**. The principle as stated therefore applies to gases as well as to liquids. Hence any body submerged in a gas loses a portion of its weight due to the buoyant force of the gas, and this loss in weight is exactly the weight of the gas displaced. Therefore bodies submerged in the earth's atmosphere weigh less than they would weigh in a vacuum by an amount which is equal to the weight of the air displaced.

Ordinarily we are unconscious of the existence of the atmospheric ocean in which we move. It offers but little resistance to a body which moves through it so long as the velocity is not very great, and because of this and similar facts it was for a long time held that the atmosphere was imponderable. That the air has weight, however, is very simply demonstrated by weighing an air-tight globe provided with a stopcock. The globe full of air is weighed on a balance. The air is now pumped from the globe and the stopcock turned. Upon weighing the globe the second time it will be found to weigh less than in the first instance. This difference in weight is undoubtedly the weight of the air which has been pumped from the globe, since everything else, including the weight of the globe itself and the buoyant force of the air in which it is submerged, remains the same. The density of the air is not very great, so that the loss in weight of a body submerged in air is

but slight as compared with its loss of weight when submerged in water. A body submerged in water loses about 772 times as much weight as it does when submerged in air.

ATMOSPHERIC PRESSURE

115. Since air has weight, it must exert a pressure upon any body submerged in it. It will be remembered that in determining the pressure due to the weight, of a liquid (Section 103) we made use of the weight of the liquid. In the same way we might determine the pressure due to the earth's atmosphere. It is not convenient, however, to calculate the pressure due to the atmosphere in this way. The density of the air is different at different levels, a fact which makes it difficult to calculate the mass of the air. In the case of the liquid considered in the section above referred to, the assumption was made that the density is the same from top to bottom of the liquid. This assumption is justified in the case of a liquid, because, as a rule, liquids are compressible only in a very slight degree. Therefore, while the lower layers of the liquid are under hydrostatic pressure due to the weight of the upper layers, they are not compressed appreciably, and the density of the lower layer is the same as that of the upper layer. The case is different with a gas. Those layers of the earth's atmosphere which lie close against the earth's surface are much more dense than those which are found higher up, due to the fact that they are compressed by the weight of the upper layers which they have to support. We find, therefore, as we ascend to greater altitudes, that the density of the air becomes less.

THE MEASUREMENT OF ATMOSPHERIC PRESSURE

116. The simplest way to determine pressure due to the earth's atmosphere at any given point is by balancing that pressure against the pressure of a column of liquid of known density. Consider, for example, the U-tube represented in Figure 76. Let it be assumed that the portion AB is filled with mercury, that the tube above A is closed and contains a vacuum. The other end of the U-tube is open, and therefore the mercury at B

is subjected to atmospheric pressure. It will be found under these circumstances that the mercury will stand high in the tube A and low in the tube B, as indicated in the figure. Under these conditions the pressure p due to the earth's atmosphere acting upon the surface of the mercury at B is balanced by the pressure due to the column of mercury AC; but the pressure due to the column AC is

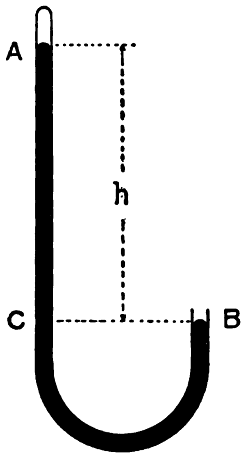

FIG. 76. — Siphon Barometer.

$$p = hdg \qquad (40 \text{ bis})$$

in which h is the height of the column, and d is the density of mercury. Since this pressure is equal to atmospheric pressure, it is of course a measure of it. Such a device for measuring the pressure of the atmosphere is called a barometer.

THE SIMPLE BAROMETER

117. Instead of the U-tube represented in Figure 76, it is oftentimes convenient to use an arrangement like that shown in Figure 77, in which AB is a straight glass tube sealed at the upper end and open at the lower end, the lower end dipping beneath the surface of mercury in the vessel C. Evidently the arrangement is equivalent to that shown in Figure 76, and the height h of the column of mercury in the tube AB is a measure of the pressure of the atmosphere which acts upon the surface of the mercury in the vessel C. If the height of the column h is expressed in centimeters, the density of the mercury in grams per cubic centimeter, and g in centimeters per second per second, then p, the pressure as given by Equation (40), is given in dynes per square centimeter.

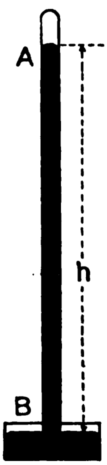

FIG. 77. — Simple Barometer.

It is found, for example, that under average atmospheric conditions at a given place the barometer column is 74 centimeters high. Tak-

ing the density of mercury at 13.6, we have for the average atmospheric pressure at this place:

$$p = 74 \times 13.6 \times 980.6$$

$$= 985000 \frac{\text{dynes}}{\text{square centimeter}} \text{ (approximately)}$$

This is roughly equivalent to 15 pounds to the square inch.

Since the mercury barometer is universally employed for the measurement of atmospheric pressure, it has become customary to express atmospheric pressure in terms of the number of centimeters or inches of mercury in the barometric column which such pressure will support. Thus, in the example given above, instead of referring to atmospheric pressure as having a value of 985,000 $\frac{\text{dynes}}{\text{square centimeter}}$ we would say the pressure of the atmosphere is 74 centimeters of mercury, meaning that it is capable of supporting a column of mercury of that height.

ATMOSPHERIC PRESSURE THE SAME IN ALL DIRECTIONS

118. By a process of reasoning similar to that employed in Sections 105 and 106, it may be demonstrated that the pressure due to the earth's atmosphere is everywhere the same in the same level (temperature effects neglected) and also that it acts equally in all directions and perpendicular to all surfaces which are exposed to its action. Thus it may be demonstrated that the atmosphere presses sidewise on a vertical wall or that it presses upward on a horizontal wall, the lower side of which is exposed to its action. Consider, for example, the apparatus sketched in Figure 78. *AB* is a cylinder to which a piston *P* is fitted air-tight. If the air contained in the cylinder *AB* is pumped away by means of an air pump, the unbalanced atmospheric pressure acting upon the lower surface of *P* will cause it to rise in the cylinder even though in so doing it is

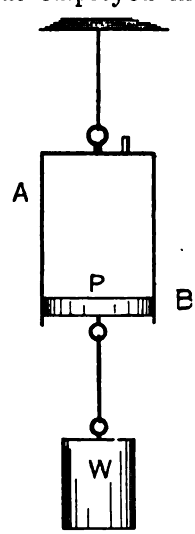

FIG. 78. — The Upward Pressure of the Air.

made to lift a weight W, as indicated in the diagram. This experiment demonstrates the upward pressure of the air.

This experiment will also serve to indicate in a general way how it is that ordinary bodies are capable of withstanding enormous force actions due to atmospheric pressure without being destroyed by it or suffering damage from it. It is evident that so long as the cylinder AB is open to the air, the air pressure acts within as well as without the cylinder, and the pressure on the lower surface of the piston P is balanced by the pressure on the upper surface. In the same way most bodies which are submerged in the atmospheric ocean are subjected to transmitted atmospheric pressure on the inside as well as to the atmospheric pressure which comes upon their surfaces. The pressures inside and outside are therefore balanced and their effects neutralized.

THE MAGDEBURG HEMISPHERES

119. An interesting demonstration of the fact of atmospheric pressure and of its equality in all directions is afforded by the Magdeburg hemispheres. Consider the halves of a hollow metallic sphere A and B, Figure 79. If the edges along which they join when placed together are ground so as to be air-tight, then when they are placed together and the air exhausted from within, the unbalanced atmospheric pressure acting upon the outer surfaces will hold the hemispheres A and B together. The force which would be required to separate one hemisphere from the other under these conditions would, of course, depend upon the size of the hemispheres and the degree of exhaustion of the air on the inside. It will be found that the force required to drag these hemispheres apart is independent of the direction of the axis EF so long as the center of the sphere remains in the same

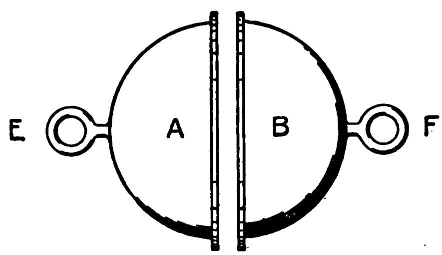

FIG. 79. — Magdeburg Hemispheres.

position, thus demonstrating that the pressure of the atmosphere is equal in all directions.

THE EXPANSIBILITY OF GASES

120. We have seen that that property of a gas by which it is distinguished from a liquid is that it always tends to expand until it completely fills the vessel in which it is placed. If two vessels, the one of which contains a gas at a definite pressure and the other a vacuum, are placed in communication by means of a pipe, a portion of the gas in the first vessel will immediately expand into and completely fill the second vessel. If the portion which fills this second vessel is removed, the first vessel will again give up a portion of its contents and the second vessel again become filled with the gas. This fact demonstrates the expansibility of gases.

THE COMPRESSIBILITY OF GASES

121. It is possible to pump a gas into a vessel which is already filled with gas at a definite pressure. For example, in "pumping up" a bicyle tire more and more air is forced into the tube until it has the desired stiffness. This fact demonstrates that gases are compressible.

BOYLE'S LAW

122. It is found that **the product of the pressure and the volume of any inclosed body of gas is a constant so long as the temperature of the gas remains unchanged.** This is known as Boyle's Law. Consider the body of air contained in the cylinder represented by *HB*, Figure 80. Let V represent the volume of this gas and P its pressure. Let it be imagined that a weight W is now placed upon the piston. The tendency of this weight is to compress the gas below the piston. Let it be assumed that the piston sinks under the action of this added

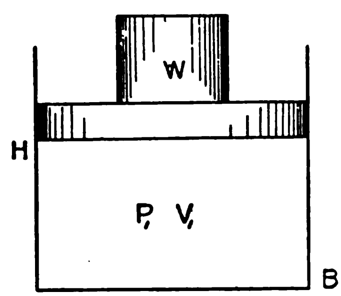

FIG. 80. — The Volume of Gas depends upon the Pressure.

volume until the new volume into which the gas is compressed is V_2 and the pressure of the gas is P_2. According to Boyle's Law

$$P_1 V_1 = P_2 \cdot V_2 \tag{45}$$

or

$$\frac{P_2}{P_1} = \frac{V_1}{V_2}$$

That is, the volumes under the two different conditions are inversely as the pressures to which the gas is subjected.

Demonstration of Boyle's Law.

Boyle's Law is very conveniently demonstrated by the following experiment. A tube represented by AC, Figure 81, having its long end A open and its short end C closed, is partly filled with mercury in such manner as to entrap in the closed end C a small quantity of dry air. Evidently the pressure to which this quantity of air is subjected is

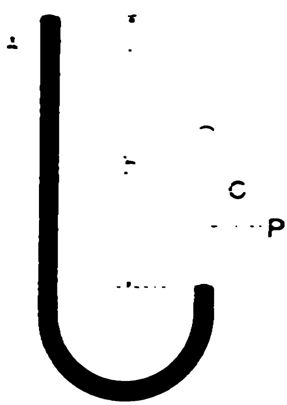

Fig. 81.—Apparatus for demonstrating Boyle's Law

$$p = hdg + B \tag{46}$$

where B stands for the pressure of the atmosphere, which is, of course, acting upon the upper end of the mercury column A, and is transmitted through the mercury to the air in C. hdg is, of course, the pressure due to the column of mercury. Let it be assumed that the height h has been measured and the volume of the air in C determined by observing the portion of the tube which it occupies. Now if more mercury is turned into the tube A, evidently the height h of the mercury column will be increased. Therefore the air in C which is subjected to this increased pressure will be still further compressed. Observing the new height of the mercury column and the new volume into which the gas was compressed, it will be found that the product of the new pressure into the new volume equals the product of the initial volume into the initial pressure.

THE MANOMETER

123. The manometer is an instrument for measuring the pressure of a gas. **The open tube manometer** is represented by AD in Figure 82. C is a vessel containing a gas the pressure of which it is desired to measure, for example, a steam boiler filled with steam. AD is a U-tube of glass partially filled with mercury and connected to the vessel C in such a manner that one end of the mercury column is subjected to the pressure p of the gas in C. The other end of the mercury column is of course exposed to atmospheric pressure. We have therefore, as in the case represented in Figure 81,

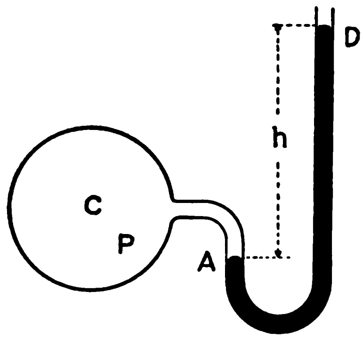

FIG. 82. — Open Tube Manometer.

$$p = hdg + B \qquad (46 \text{ bis})$$

Evidently the height of the column of mercury in the open tube manometer is a measure of the difference between the pressure in the vessel to which it is connected and the pressure of the atmosphere.

The closed tube manometer represented by AD, Figure 83, is like that shown in Figure 82, except that the upper end D is sealed and contains a small quantity of dry air. Evidently the volume of this entrapped air may be used as a measure of the pressure p_1 to which the D end of the mercury column is subjected. The pressure p in the vessel C is evidently

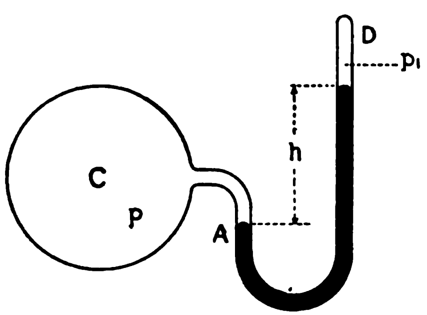

FIG. 83. — Closed Tube Manometer.

$$p = p_1 + hdg$$

Therefore, if p_1 were known, p would be determined. p_1 is determined by comparing the volume of the gas in D when subjected to the pressure

p_1 with its volume when subjected to atmospheric pressure according to Boyle's Law.

Problems

1. A column of mercury is 76 cm. high. What is the pressure at the bottom of this column due to the weight of the liquid, (*a*) in dynes per square centimeter, (*b*) in pounds per square inch?

2. What is the pressure in pounds per square inch 1000 ft. below the surface of the sea? Density of sea water = 1.03.

3. A vessel contains 20 cm. of mercury, 30 cm. of water, and 50 cm. of oil. What is the pressure in the bottom of the vessel due to the weight of the liquids? Density of mercury = 13.6, of water = 1.0, of oil = 0.8.

4. The weight of a body in air is 40 g. weight; in water, 30 g. weight. What is the specific gravity of the body?

5. A body weighed in water loses 20 g. weight; when weighed in a second liquid, it loses 50 g. weight. Find the specific gravity of the second liquid.

6. A stone has a mass of 500 g. and a volume of 90 cc. What is its apparent weight when submerged in kerosene having a density of 0.89?

7. The specific gravity of a block of wood is 0.9. What proportion of its volume will be under water when it floats?

8. What force will be required to hold a ball of iron having a mass of 500 g. submerged in mercury?

9. A block of paraffin weighs in air 800 g. A sinker in water weighs 1000 g. The paraffin block and sinker together in water weigh 800 g. What is the specific gravity of the paraffin?

10. A certain mass of gas has a volume of 50 cc. when subjected to a pressure of 15 lb. per square inch. What will be its volume when subjected to a pressure of 100 lb. per square inch?

11. The volume of an air bubble at the bottom of a pond 600 cm. deep is 0.2 cc. Find its volume just as it rises to the surface. Atmospheric pressure = 76 cm. Hg.

12. A room 10 × 10 × 10 ft. is closed when the barometer reads 29 in. What total outward pressure must each wall of this room sustain if the barometer suddenly falls to 28 in.?

FLUIDS IN MOTION

CHAPTER XI

THE LIFT PUMP

124. The most common form of pump used for elevating liquids is the **lift pump.** This form of pump is represented in Figure 84. It consists essentially of a cylinder *AB* in which moves a tight-fitting piston. From the lower end of the cylinder *AB* a pipe *BC* connects with the reservoir from which the liquid is to be lifted, for example, a well or a cistern from which water is being pumped. Two valves are provided, the one in the bottom of the cylinder at *B* and the other in the piston as indicated. Both of these valves open upward. Let it be assumed that the cylinder, and the valves are wet, but that otherwise there is no water in the pump. Then on the upstroke of the piston, the piston valve *D* being held shut by atmospheric pressure from above, a partial vacuum will be produced in the lower end of the cylinder *AB*. The pressure of the atmosphere acting upon the surface of the water in the cistern will force some of it up into the pump. As the piston descends, the pressure beneath the piston closes the valve *B* and opens the valve *D*. The piston therefore passes below

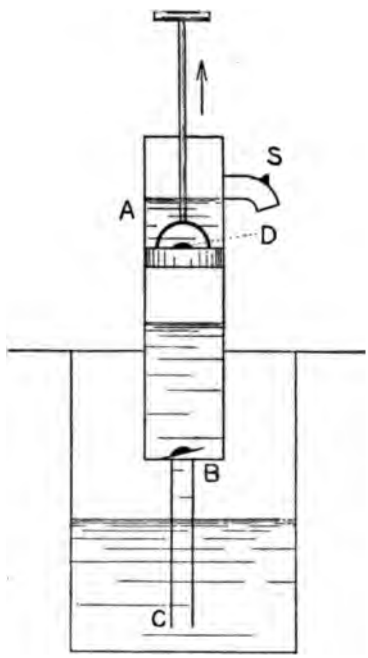

FIG. 84. — The Lift Pump.

the water which was in the lower part of the cylinder. Upon the next upstroke this water is lifted to the top of the cylinder and emptied through the spout *S*, while the lower part of the cylinder is again being filled by the pressure of the atmosphere upon the water in the cistern.

It is evident that there is a limit to the height to which a liquid can be raised by this device. Suppose, for example, the pump were being used to lift mercury. In that event the distance *BC* would of necessity be less than the height of the barometric column; otherwise the pressure of the atmosphere on the surface of the liquid in the cistern would not be able to lift the liquid to the cylinder. If the pump is being used for lifting water, the maximum distance possible between *B* and *C* would be about 13.6 times the height of the barometric column, or about 34 feet, since the density of mercury is 13.6 times that of water. The distance *BC* above referred to is the vertical distance of the cylinder from the free surface of the liquid. The practical limit of the lift pump is considerably less than 34 feet, owing to the fact that a perfect vacuum in the cylinder is never attained.

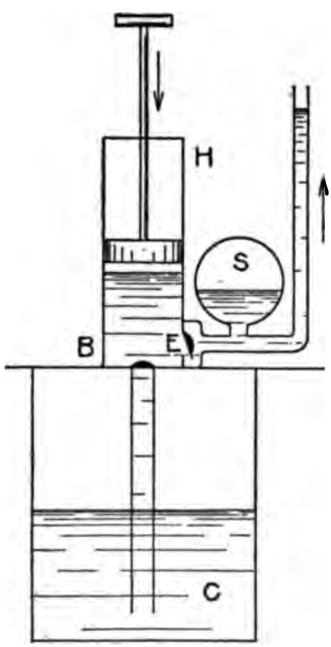

FIG. 85. — The Force Pump.

THE FORCE PUMP

125. When water is to be raised to great heights, the force pump is used. The essential parts of the force pump are the same as those of the lift pump, except that the piston is solid and the liquid is delivered through a side tube at or near the bottom of the cylinder, represented at *E* in Figure 85. This side tube *E* is provided with a valve opening outward from the cylinder which closes upon the upstroke of the piston and opens as the piston descends. The liquid

in the lower part of the cylinder is thus forced by the piston through the side opening E. The side tube E may connect with an elevated tank. Thus the liquid drawn from the cistern may be forced up into the tank, whatever its elevation may be. In fact, the only thing which limits the pressure against which the force pump will work is the strength of the pump itself and the force available to push the piston down.

The **air chamber** S, Figure 85, is used for the purpose of making the flow of water more nearly continuous than it would be if the air chamber were not connected. On the downstroke of the piston energy is stored in the compressed air in S, which is returned to the water, thus tending to maintain its motion, during the upstroke of the piston.

AIR PUMPS

126. The mechanical air pump is not essentially different from the lift pump used for liquids. If the tube CB, Figure 84, instead of dipping into a cistern of water, is connected to a vessel filled with air, upon the upstroke of the piston the air in the vessel will expand through the tube CB and a portion of it will pass into the cylinder of the pump. Upon the return stroke of the piston the portion of air will pass above the piston through the valve D, and will therefore have been pumped out of the vessel in much the same way that the water has been pumped out of the cistern in the operation of the pump as described above.

In the practical operation of an air pump of this type one of the difficulties encountered is the failure of the valves to operate. That is, after a large part of the air has been pumped from the vessel to which the pump is connected, the remaining pressure is not sufficient to lift the valve at B, even though there is a fairly good vacuum above it. In some forms of the mechanical air pump this defect is in a measure overcome by operating the valves mechanically in some such manner as the valves of an engine are operated. Another disadvantage is that there are always small spaces at the end of the cylinder into which the piston does not fit closely and from which the air cannot be driven out. This space is known as "clearance."

GERYK PUMP

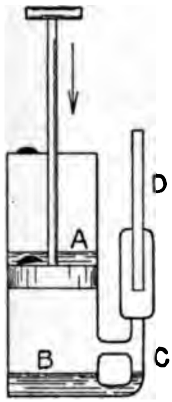

FIG. 86.—The Geryk Pump.

127. The defects of the earlier form of mechanical air pump have been largely eliminated in the Geryk pump. In this pump a layer of oil *A*, Figure 86, is carried on the piston and another *B* covers the bottom of the cylinder. The tube *D* is connected to the vessel from which the air is to be pumped. The double side tube connection *C* allows the air to flow from *D* into the cylinder and the oil *B* to flow over the top of the piston when it is at the bottom of the cylinder. The layer *A* closes and seals the piston valve, and by filling the top of the cylinder when the piston reaches the upper end of its stroke forces out practically all of the air, thus avoiding clearance effects.

THE SPRENGLE AIR PUMP

128. When it is desired to secure a very high vacuum, the mercury air pump is used. Of the numerous forms, that shown diagrammatically in Figure 87 is one of the best. *A* is a funnel filled with mercury which connects with a long vertical tube *BC*, down which the mercury tends to flow under the action of gravity. The stopcock at *B* is so turned that the mercury flows but slowly. If the tube *BC* is quite narrow, the mercury, instead of flowing in a steady stream, will pass down the tube in small globules which, acting like pistons, will carry small portions of air down the tube *BC*. Thus a vessel *D*, which is connected to the tube *BC* near its upper end, will be gradually exhausted. When *D* is exhausted, a solid column of mercury (the barometric column) will stand in the lower part of the tube *BC*. Such a device, while working slowly, is capable of producing a very high vacuum.

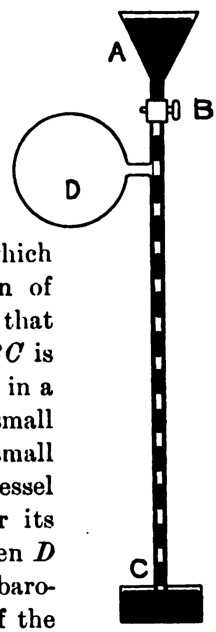

FIG. 87.—The Sprengle Air Pump.

MEASUREMENT OF VACUUM

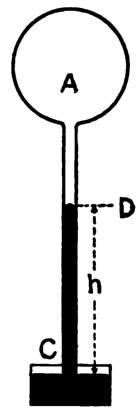

FIG. 88. — The Measure of a Vacuum.

129. Engineers often describe a vacuum by giving the **difference between the pressure of the gas and that of the outside air.** Thus, in Figure 88, the vessel A contains a partial vacuum. It has a stem DC which dips into mercury at C. The mercury rises to the point D. The engineer would say the vacuum in A is a "**vacuum of h inches,**" h being the height of the column CD which measures the difference between the pressure in A and that of the outside air.

The scientific expression for the vacuum is

$$p = B - hdg \quad \text{(Compare Equation 46).}$$

PASCAL'S LAW

130. The pressure applied at any point to a liquid in a closed vessel is transmitted undiminished to every portion of the vessel. This is known as the Law of Pascal. It has many important applications, one or two of which will be given.

One of the applications of Pascal's Law is found in the **hydraulic press,** a simple form of which is shown in Figure 89. GH represents a large cylinder with a tight-fitting piston P. IJ is a small cylinder also provided with a tight-fitting piston K, which communicates with the large cylinder GH by means of the pipe JH. The cylinder and pipe are filled with water. Let it be assumed that a force f is applied to the piston K. This will give rise to a pressure p, equal to $\frac{f}{a}$, where a is the area of the piston K. This pressure is transmitted to all parts of the communicating vessels. Therefore, the total upward force on the piston P would be given by

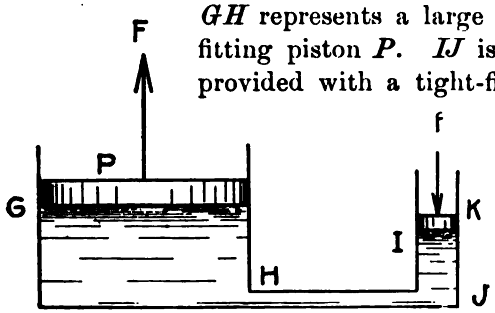

FIG. 89. — The Hydraulic Press.

$$F = p \cdot A$$

in which A is written for the area of the large piston P. That is,

$$F = \frac{f}{a} \cdot A$$

or

$$F = \frac{A}{a} \cdot f \tag{47}$$

Let it be assumed that the area of piston p is 1000 times the area of piston K, *i.e.*

$$\frac{A}{a} = 1000$$

$$\therefore F = 1000 \times f$$

That is to say, the application of the force f at K will give rise to a force 1000 times as great, tending to lift the piston P. The hydraulic press may therefore be used for lifting large weights, or for any other operation in which large forces are required. The mechanical advantage (Section 76) is given by the ratio of the piston areas. Thus in the example given above the theoretical mechanical advantage is 1000.

In the practical form of the hydraulic press a force pump similar to that represented in Figure 85 is made use of in place of the cylinder EF as described above.

THE HYDRAULIC ELEVATOR

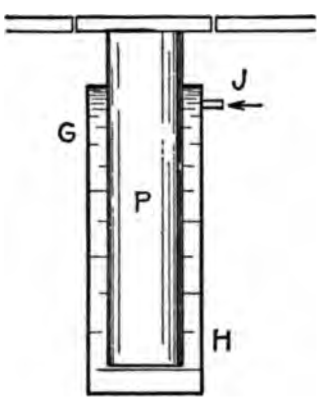

FIG. 90. — The Hydraulic Elevator.

131. Evidently, if the cylinder GH were sufficiently high the arrangement shown in Figure 89 might be used as an elevator, say, for transferring goods from one floor to another in a warehouse. This use of the apparatus as described is quite common, the cylinder GH and its piston P being arranged in some such manner as that represented in Figure 90. If the elevator is to be operated from the ground floor, a deep hole is dug

in the ground to accommodate the long cylinder *GH*. The piston *P* is a cylinder of metal which is more or less completely immersed in the water in cylinder *GH*, depending upon the position of the elevator platform. The water for operating the hydraulic elevator is pumped in through some conveniently located pipe *J*. In some hydraulic elevators the car or cage is attached to the piston *P* by means of an inverted set of fixed and loose pulleys (Figure 53), and the necessity of having a cylinder in length equal to the total distance through which the elevator travels is obviated. This arrangement possesses another advantage in that it is possible to secure a much more rapid motion of the elevator car than the piston of the press possesses.

PASCAL'S LAW AS APPLIED TO GASES

132. The law of Pascal applies to gases as well as to liquids, providing the transmitted pressures are measured after the compressed gas has come to rest. For example, referring again to Figure 89, the space *GHIJ* below the pistons might contain air instead of a liquid. If, under these circumstances, the piston *K* is forced downward by an external force action, the compression which the force *f* brings about just beneath the piston *K* equalizes itself throughout the entire system of communicating vessels; and when this pressure has become equalized, the upward pressure on *P* is exactly the same as that beneath the piston *K*.

HYDRAULIC TRANSMISSION OF POWER

133. By application of the above principle, power may be transmitted to a distance. Consider a long pipe filled with water connected at one end to a force pump and at the other to a hydraulic engine. When the pump is operated, the engine at the other end of the pipe will be fed with water under pressure very much as a steam engine is fed with steam from a boiler. In this manner power may be transmitted from the pump to the hydraulic engine.

TRANSMISSION OF POWER BY COMPRESSED AIR

134. The transmission of power by compressed air is extensively employed in mines and factories. The air compressor

maintains the pressure in the mains which are connected to engines similar to steam engines, which may be located at any distance from the compressor.

THE SIPHON

135. The siphon is a bent tube which is used for carrying a liquid from a higher to a lower level over some intervening obstacle. If, for example, it is desired to transfer the liquid in the vessel A to the vessel B, Figure 91, this may be accomplished by means of a bent tube C placed in the position shown in the figure. Assuming that the tube C is filled with the liquid, its action may be understood from the following discussion: Consider a portion of the liquid at the highest part C of the tube. This experiences **a pressure urging it toward the right which is equal to the pressure acting on the surface of the liquid in the vessel A minus hdg,** in which h is the height of this portion of the tube above the level of the liquid in A. This same portion of **liquid at C is urged toward the left by a pressure which is equal to the pressure acting upon the surface of the liquid in B minus h_1dg** in which h_1 is the height of C above the free surface of the liquid in the vessel B. If the pressure P acting upon the surface of the liquid in A is the same as that which acts upon the surface of the liquid in B, we have

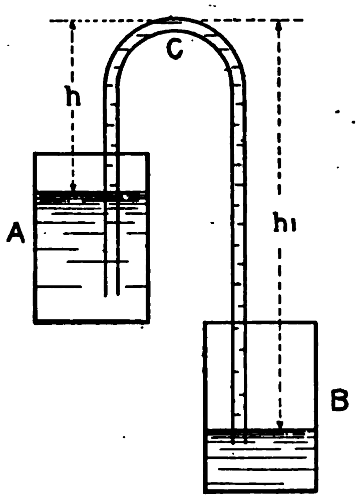

FIG. 91. — The Siphon.

$$p_1 = P - hdg$$
$$p_2 = P - h_1dg$$

where **p_1 stands for the pressure urging the C portion of the liquid to the right and p_2 for the pressure urging this same portion of liquid toward the left.** We have, therefore,

$$p_1 - p_2 = (h_1 - h)dg$$

where $p_1 - p_2$ is the excess of pressure acting to move the portion C of the liquid in the tube toward the right. Thus at all times there will be an unbalanced pressure $(h_1 - h)dg$, urging that part of the liquid which is at the top of the tube toward the right. In other words, so long as h_1 is greater than h, there will be a flow of liquid from the vessel A to the vessel B. Evidently, when the liquid comes to the same level in both vessels so that h_1 is equal to h, the flow will cease. Furthermore, in the event of h_1 becoming less than h, the liquid will flow from the vessel B into the vessel A.

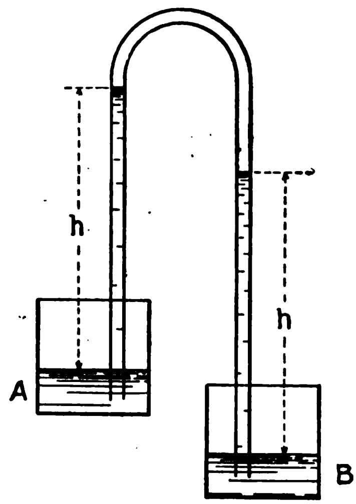

FIG. 92. — The Siphon fails if $hdg = P$.

The siphon cannot be used in case the distance h is greater than the height of the barometric column. In this case the condition of affairs would be as represented in Figure 92, in which the liquid is represented as standing at the height of the barometric column in each of the vertical portions of the siphon, there being, of course, in the C portion of the tube a vacuum. Thus, for mercury, the limiting value of h is 74 centimeters; for water about 34 feet.

THE FLOW OF LIQUIDS

136. In any case of motion in a liquid, force action must be present to account for that motion. If the velocity of the liquid is changing, there is a force acting which is doing work in accelerating the mass of the liquid which is moving. If the liquid is moving with uniform velocity, there must be present a force sufficient to overcome the frictional resistance to flow encountered by the liquid. We may, therefore, conclude that liquids move only under the action of sufficient force. A more convenient way of stating the same thing is that **a given portion of liquid will be set in motion when it is acted upon by un-**

balanced pressure. Thus, for example, water flows in a pipe only when there is a difference of pressure between the two ends of the pipe. If the pressures at the two ends of the pipe are equal, their tendencies to move water in the pipe neutralize one another, and the water under the combined influence of the two pressures remains at rest.

EFFLUX. TORRICELLI'S THEOREM

137. The velocity with which a liquid will escape through an opening in the side of a vessel when acted upon by the weight of the liquid alone is given by the following formula:

$$v = \sqrt{2\,gh}$$

in which v is the velocity of the escaping liquid, g is the acceleration of gravity, and h is the height of the free surface of the liquid above the opening through which the liquid is escaping. This is known as the theorem of Torricelli. It may be demonstrated in the following manner. Referring to Figure 93, let AB represent a vessel filled with a liquid and having a narrow opening at B through which the liquid escapes as indicated. Let us call the height of the free surface of the liquid above the orifice h. This distance is commonly referred to as the **head.** Consider that which takes place with reference to the energy of the system when a portion of the liquid escapes through the orifice B. Let it be assumed that in the interval considered the level falls from A to A_1. It will be evident that every layer of liquid in the vessel will have fallen through precisely this distance AA_1. Thus **the body of liquid has lost an amount of potential energy which is equal to the total weight of the liquid contained in the vessel, multiplied by the height** AA_1. But this is equivalent to the potential energy which would be lost by the layer AA_1 in falling from the free surface

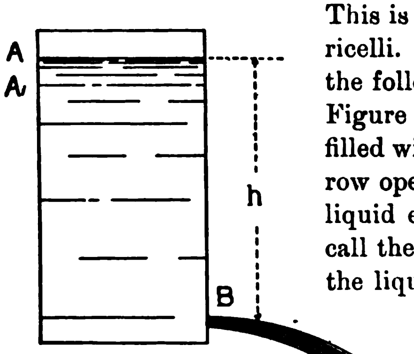

FIG. 93. — The Liquid escapes at B with a Velocity $= \sqrt{2\,gh}$.

of the liquid to the orifice B. We have, therefore, for the potential energy lost by the liquid

$$E_p = mgh$$

in which m stands for the mass of the liquid in the layer AA_1. On the theory of the conservation of energy, the kinetic energy of the liquid which escaped in the interval under consideration must be equal to this loss of potential energy by the layer AA_1, but since the mass of liquid which has escaped is the same as that of the layer AA_1 we have, for the kinetic energy of the liquid which escaped,

$$E_k = \tfrac{1}{2} mv^2$$

in which v is the velocity of efflux, that is, the velocity with which the liquid escapes from the orifice. Equating these two energy expressions, we have

$$\tfrac{1}{2} mv^2 = mgh$$

or

$$v = \sqrt{2gh} \qquad (48)$$

It will be noted that this expression is the same as that for the velocity of a body which has fallen freely through the distance h, under the action of gravity. (Compare Equation 9.)

FRICTION HEAD

138. The loss of effective head due to the friction effect in a pipe is called the **friction head**. A simple example will make the meaning of this expression clear. In Figure 94, A represents a tank filled with water. Communicating with this tank is a narrow pipe having three orifices B, C,

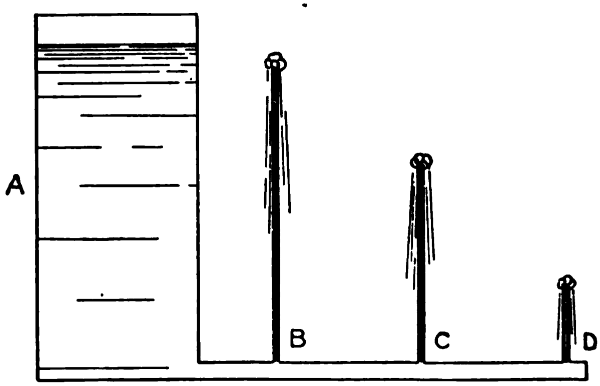

FIG. 94. — Friction Head.

and D on the upper side as indicated. The water will spout through the opening B to a height which is nearly equal to the distance of the orifice from the free surface of the liquid. From the orifice C it spouts to a distance considerably less. Thus, while the orifices B and C are apparently subjected to the same head of water, the **effective head** is different. This is explained by saying that a part of the head (pressure) is used in overcoming the friction in the pipe between the orifices B and C. That this loss of head between the points B and C is necessarily present, is evident from the general statements which have been made above with reference to the flow of liquids. **If there were no difference in pressure between the points B and C there would be no flow of liquid between these two points.** But if a certain amount of pressure is used up in this way, evidently a portion only of the original pressure will be available at the orifice C.

EFFLUX FROM AIR-TIGHT SPACES

139. In the discussion given in Section 137, the assumption was made that pressure on the upper surface of the liquid in the vessel AB, Figure 93, remains constant and that the liquid escapes into a region B the pressure of which is the same as that at A. Evidently the velocity with which the liquid escapes under these assumptions depends upon the head of the liquid alone and is independent of the pressure above referred to. If, however, the pressures at A and B are different, this difference in pressures must be taken account of in the discussion of the efflux. It is conceivable that the pressure at B acting from without might be larger than the pressure acting upon the surface A by just that amount (hdg), which is due to the weight of the liquid. Under these circumstances there would be no flow of the liquid through the orifice. This condition of affairs is actually reached in case the efflux takes place from an air-tight vessel. Consider the flow of liquid from the vessel represented in Figure 95. The conditions are supposed to be the same as those represented in Figure 93, except that the vessel is closed at the top, the closed space above the liquid containing air at a pressure p_1. Let it be assumed, to begin with, p_1 is

equal to p_2, the pressure on the outside at the orifice. Then the velocity with which the liquid **begins** to flow through the orifice is given by Equation (48). As soon, however, as an appreciable amount of liquid has passed the orifice, p_1 becomes less than p_2, since the air inclosed must expand to fill that space which is emptied by the liquid which flows out of the vessel. Expanding into the larger volume this air will, according to Boyle's Law, have a lower pressure. To determine whether or not under these circumstances the liquid will actually flow from the orifice, we have the following considerations. The pressure acting upon a given body of the liquid at the orifice, which tends to move that volume to the right, is $p_1 + hdg$. The pressure acting upon the same body of liquid which tends to move it toward the left through the orifice is p_2. Evidently, therefore, when

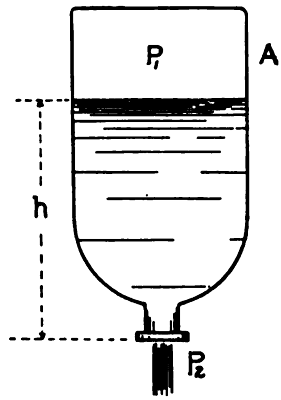

FIG. 95. — Intermittent Efflux from Air-tight Space.

$$p_1 + hdg = p_2$$

the liquid at the orifice will have no tendency to flow in either direction. In case

$$p_1 + hdg > p_2$$

there is an unbalanced pressure urging the liquid at the orifice toward the right. In case

$$p_1 + hdg < p_2$$

the liquid in the orifice will move toward the left and air will pass into the vessel from the outside.

Under the conditions assumed, therefore, the liquid will escape from the vessel **at first** with a velocity $v = \sqrt{2gh}$. Afterwards v will gradually become less, finally reaching the value zero. It usually happens, however, that the kinetic energy of the liquid which is flowing towards the orifice carries the efflux beyond this point, so that

$$p_1 + hdg < p_2$$

Hence, when the stream stops, air will pass into the vessel. This air will increase p_1 so that again

$$p_1 + hdg > p_2$$

and efflux will once more begin, and so on. It is evident that if it is desired to secure an uninterrupted and steady flow of the liquid from the orifice, it will be necessary to provide some means for maintaining p_1 constant. This is conveniently done by making a small hole in the top of the vessel as at A, through which the air may be allowed to flow in as the liquid escapes through the orifice.

THE HYDRAULIC RAM

140. According to Newton's second law of motion any body of mass m having acceleration a is necessarily acted upon by a force $f = ma$. If the body is increasing in velocity, the force producing the acceleration is in the same direction as the velocity. If the velocity of the body is decreasing, the force which acts to give the body negative acceleration, that is to say, which tends to stop the body, is acting in a direction opposite to that of the velocity of the moving body. This statement will of course hold for liquids equally as well as for solids. Therefore a quantity of liquid of mass m moving with a velocity v which is suddenly brought to rest must be acted upon by a force which is given by the product of the mass of the liquid stopped and the acceleration which it experiences. Since action and reaction are equal, we may say that the liquid in stopping exerts upon the restraining vessel a force action or pressure which is proportional to the mass of the moving liquid and the acceleration which it has while stopping.

This effect is taken advantage of in the "hydraulic ram," the operation of which will be understood by reference to Figure 96. A is a reservoir containing water; BC is a pipe of large dimensions which leads from the reservoir to the point C. Evidently if the conditions were as represented in the diagram, the liquid in the reservoir would flow along the pipe BC, escaping past the valve D. Imagine that D is so adjusted that when the liquid acquires a sufficient velocity it will carry the valve along with it, that is to say, it will lift the valve and close the

opening at *D*. This, of course, brings the column of liquid *BC* suddenly to rest. This will give rise to the "water hammer" or "water ram" effect referred to above, that is to say, as the liquid is suddenly checked in its motion it will exert a large pressure upon the walls of the pipe at *DC*. Imagine that a vertical tube *EF* is attached as indicated. This tube being enlarged at *E* is provided with a valve opening upward. At the moment at which the water ram occurs a small portion of the liquid in *DC* will be forced up into *EF*. This will take place even though the head of water in the pipe *EF* is greater than that in the pipe *BC*. As soon as the flow of water in the pipe *BC* is checked the valve *D* will open by its own weight and the operation will be repeated. Each time this operation is repeated a portion of the liquid which flows from the reservoir *A* along the pipe *BC* is lifted to the reservoir *G*. Considered from the standpoint of the theory of the conservation of energy it will be understood that the quantity of water elevated in the pipe *EF* must be smaller than the quantity which flows along the pipe *BC*. If w represents the weight of water raised through the pipe *EF*, h represents the height through which it is raised, and W is taken to represent the weight of water which flows along the pipe *BC*, and H the distance through which it falls, we have, evidently,

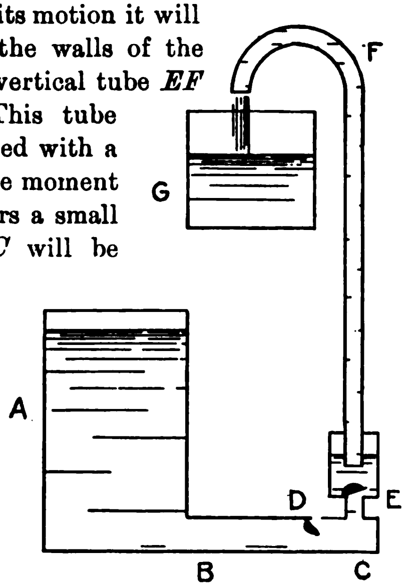

Fig. 96. — Illustrating the Action of the Hydraulic Ram.

$$wh < WH$$

wh is the potential energy of the raised water, WH is the potential energy of the water before it flows along the pipe *BC*. And since some energy is used in overcoming friction, etc., wh can never quite equal WH.

THE DIMINUTION OF PRESSURE IN REGIONS OF HIGH VELOCITY

141. In a stream of moving fluid **the pressure is greatest in the wide portions of the stream where the velocity is least, and is least in the narrow portions of the stream where the velocity is greatest**; that is to say, the narrow portions of any stream of fluid correspond to regions of relatively low pressure. That this must be the case will be evident from the following consideration. Consider a pipe *AB* which is wide in the portions *A* and *B* (Figure 97), and narrow at the part *C*. Let it be assumed that a stream of air is flowing through this tube. It will be evident that the quantity of air which passes the narrow part *C* of the tube per unit time must be equal to the quantity which passes the wide part *A*. If this were not the case, air would accumulate in larger and larger quantities between the points *A* and *C*. If the quantity of fluid passes each of the sections *A*, *B*, and *C*, it will move with greater velocity through the narrow part of the tube at *C* than it will in passing the wide parts of the tube at *A* and *B*. Thus *C* is a region of high velocity in the moving stream; *A* and *B* are regions of low velocity, therefore **the kinetic energy of the moving stream at *C* is greater than at *A* or at *B*.**

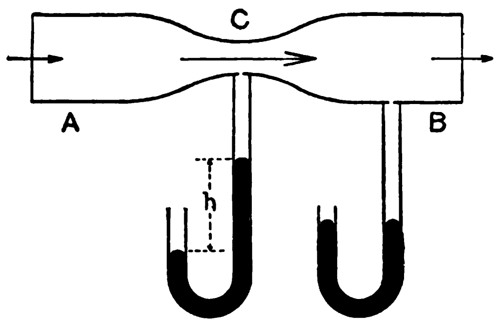

FIG. 97.—The Narrow Part of the Tube is a Region of Low Pressure.

It follows, therefore, that **the fluid has accelerated motion as it passes from *A* to *C* and again as it passes from *C* to *B*. But this acceleration can only take place under unbalanced pressure.** Therefore the pressure at *A* must be greater than the pressure at *C* (velocity of fluid increasing), and the pressure at *C* must be less than that at *B* (velocity of fluid decreasing).

That the pressure at *C* is less than the pressure at *B* may be demonstrated by attaching open tube manometers (Section 123)

as indicated in Figure 97. It will be found that the liquid in the branches of the manometer at C will stand at different levels as shown, thus indicating that the pressure at C is less than that of the atmosphere, while those in the manometer at B will remain practically at the same level, the tube AB at this end being assumed to be open to atmospheric pressure.

Various phenomena are explained by the application of this principle of the **diminution of pressure in the contracted portion of a stream.** In Figure 98 (a) is shown a device which is used for illustrating one of them. It consists of a vertical tube AB terminating in a flat disk B. Just below this is placed a second disk of paper or thin metal at C. If now a stream of air flows down the tube from A to B, it will escape laterally in a thin stream between the two disks as indicated by the arrows. Since the stream of air is narrow in this region, the velocity will be high and the pressure will be correspondingly low. A portion of the atmospheric pressure which acts on the disk C from below will be unbalanced and the disk C will be held firmly against the disk B.

Another illustration of the same effect is that afforded by the "**ball nozzle**" represented in Figure 98 (b). It consists of a tube DE having a conical nozzle F within which is placed a ball G as indicated in the figure. A stream of air or other fluid passing down the tube DE escapes in a thin stream around the ball G. This thin portion of the stream is accordingly a region of diminished pressure, a portion of the atmospheric pressure acting upon the ball from the lower side is therefore unbalanced and tends to hold the ball firmly against the nozzle.

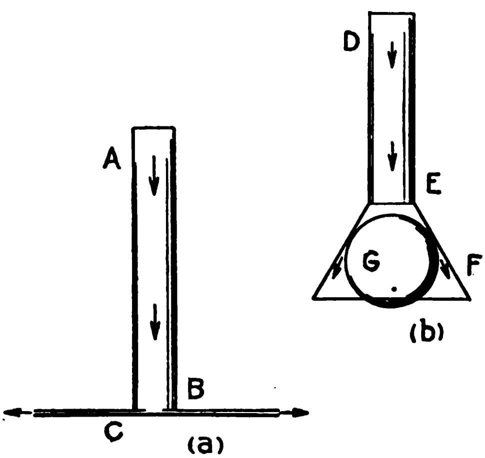

FIG. 98. — Principle of the Ball Nozzle.

Another example is afforded by the **ball and jet.** Consider the arrangement shown in Figure 99. A is a nozzle from which a small stream of fluid, let us say air, is escaping at high velocity. B is a light ball which is in contact with the jet as shown. Under these conditions the ball B will remain at rest, that is to say, the forces acting upon it will be in equilibrium. There are three forces present, as follows: first, the weight of the ball represented by W; second, the pressure of the stream against the ball due to the impact of the particles of fluid against the ball represented by I; third, an unbalanced portion P of the atmospheric pressure acting from the right as indicated. This unbalanced part of the atmospheric pressure arises from the diminution of pressure on the left hand side of the ball due to the velocity of the stream in that vicinity. It can be seen that these three forces may be in equilibrium. Their relative values are represented by the sides of the vector triangle C. A stream of water may be substituted for the stream of air with similar results.

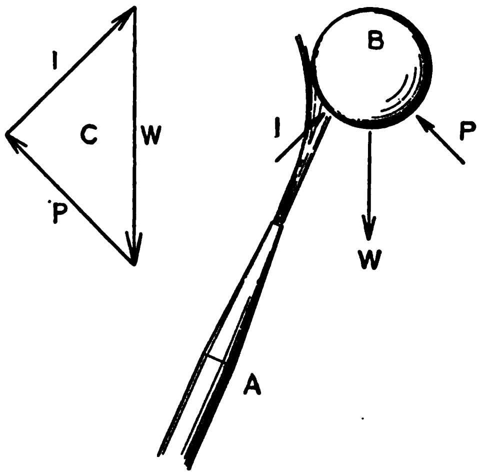

FIG. 99. — The Ball and Jet.

EXPLANATION OF THE CURVING OF A TENNIS BALL OR BASEBALL

142. If a ball is projected forward in such manner that it has in addition to its motion of translation a motion of rotation about an axis at right angles to its motion of translation, then, instead of moving along the path followed by an ordinary projectile, it will curve up or down, to the right or left, according

to the direction of the axis about which it is rotating. Consider the case shown at *a* in Figure 100. *B* represents a ball moving forward in a straight line with a velocity *V* and at the same time rotating in such manner that its surface is moving with a velocity *v* as indicated by the curved arrow. So far as any tendency to move sidewise is concerned, the conditions assumed are exactly equivalent to those represented in Figure 100 (*b*), in which the ball is assumed to be at rest so far as linear motion is concerned, possessing only the rotatory motion represented by the curved arrow, and having a stream of air moving from left to right past the ball with a velocity *V* as indicated. Evidently the velocities *V* and *v* are in the same direction at the upper side of the ball and in opposite directions at the lower side of the ball. This results in a larger diminution of pressure at the upper side of the ball than at the lower. There will therefore be, under the assumed conditions, an **unbalanced pressure acting upon the ball from below** which will tend to move it in an upward direction, or what is the same thing, referring again to *a*, Figure 100, the ball *B* as it moves forward will "curve" as indicated by the dotted line.

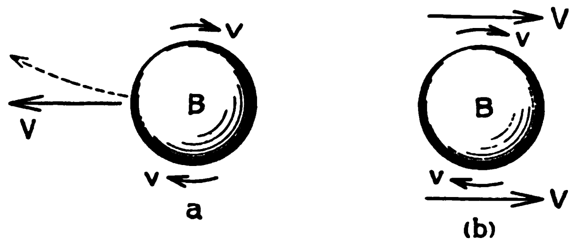

FIG. 100. — Curving of a Tennis Ball.

Problems

1. The piston areas of an hydraulic press are 1 sq. ft. and ⅕ sq. in. The force acting on the small piston is 20 lb. weight. What force acting on the large piston will maintain equilibrium?

2. A cylindrical boiler shell is 3 ft. in diameter and 10 ft. long. The pressure of the steam as measured by an *open tube* manometer is 50 lb./sq. in. What is the total force tending to push out one end of the boiler?

3. In the cylindrical shell of the boiler of problem 2 there are two rows of rivets diametrically opposite to one another. The rivets are 3 in. apart. What is the shearing force on each rivet?

4. Two Magdeburg hemispheres, having an inside diameter of 10 centimeters, are carefully fitted together and the air within exhausted to a pressure of 10 cm. of mercury. If the barometer reads 76 cm. of mercury, what force would be required to separate the hemispheres?

5. What is the limiting height of an obstacle over which mercury can be siphoned?

6. The bend of a siphon is 10 ft. above the surface of the water in the vessel being emptied. What is the internal pressure at the bend?

7. What is the limiting height over which water can be siphoned in a region where the barometric pressure is 40 cm. of mercury?

8. With what velocity will water escape from an orifice 10 ft. below the surface of the liquid in a thin-walled tank?

9. What will be the head of water in the tank of problem 8 when the velocity of efflux has decreased in the ratio of 4 to 3?

SURFACE TENSION

CHAPTER XII

MOLECULAR FORCE ACTION

143. The adjacent layers of molecules in any body are held together by a mutual attraction which is called **cohesion**. To break a given body, or to tear adjacent portions asunder, requires of course the overcoming of these forces of cohesion. We may convince ourselves that these cohesive force actions are large by attempting to pull an object to pieces. Different substances exhibit these cohesive force actions in different degrees, but in all cases we find the forces to be relatively large.

Once an object has been broken in two it may be mended by putting the pieces together in such manner that they come into the same intimate contact which existed before they were broken asunder. Whenever this can be done it is found that the broken object is mended and is quite as strong at the place at which the break occurred as it was before it was broken. The reason that we find it difficult to mend objects in this way is that in the usual case it is quite impossible to put the pieces back into intimate contact with one another. There are various examples, however, of this method of making of two pieces one solid whole. In the filling of teeth, pieces of gold are hammered together and are thus brought into intimate contact. They become "welded" together and form one solid body. This operation is found to be readily possible with gold because the surface of the pieces used are easily kept clean, and the metal being very malleable, they are easily driven or pressed into intimate contact.

The **welding** of two pieces of iron is accomplished by first softening the pieces of iron by heating them. They are then covered with a "flux," the object of which is to prevent the hot iron from oxidizing by contact with the air, since a layer of

oxide on the surfaces of the pieces of iron would prevent their being brought sufficiently close together. When hammered together, they are brought into intimate contact, the cohesive force actions come into play, and the two pieces of iron become as one.

The molecular force action between unlike molecules is called **adhesion**. Thus pitch will adhere to glass, glue will adhere to wood, and so on. The forces of adhesion may oftentimes be as great or greater than the forces of cohesion within the adhering substances. A good example of this is the following: If a piece of plate glass is carefully cleaned and covered with a layer of fish glue, as the glue dries and shrinks it will tear bits of glass from the surface of the glass plate. Evidently in this case the adhesion between the glue and the glass is greater than the cohesion within the glass.

MOLECULAR FORCES ON "INNER" AND ON SURFACE MOLECULES

144. From the above discussion it would appear that two molecules must be brought into quite intimate contact before the force actions referred to come into play. The question very naturally arises as to what distance they may be separated and still have this molecular force action present. This is found to be extremely small. For our present purposes it is sufficient to say that there is a maximum distance for two given molecules beyond which they must not be separated, if they are to continue to exert this cohesive action upon one another. Thus we may say that about each molecule in any body there is a **sphere of molecular action**, an imaginary sphere drawn about the molecule as a center and of such dimensions that all molecules within the sphere are acted upon by the molecule at the center. All molecules outside the sphere are beyond the range of this molecular force action from the molecule at the center. It will be apparent, therefore, that molecules which are in the center of a body, let us for the sake of brevity call them "inner" molecules, are conditioned in quite a different manner from those which are on the surface of a body.

Referring to Figure 101, let AB represent a vessel containing a liquid. Consider the particles a and b. The circles are

drawn to represent the spheres of molecular action. The arrows within these circles represent the forces with which the various particles within the sphere act upon the center particles. It will be evident from symmetry that the forces acting upon the particle *A* will be in equilibrium so that *a* is urged equally in all directions and experiences no unbalanced molecular force action.

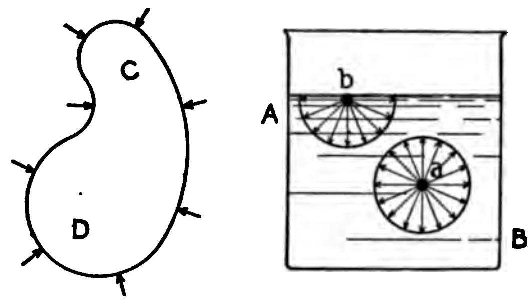

FIG. 101. — The Sphere of Molecular Action.

On the other hand, the *b* particle is acted upon by an unbalanced molecular force action urging it downward, that is, toward the body of the liquid on the surface of which it is supposed to be situated. This resultant downward force is the vector sum of all of the molecular forces in the hemisphere below it, as indicated in the figure. We thus arrive at the conclusion that a **surface particle** on any portion of matter experiences a resultant force action which tends to draw it in toward the body of that portion of matter on the surface of which it is found. See *CD*, Figure 101.

SURFACE TENSION

145. Since all surface particles on any given portion of matter experience these resultant molecular force actions tending in a general way toward the center of the body, it is evident that there is a general tendency of the surface to contract as far as possible. Another statement of the case would be that there is a tendency for the portion of matter referred to to be molded by these molecular force actions into such form that the surface is as small as possible consistent with the volume contained. Since the sphere has the smallest surface for a given volume of any of the geometrical forms, it is clear that under the molding action referred to, all portions of matter tend to assume the spherical form.

A fluid surface acts therefore like a stretched elastic membrane. This effect is called surface tension. The numerical value of the **surface tension is the force required to balance the tendency to contract in a strip of surface film one centimeter in width.**

It quite often happens that other and greater force actions are present which partially, if not completely, mask this molding tendency of the molecular forces. Thus a cupful of water turned out upon the floor, instead of assuming this spherical form as it would do if these molecular forces were the only forces acting upon it, spreads out into a thin horizontal layer under the influence of the weight of the water, which is of course a large force as compared with those under discussion. If, however, instead of a cupful of water a small drop is placed upon the floor, the surface of which is dusty or oily, so that adhesion between the drop of water and the floor is prevented, it will be found that the drop of water assumes a form which is very approximately that of the sphere. Thus drops of dew which gather upon leaves of plants, and drops of rain which fall on a dusty roadway, are found to take on the spherical form. This explanation applies also to water which falls slowly from a small opening in a vessel in the form of drops. These upon examination will be found to be more or less nearly spherical except as they are distorted by external force actions.

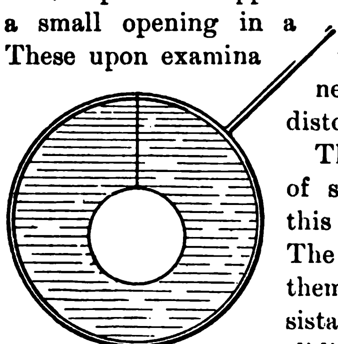

FIG. 102. — Contracting Soap Film.

The molecules in the surface layers of solids experience in the same way this unbalanced molecular force action. The solid, however, is not molded by them because of its rigidity or the resistance which the molecules offer to sliding upon one another. If the solid is softened, the molding action of surface tension immediately comes into play. Thus when the end of a glass rod is placed in the flame of a Bunsen burner, it becomes rounded, all sharp corners disappearing, indicating that this molding action is present.

A beautiful illustration of the tendency of a film to contract is the following: A soap film is placed upon a wire ring to

which a silk thread is attached in such manner as to form a loop resting upon the film. If the film inside the loop is broken, the loop is at once drawn into a circular form, as shown in Figure 102. The film outside the loop in contracting to the least area possible under the limiting conditions extends the loop so as to include the largest area possible under the limiting condition of the fixed length of string. This largest possible area is attained, of course, when the inclosed area is circular.

CAPILLARITY

146. The effects of these unbalanced molecular forces in the surface layers of bodies give rise to various phenomena, among which is that of the rising of a liquid within a small tube the walls of which are wetted by the liquid. This phenomenon, known as **capillarity**, is explained in the following manner: Let AB, Figure 103, represent a narrow tube the lower end of which dips into a liquid which wets the tube. Under these conditions the liquid will rise in the tube as indicated in the figure. Consider the surface film at the top of the column. Since the liquid wets the walls of the tube, this surface film will have a form something like that of a glove finger. If the radius of the tube is r, the width of film measured around the tube is $2\pi r$.

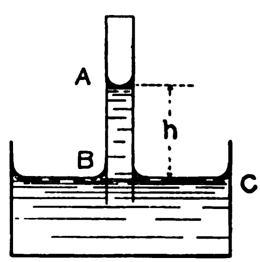

FIG. 103. — Elevation of a Liquid in a Narrow Tube.

Therefore the total upward pull due to the surface tension in this film is $2\pi rT$, in which T is the surface tension. The weight of the column of liquid is $\pi r^2 hdg$, in which h is the height of the column and d the density of the liquid. Since the tension in the film is balanced by the weight of the column, therefore,

$$2\pi rT = \pi r^2 hdg$$

or

$$h = \frac{2T}{rdg} \tag{49}$$

which gives the height to which the liquid will rise, in terms of the surface tension and density of the liquid and the radius of the tube.

The tendency of the liquid to rise against the wall of the

containing vessel is to be explained in the same way. When the film stretches across the angle between the wall and surface of liquid, a certain amount of water is lifted and supported by the film. The liquid rises under the film until the weight of water supported is sufficient to balance the tendency of the film to contract.

In case the tube or wall of the vessel is not wetted by the liquid, there will be a depression instead of an elevation of the liquid. The forces are in equilibrium when the upward pressure of the depressed liquid balances the tendency of the film to contract.

THE DIFFERENCE OF PRESSURE BETWEEN THE TWO SIDES OF A CURVED SURFACE FILM

147. Consider the difference of pressure between the inside and the outside of a soap bubble. Call this difference of pressure p. Let it be required to find an expression for the balance of force actions on a hemispherical soap bubble on the surface of a soap solution (see Figure 104). Evidently the surface tension in the film at the surface of the solution resists the separation of the bubble from the solution, which the pressure within the bubble tends to bring about. These force actions are numerically equal, since they are oppositely directed and their combined effect is zero; that is, the bubble remains stationary under their joint action. If the surface tension per centimeter along the line $CDEF$ is T, evidently the molecular force action for a single film is $2\pi rT$, and since the soap film has two surfaces, the total molecular force action is $4\pi rT$. r is the radius of the bubble. The total force tending to separate the bubble from the liquid is $\pi r^2 p$, since this is the total force with which the pressure p acts upon the surface of the liquid. Equating these two force actions, we have

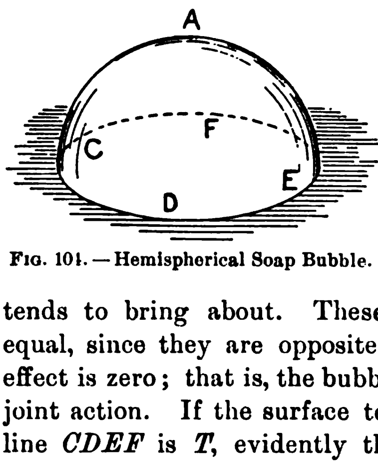

FIG. 104. — Hemispherical Soap Bubble.

$$4\pi rT = \pi r^2 p$$

or

$$p = \frac{4T}{r} \qquad (50)$$

That is, the excess of pressure on the inside of the spherical film varies inversely as the radius of that spherical surface.

Consider the case of a cylindrical surface. Let *ABCD*, Figure 105, represent a portion of a cylindrical soap bubble. The force actions on a strip *BC* 1 centimeter in width, are as follows: a force *T* acting downward at each end of the strip giving a total downward force of 2 *T*, or for a double film 4 *T*, and an upward force action due to the excess of pressure on the inside of the film of $2rp$, since the area of the surface under the strip $= 2r$.

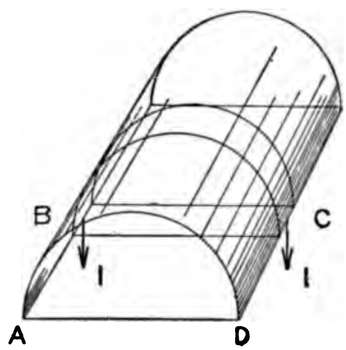

FIG. 105. — Cylindrical Soap Bubble.

Therefore, we have

$$4T = 2rp$$

or

$$p = \frac{2T}{r} \qquad (51)$$

Comparing this result with that obtained above, it will be seen that the excess of pressure inside a spherical film is twice that of a cylindrical film of the same radius.

THE MEASUREMENT OF SURFACE TENSION

148. A convenient way to measure surface tension is by means of the apparatus shown in Figure 103 and Equation (49). Solving this equation for *T*, we have

$$T = \frac{rhdg}{2}$$

Therefore, knowing the radius of the tube and the density of the liquid, the height *h* to which the liquid rises in the tube may be observed and the equation above solved for the value of *T*.

BEHAVIOR OF OIL ON WATER

149. A drop of kerosene placed on water will spread indefinitely, owing to the fact that the surface tension in the water acting on the edge of the drop of oil is greater than the surface tension of the oil.

EFFECT OF TEMPERATURE ON SURFACE TENSION

150. Heat reduces the surface tension effect. A layer of grease on a hot plate is drawn away from the hotter portions because of the diminished surface tension at these points. A drop of grease, which spreads over a large area of water when the water is cold, will contract when the temperature of the water is raised. This is due to the fact that the surface tension of water diminishes more rapidly than the surface tension of oil with a rise of temperature. This effect is illustrated by the behavior of drops of grease on hot soup, drops of oil on hot coffee, etc.

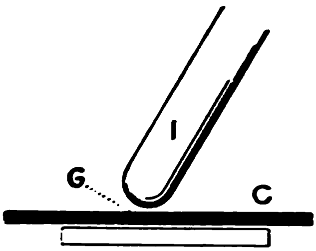

FIG. 106.—Removal of Grease Spot by Hot Iron.

The removal of grease from cloth by means of a hot iron affords a good example of this effect. Let C, Figure 106, represent a piece of cloth containing a grease spot G. If a hot iron I is brought near, the surface tension of the grease will be reduced on the heated side, and the unbalanced surface tension of the colder parts will draw the grease to the opposite side of the cloth, whence it may be removed by a blotter B.

Problems

1. How much does the pressure on the inside of a soap bubble 20 cm. in diameter exceed that on the outside? Assume surface tension is 80 dynes per centimeter.

2. Two soap bubbles are connected by means of a glass tube. The diameter of one bubble is 5 cm., that of the other is 2 cm. Which bubble will increase in size?

3. What is the pressure due to surface tension in a drop of water 2 mm. in diameter? Assume surface tension of water to be 81 dynes/cm.

4. The surface tension of pure water is 81 dynes/cm. How far will water rise in a capillary tube $\frac{1}{20}$ mm. in diameter?

5. What is the surface tension of a liquid which rises 25 cm. in a capillary tube $\frac{1}{10}$ mm. in diameter. Density of the liquid = 0.8.

6. If alcohol and water are made to form drops from the end of the same pipette, it will be found that the water drops are larger than those of alcohol. Explain.

First Semester 1914

PART II

HEAT

HEAT

CHAPTER XIII

THE NATURE OF HEAT

151. We have seen that in the operation of any machine or mechanical device a certain amount of mechanical energy is always transformed into heat; that is to say, from the friction effects, which are unavoidably present in such devices, heat is developed whenever such devices are operated. We are thus led to the conclusion that **heat is a form of energy.**

It is thought that the molecules of a hot body are in a state of rapid vibration, and that the hotter the body, the more rapid is this vibratory motion of its molecular parts. **Heat is** therefore defined as **the energy possessed by a body in virtue of the vibratory motion of its molecular parts.**

TEMPERATURE

152. The temperature of a body should be carefully distinguished from the quantity of heat which it possesses. While it is true, in many cases, that hot bodies possess relatively large amounts of heat, it is equally true that a cold body may actually contain more heat than a hotter body. The general notions which we have of temperature are derived from our temperature sense. We determine by feeling a body whether it is hot, warm, cool, or cold, and it is true that, in a general way, we are enabled by this means to measure temperature roughly. However, our temperature sense may very easily lead us into error in estimating the temperature of objects. An experiment which may be performed for the purpose of illustrating this point is the following:

Let the right hand be held for a moment in cold water, the left meanwhile being held in hot water. Then let the hands

be dipped together into a vessel of tepid water. To the hand which was in the cold water the tepid water will seem hot, while to the hand which was placed in the hot water, the tepid water will seem cold. This experiment gives us a hint as to the real significance of temperature sensation. **We experience the sensation of cold when heat passes from the body to surrounding objects. We experience the sensation of heat when heat passes from surrounding objects to the body.** Furthermore, the degree in which the sensation is felt depends upon the rapidity with which heat passes to or from the body. The various objects in a room on a cold morning will appeal to one as if some were colder than others. If the hand is placed upon a woolen blanket, and then upon a piece of wood, and then upon a piece of metal, the metal will seem colder than the wood, and the wood colder than the woolen blanket, even though the three objects are at exactly the same temperature. The explanation is that the heat flows from the hand to the iron and off through the iron more readily than it does through the wood and more readily through the wood than it does through the woolen cloth.

Temperature may in a rough way be likened to pressure or "head" in hydraulics. Heat tends to flow from a region of high temperature to a region of low temperature, just as water tends to flow from a region of high pressure to a region of low pressure. Reasoning from this principle that a flow of heat can only take place between two bodies having a difference of temperature, we may say that **two bodies have the same temperature if when they are brought into contact there is no interchange of heat between them.**

THE EFFECTS OF HEAT

153. When heat is imparted to a body, one or more of the following effects are observed:

1. Rise of temperature.
2. Change in size.
3. Change of state.
4. Chemical change.
5. Electric effect.

The temperature of a body usually rises when heat is imparted to it, although this is by no means the invariable rule. For example, when water is boiling freely, it makes no difference how much heat is imparted to it or how rapidly, it is not possible to change the temperature of the boiling water until it is entirely boiled away. In the same way the temperature of a mixture of pure ice and water will remain constant, no matter how rapidly heat may be imparted to the mixture, until all of the ice is melted. In the first example cited, the heat imparted is used in evaporating the water or changing it over into the form of steam. In the second case the heat imparted is used in melting the ice. It is assumed, in the examples given, that the pressures acting remain the same throughout.

The general rule as to change in size is that the hotter a body becomes, the greater is its volume. There are some exceptions to this rule. For example, water is found to decrease in size when it is heated from the freezing point to a few degrees above the freezing point (0° C. to 4° C.).

It is a matter of common observation that by imparting heat to a solid it may be converted into a liquid, as, for example, in the melting of ice. Also, that by imparting heat to a liquid it may be converted into a vapor, as for example, in the generation of steam from water. This is commonly referred to as a change of state.

Heat facilitates chemical change. One of the best examples of this effect is in the burning of coal. Before the carbon of the coal will combine with the oxygen of the air it is necessary to apply heat. Once the action is started, the heat developed by this chemical combination is sufficient to maintain the chemical action. Hence chemical change is one of the most important effects of heat.

It is found that under suitable conditions an electric current may be caused to flow in a wire by heating it, hence this is included among the effects of heat.

THERMOMETERS

154. A thermometer is a device for measuring temperature. Referring to the last section and noting that the first two

effects of heat mentioned are a rise in temperature and increase in size, the thought will very naturally occur to one that it might be possible to measure the rise in temperature of a body by taking note of its increase in size. The increase in size of a body would, of course, be an accurate measure of the rise in temperature provided the one were proportional to the other. The increase in size of certain bodies is quite accurately proportional to the rise in temperature, such bodies naturally offer themselves as suitable thermometric substances.

THERMOMETRIC SUBSTANCES

155. Solids. In certain devices used for the measurement of temperature the expansion of a metal with a rise of temperature is taken advantage of. By knowing the increase in length of a bar as it is heated, its rise in temperature may be estimated. This form of thermometer is not adapted to ordinary temperature measurements for the reason that the increase in length of a bar of metal is very small as compared with its length, so that very long bars would have to be made use of in order that the increase in length might be readily observed and measured.

Liquids. Certain liquids are found to be well adapted to use as thermometric substances. Alcohol increases in size quite uniformly with an increase in temperature, so that by filling a suitably shaped glass vessel, having a large bulb with a long narrow stem, with alcohol a very convenient thermometer is secured. Alcohol possesses the advantage of freezing only at very low temperatures, so that it may be used in cold climates and in very cold weather.

Mercury is perhaps the best liquid thermometric substance known. It possesses several distinct advantages as follows: It expands quite uniformly over a large range of temperature ($-40°$ C. to $+330°$ C.). It is opaque, so that a fine thread of mercury in a glass tube is easily visible. It does not wet glass, and hence does not stick to the walls of the containing vessel. Its expansion for a given rise in temperature is relatively large.

The last topic mentioned in the preceding paragraph is of some importance, since it will be quite evident that the apparent expansion of mercury in a glass bottle is in reality the difference between the absolute expansion of the mercury and that of the bottle. If, therefore, the bottle and contained liquid expanded equally with a given rise of temperature, the apparent expansion would be 0, that is to say, a thermometer made up in this manner would give the same reading independent of the temperature to which it was subjected.

Gases. A gas lends itself very readily for use as a thermometric substance. It is found to expand quite uniformly through great ranges of temperature. A gas used in this manner is usually inclosed in a glass or porcelain bulb with a long narrow stem, the stem being graduated in the customary way.

THERMOMETER SCALES

156. There are two thermometer scales in common use. That which is most widely used and quite universally adopted for scientific purposes is known as the **Centigrade** scale. A thermometer to have such a scale is graduated in the following manner. The thermometer is placed in melting ice and a scratch is made upon the stem at the point at which the column of mercury comes to rest under these conditions. This point is marked 0° C. The thermometer is next placed in boiling water, or, better still, in a closed space just above boiling water, the pressure upon the surface of the boiling water being that which is known as the standard atmosphere (76 centimeters of mercury). The point to which the mercury rises under these circumstances is marked 100° C. The distance between the "ice point" and the "steam point" determined in this manner is divided into 100 equal parts. These parts are known as Centigrade degrees.

Another thermometer scale which is very widely used for domestic purposes is known as the **Fahrenheit** scale. To graduate a thermometer according to this scale the ice point is marked 32° F. and the steam point 212° F. The distance between the two points in this case is divided into 180 equal

parts, and these are known as Fahrenheit degrees. In Figure 107 the two scales are compared. To reduce a temperature as read upon one of these thermometers to the corresponding reading upon the other, we have the following general considerations. Let it be assumed that the mercury stands on the line AB, and that its reading corresponds to c degrees on the centigrade scale. It is required to find the corresponding reading on the Fahrenheit scale. Call this reading f, then the reading on the same scale **measured from the ice point** will be $f - 32$. Then since

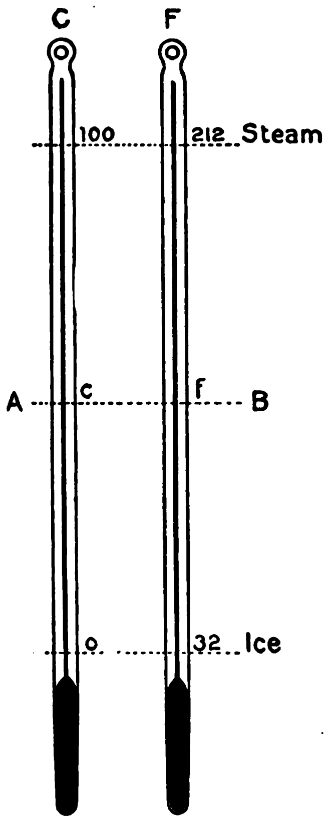

Fig. 107. — Centigrade and Fahrenheit Thermometers.

100 Centigrade degrees = 180 Fahrenheit degrees

$$c : f - 32 = 100 : 180$$

or,

$$\frac{c}{100} = \frac{f - 32}{180}$$

$$\therefore c = \tfrac{5}{9}(f - 32) \quad (52)$$

and

$$f = \tfrac{9}{5} \cdot c + 32 \quad (53)$$

From this relation a reading on either scale may be readily converted into the corresponding reading on the other.

SIMPLE AIR THERMOMETERS

157. The air thermometer is used in scientific work requiring temperature measurements of great accuracy or wide range. When used for such purposes, the air in the bulb is kept at **constant pressure** and the changes in volume observed, or at **constant volume** and the changes in pressure observed.

In Figure 108 are shown two simple forms of air thermometers. In A the inclosed air is separated from the out-

side air by means of an index of mercury or other liquid. A motion of the index toward the bulb indicates a fall in temperature of the inclosed air. If the bulb of this instrument is grasped by the hand, the index will move away from the bulb, showing the expansion of the inclosed air due to the heat received from the hand.

B is a glass bulb attached to a long tube which dips into a liquid. The liquid is caused to stand in the tube at some convenient height *h* above the liquid in the vessel below. If the bulb is heated, the pressure of the inclosed air is increased and the liquid column falls. If the air in the bulb is chilled, the column rises. Evidently both forms of air thermometer shown in Figure 108 are dependent upon the pressure of the outside air. They can therefore be used as temperature indicators only so long as the outside air pressure is constant.

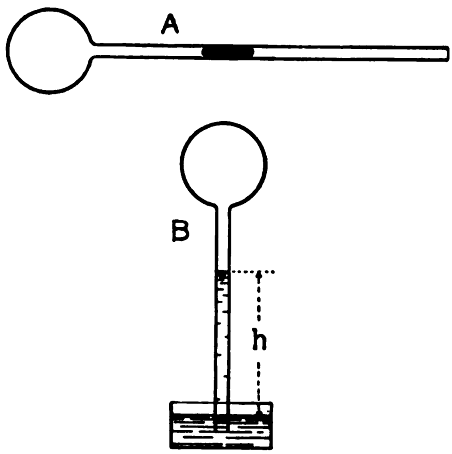

FIG. 108. — Simple Air Thermometers.

LINEAR EXPANSION

158. If a bar of metal is heated, it will increase in length, the increase in length being proportional to the rise in temperature and to the original length, that is,

$$\begin{aligned}\text{increase in length} &\propto L_0 t \\ &= \alpha L_0 t\end{aligned}$$

in which L_0 is the original length, t is the rise in temperature, and α is a constant for that particular substance of which the bar is made and is known as the "coefficient of linear expansion."

The coefficient of linear expansion of any substance is the increase in length for each unit of length for each degree of

rise in temperature. This definition comes at once from the above expression if we make L_0 and t each equal to unity.

The final length of the bar L_t is therefore given by the following equation:

$$L_t = L_0 + \alpha L_0 t$$

or

$$L_t = L_0(1 + \alpha t) \tag{54}$$

L_0 should be taken as the length of the bar at 0°. The increase in length in a bar as it is heated from $t°$ to $t_1°$ would be

$$L_0(1 + \alpha t_1) - L_0(1 + \alpha t) = \alpha L_0(t_1 - t)$$

Evidently Equation (54) may be used for the determination of the coefficient of linear expansion. In this case the initial and final lengths are accurately measured and the rise in temperature noted by means of suitable thermometers. Then α is calculated from the equation. In case α is known, Equation (54) may be used for estimating the length which a bar would have at a temperature t, its length at 0° C. being known.

Equation (54) is based upon the assumption that the increase in length of a bar of metal is strictly proportional to its rise in temperature. This is not exactly true. The increase in length is **very nearly** proportional to the rise in temperature, and therefore for most practical purposes the relation may be regarded as exact. The error resulting from the use of Equation (54) is negligible if α has been determined from this relation by observing values of L_0 and L_t with t comparable in value with the range of temperatures over which the equation is to be used. Such a determination of α gives, of course, the **average value** of the coefficient between 0° and $t°$.

Table of Coefficients of Linear Expansion

Substance	α
Copper	0.0000178
Iron	0.0000116
Glass	0.0000085
Platinum	0.0000085
Lead	0.000028
Tin	0.000022

APPLICATIONS

159. The fact that different solids expand at different rates, that is to say, that different solids have different coefficients of linear expansion, is made use of in various ways. For example, in the compensated clock pendulum this principle is employed for maintaining the length of the pendulum constant. We have seen that the time of vibration of a pendulum varies as the square root of its length (Section 51). Evidently, therefore, any change in the length will be accompanied by a change in the period of the pendulum. If the pendulum is made longer, it will vibrate more slowly. If it is made shorter, it will vibrate more rapidly. It will therefore be evident that a pendulum clock in order to keep good time would have to be provided with a pendulum of invariable length. The pendulum which is not compensated for temperature effects grows longer in warm weather and shorter in cold weather. Thus the clock runs too fast in cold weather and too slow in warm weather. To obviate this difficulty the "gridiron pendulum" is sometimes employed. This pendulum is represented diagrammatically in Figure 109. Let O represent the point from which the pendulum is suspended, and a a rod of some suitable metal supporting a crossbar AB. Suspended from this crossbar are two bars c and d of the same metal as that used in a. Supported by the bars c and d are the crossbars C, D. Standing upon these crossbars are two bars e, f, of a metal different from that used in a. From the upper end of these bars hangs the bob as indicated in the diagram. If the lengths of the bars used are properly chosen, it will be found that the distance OE of the center of mass of the pendulum from the point of support will remain unchanged through wide ranges of temperature. Evidently, the lengths of the rods a, c, d, and g, which may be, for example, of iron,

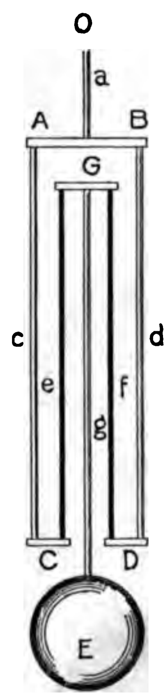

FIG. 109. — The Gridiron Pendulum.

which may be.

of $a-c+g$

a form of

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figure 115). It

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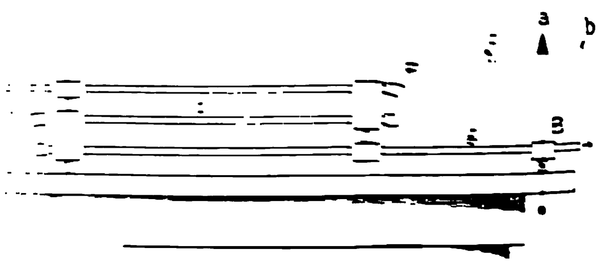


Z in a similar manner at

support attached to the

fastened to p, which rests

roller lying upon the

roller is fastened a pointer a, which

of the small roller under the

B. A small roller is connected by means of a piece

of rubber tubing to G at S. When steam is generated in the boiler and caused to flow through the tubes, the following results are observed. As the steam flows through G and causes a rise in its temperature, it expands and moves the system RB to the right. The pointer moves from a to b. When the steam passes into Z and causes it to expand, that portion of the system lying beyond Z, that is, rB, is moved to the left. The pointer moves from b to some such position as c. Finally, when the steam enters g, the expansion in this tube will move the block B once more to the right, and the pointer will move from c toward a.

Evidently the expansion effects of G and g are in the same direction and opposed to that of Z. It follows, that if the lengths of Z, G, and g are properly chosen, the net result will be zero, that is, the block B will be at the same distance from C when the three tubes are equally heated that it was when the tubes were cold. In the experiment this will be indicated by a return of the pointer to the position a when steam is flowing freely through all of the tubes.

Now the coefficient of linear expansion for glass is 0.0000085 and for zinc 0.000029. Call the length of the zinc tube L. The increase in length of the zinc tube is therefore

$$0.000029 \times Lt$$

in which t is the rise in temperature of the zinc tube when steam flows through it.

The increase in length of the glass tubes is

$$0.0000085 \times L_1 t$$

in which L_1 represents the added lengths of G and g, and t their rise in temperature.

Now if the distance CB is to remain unchanged, the increase in length of the zinc tube must equal the increase in length of the glass tubes, that is,

$$0.000029 \times Lt = 0.0000085 \times L_1 t$$

or since the temperature rise is the same for all tubes

$$290\,L = 85\,L_1$$

i.e. $$L_1 = 3.4\,L \text{ (approximately)}$$

That is, the lengths of G and g together must be 3.4 times that of Z.

The arrangement of tubes in this apparatus corresponds to that of the rods in a gridiron pendulum. C corresponds to the point of suspension in the pendulum and the block B to the pendulum bob.

CUBICAL EXPANSION. CHARLES' LAW

161. If a given volume of any substance has its temperature increased, it is found in general to increase in volume. The increase in volume is proportional to the original volume and to the rise in temperature, that is,

$$\text{increase in volume} \propto V_0 t$$
$$= \beta V_0 t$$

in which V_0 is the original volume (at 0° C.), t is the rise in temperature, and β is the coefficient of cubical expansion for that particular substance.

The coefficient of cubical expansion is the increase in volume for each unit of volume for each degree of rise in temperature. This definition comes from the above expression at once if we make V_0 and t each equal to unity.

The final volume V_t is therefore

$$V_t = V_0(1 + \beta t) \qquad (55)$$

Since the increase of the volume of a body depends upon the increase of its three linear dimensions, it follows, of course, that different substances, for example, brass, iron, glass, etc., have different coefficients of cubical expansion. (See values of α, Section 158.)

It is found that **the coefficient of cubical expansion for all gases at constant pressure is the same** and has a value of

$$\beta = 0.00367$$
$$= \tfrac{1}{273}$$

of ~~what value~~ at 0° C

This fact was first discovered by Charles and is commonly known as **Charles' Law.** It is also sometimes known as the Law of Gay-Lussac, the name of the man who first put the law to experimental test.

THE GENERAL LAW FOR THE EXPANSION OF A GAS WITH CHANGE IN PRESSURE AND TEMPERATURE

162. It will be remembered that in stating Boyle's Law (Section 122) the assumption was made that the temperature of the gas remains constant and the statement was made that **Boyle's Law would apply only under the condition of no change in temperature.** Boyle's Law, therefore, specifies the manner in which the volume of a gas changes with pressure when its temperature remains constant. Charles' Law, on the other hand, specifies the manner in which the volume changes with the temperature so long as the pressure remains constant. It is convenient to have these two laws combined in one general statement. This is readily obtained in the following manner. In Figure 111 let the rectangle V_0 represent the volume of a certain mass of gas, the pressure of which is p_0 and the temperature 0° C. Let it be assumed that the gas is heated until its temperature rises to t° C., **the pressure remaining the same.** According to Charles' Law the volume will increase such that

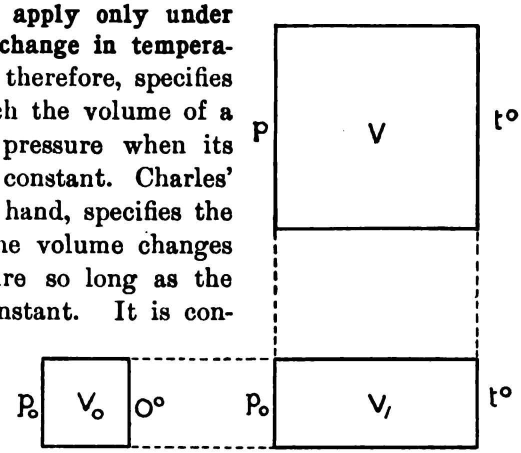


FIG. 111. — Illustrating the General Law of the Expansion of Gases.

$$V_1 = V_0(1 + \beta t)$$

in which V_1 is the new volume. Let it be assumed that the pressure is now decreased, **the temperature being held constant.** Then according to Boyle's Law there will be a still further expansion of the gas such that the new volume is given by the following equation:

$$V \cdot p = V_1 p_0$$

In which V is the new volume and p the new pressure. Eliminating V_1 by combining the expression for V_1 in terms of V_0, and that for V in terms of V_1, we have at once

$$V = \frac{p_0 V_0}{p}(1 + \beta t) \qquad (56)$$

This equation expresses the volume of the gas under the new temperature and pressure in terms of its volume under the old conditions of temperature and pressure. This is known as the general law of the expansion of a gas.

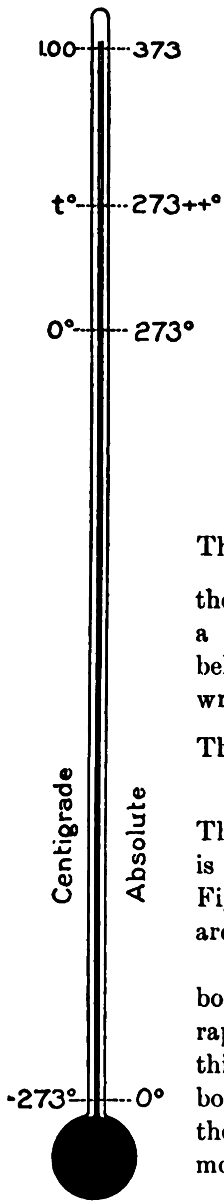


Fig. 112. — Scale of Absolute Temperatures.

THE ABSOLUTE ZERO

163. Since β for all gases is equal to $\frac{1}{273}$, Equation (56) may be written in the following form:

$$pV = p_0V_0\left(1 + \frac{t}{273}\right)$$

$$= \frac{p_0V_0}{273}(273 + t)$$

The fraction $\frac{p_0V_0}{273}$ is a constant, call it R, and the factor in parentheses represents evidently a temperature as measured from a point 273° below zero on the Centigrade scale. Let us write

$$T = 273 + t$$

Therefore, we have

$$pV = RT \tag{57}$$

This new zero, 273° below 0° C., from which T is measured, is called the **absolute zero.** In Figure 112 the Centigrade and absolute scales are compared.

On the theory which assumes that in hot bodies the molecular parts are in a state of rapid vibratory motion, while in cold bodies this motion is less marked, it follows that if a body were cooled down to 273° below zero on the Centigrade scale this molecular vibratory motion would cease. That is to say, at this temperature the body would have no heat. This result is reached as follows. From the equation just written it is evident that the

pressure of the gas on the walls of the containing vessel is proportional to the absolute temperature. That is to say,

$$p \propto T$$

upon the assumption that the volume of the gas is constant. But the pressure of a gas is supposed to be due to the impact of its molecular parts as they vibrate to and fro. The pressure therefore can only be zero when the vibratory motion ceases entirely. Therefore the **temperature 273° below zero on the Centigrade scale is that temperature at which the molecular parts of a body are entirely without vibratory motion. It is therefore the lowest possible temperature.**

The absolute zero of temperature has never been reached experimentally. It is, however, interesting to note in this connection that a temperature of -268.5 on the Centigrade scale has actually been attained.

THEORY OF THE AIR THERMOMETER

164. The essential parts of a constant volume air thermometer are shown in Figure 113. The bulb *B*, which may be made of glass or porcelain, contains the thermometric substance, dry air or hydrogen. This bulb is connected by a tube of small bore to the U-tube *AC* partially filled with mercury. The upper end of the tube *AC* is sealed and contains a vacuum, and the pressure of the gas in *B* is therefore measured by the difference in height *h* of the mercury columns in *A* and *C*. By raising or lowering the vessel *V*, the mercury in the tube *AC* may be brought or held to a marked point in *A*, and the volume of the gas in the bulb is in this manner kept constant.

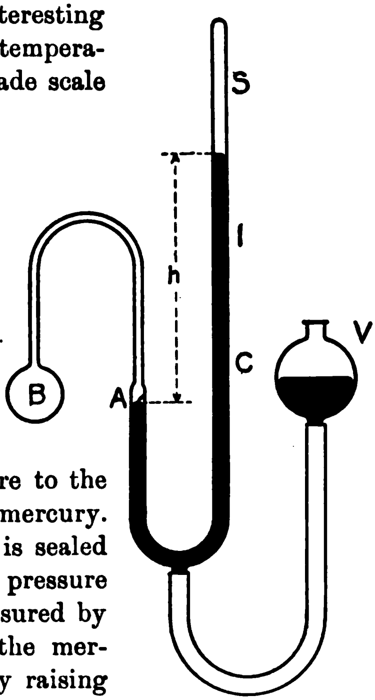


FIG. 113.— Air Thermometer.

Now from the general law of expansion of gases,

$$pv = RT$$

it follows that if the volume of any body of gas v is kept constant, its pressure will vary as its absolute temperature, *i.e.*

$$p \propto T$$

so that if the pressure of the gas at an absolute temperature T is p, its pressure at an absolute temperature T' will be p', such that

$$\frac{p}{p'} = \frac{T}{T'}$$

or

$$T' = T \cdot \frac{p'}{p}$$

The air thermometer may be used as follows: First surround the bulb with melting ice, temperature 273° absolute, and by adjusting the height of V bring the mercury to A and observe the difference in height h of the mercury columns. Then place the bulb in the region of the temperature T' which is to be measured, and when the mercury is again brought to A, observe the difference in height of the mercury columns as before. Call this difference h'. Then

$$T' = 273 \cdot \frac{h'}{h}$$

EXPANSION OF LIQUIDS

165. The determination of the expansion of a liquid by ordinary methods is complicated by the fact that the containing vessel expands and contracts with rising and falling temperature, so that accurate knowledge of the change in volume of the vessel with changing temperature is necessary, in order that proper allowance for it may be made. In other words, the observed expansion is an **apparent expansion** which depends upon the expansion of the containing vessel as well as upon the absolute expansion of the contained liquid.

Regnault devised a method for determining the absolute expansion of a liquid, which is independent of the expansion of the containing vessel. The apparatus used in this method consists essentially of two glass tubes, AB, connected as shown

in Figure 114, the connecting tube being of small diameter. These tubes are filled with the liquid to convenient heights.

Let it be assumed that the tube A is kept at the temperature of melting ice, and the tube B at a temperature $t°$ C. Let h_0 be the height of the column in A and h_t the height of the column in B. Then

$$h_0 d_0 g = h_t d_t g$$

since the pressures right and left balance one another.

$$\therefore h_0 d_0 = h_t d_t \qquad (a)$$

Now from Charles' Law we have

$$V_t = V_0(1 + \beta t)$$

or since $$V = \frac{M}{d}$$

$$\frac{M}{d_t} = \frac{M}{d_0}(1 + \beta t)$$

that is, $$d_t = \frac{d_0}{1 + \beta t} \qquad (b)$$

Combining (a) and (b), we obtain

$$\beta = \frac{h_t - h_0}{h_0 t} \qquad (58)$$

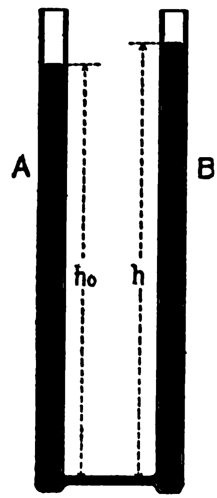


Fig. 114.—Regnault's Apparatus for Cubical Expansion.

from which β, the coefficient of cubical expansion, may be calculated from observed values of h_0, h_t, and t.

The value of β obtained from such calculation is the average value of the coefficient of cubical expansion throughout the temperature range 0° to $t°$.

THE EXPANSION OF WATER

166. Water is remarkable in that it forms an exception to the general law that liquids expand with a rise of temperature. If water at the temperature of melting ice is heated, **it contracts as its temperature rises to about 4° C.** At this point it reaches its minimum volume, and therefore its maximum density. If its temperature is raised above this point, it expands, at first

slowly and then more rapidly as its temperature rises, until the boiling temperature is reached.

The curve shown in Figure 115 exhibits the changes in volume which a given mass of water undergoes when its temperature rises from 0° C. to 100° C.

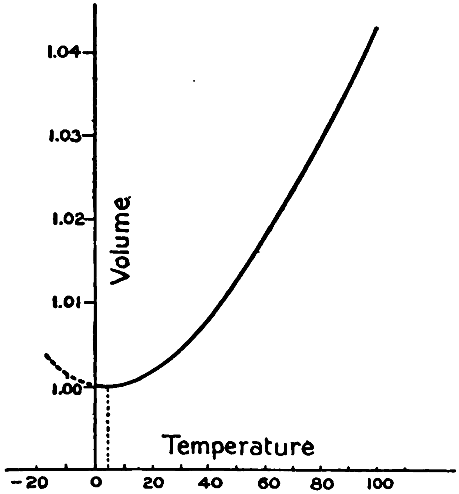


Fig. 115. — Curve showing Change in Volume of a Given Mass of Water with Change of Temperature.

THE THERMO COUPLE

167. One of the effects of heat as given in Section 153 is the electric effect. This effect may be briefly described as follows: If two dissimilar metals, for example, copper and iron, are joined together in such manner as to form a complete metallic circuit, and the two junctions of such are at different temperatures, it is found that there is present in the circuit, **because of this difference of temperature**, an electric current. Such a circuit is represented in Figure 116. The different parts of the circuit are indicated in the diagram. The points marked "Hot" and "Cold" are the junctions, that is to say, the points of contact, between the dissimilar metals. *G* is a galvanometer, that is, an instrument for detecting the presence of the electric current. In such a circuit there is no ten-

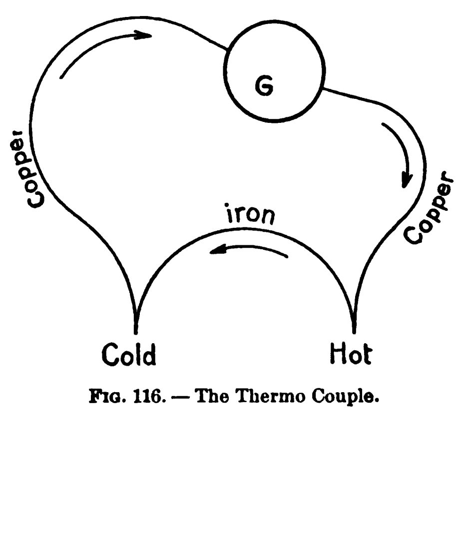


Fig. 116. — The Thermo Couple.

dency for an electric current to flow so long as the junctions are at the same temperature. If, however, there is a difference of temperature between the junctions, a current will flow in the circuit, and the magnitude of the current is proportional to the difference of temperature between the junctions. It will be evident that such a device might be used for measuring differences of temperature.

Problems

1. Reduce to Fahrenheit readings the following Centigrade temperatures: 40°, 21°, – 20°.

2. Reduce to Centigrade readings the following Fahrenheit temperatures: 110°, 32°, 0°.

3. At what temperature will Fahrenheit and Centigrade thermometers give the same reading?

4. A platinum wire is 5 m. long at 0° C. What is its length at 100° C.?

5. An iron pipe is 5 m. long at 20° C. What is its length at 0° C.?

6. The length of a copper wire at 30° C. is 20 m. What is its length at 10° C.?

7. The area of a sheet of iron is 15 sq. m. at 0° C. What is its area at 40° C.?

8. A mass of gas at 0° C. occupies 150 cc. What volume would it occupy at the same pressure if its temperature were increased to 100° C.?

9. A mass of gas at 0° C. and a pressure of 760 mm. of mercury occupies a volume of 500 cc. Find its volume when the pressure is increased to 1000 mm. of mercury and the temperature to 40° C.

10. A given mass of gas has a volume of 400 cc. when subjected to a temperature of 20° C. and a pressure of 100 cm. of mercury. At what temperature will it have a volume of 500 cc. at a pressure of 90 cm. of mercury?

CALORIMETRY

CHAPTER XIV

THE UNIT OF HEAT

168. Although heat is a form of energy and may therefore be measured in ordinary units of energy, it is convenient for a good many purposes to employ a unit which is based upon the effect of heat in raising the temperature of water.

The *calorie* is the unit of heat in the c. g. s. system and is defined as the quantity of heat required to raise the temperature of one gram of water from 4° to 5° on the Centigrade scale.

The British Thermal Unit (B. T. U.) is the unit of heat in the f. p. s. system and is defined as the heat required to raise the temperature of one pound of water from 60° to 61° on the Fahrenheit scale.

SPECIFIC HEAT

169. The specific heat of any substance is the quantity of heat required to raise the temperature of one gram of the substance one degree.

From the definition for the unit of heat given in the last section it follows that the specific heat of water at 4° Centigrade is unity. The specific heat of water at other temperatures is very nearly unity. In fact, it is so near unity at all temperatures between the ice point and the steam point that for most purposes in heat measurement this value may be assumed to be correct. Strictly speaking, however, the specific heat of water is unity only at 4° C.

The specific heat of a substance in general depends upon its temperature. Therefore, in specifying the specific heat of a substance, to be rigidly exact we should always give the temperature at which the specific heat is supposed to be measured.

The specific heats of some of the more common substances are given in the following tables, in calories per gram per Centigrade degree.

SPECIFIC HEATS OF SOLIDS

Aluminum	0.212
Brass	0.094
Copper	0.095
Glass	0.195
Ice	0.504
Iron	0.112
Lead	0.031

SPECIFIC HEATS OF LIQUIDS

Alcohol	0.547 at 0° C.
Ether	0.529 at 0° C.
Mercury	0.033 at 30° C.

Gases have two specific heats, — **the specific heat at constant volume** and **the specific heat at constant pressure**; that is to say, the heat required to increase the temperature of one gram of the gas one degree without changing its volume, and the amount of heat required to raise the temperature of one gram of the gas one degree as it expands without change of pressure. The specific heat at constant pressure is the greater since an expanding gas does work at the expense of the heat contained by it.

THERMAL CAPACITY

170. The thermal capacity of a body is the heat required to raise its temperature one degree. The thermal capacity of a body is equal to the product of the mass of the body and the specific heat of the substance. That is,

$$\text{thermal capacity} = M \cdot S$$

in which M is the mass of the body and S the specific heat of the substance of which the body is composed. This relation is apparent, since from the above definition **the specific heat of a substance is its thermal capacity per unit mass.**

The following experiment is often made to demonstrate the difference in the specific heats of various metals. A number

of balls of equal size (equal volume) of different metals are heated to some convenient temperature and placed side by side upon a cake of wax, Figure 117. The balls melt their way into

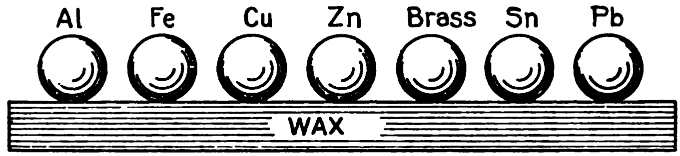


FIG. 117.—Illustrating Difference in Thermal Capacity per Unit Volume.

the wax; the depth to which each ball sinks being determined by the amount of heat it can give up as its temperature falls to that of melting wax. Therefore, the depth to which a given ball sinks is a measure of its **thermal capacity**. Now the thermal capacity of a body is given by the product of its mass and the specific heat of the substance of which the body is composed. Equal volumes of two different metals may therefore differ appreciably in thermal capacity, although having nearly the same specific heat, unless their densities are also equal.

In the table below are given the specific heat, density, and thermal capacity per unit volume of a number of metals:

Metal	Specific Heat	Density	Thermal Capacity per Unit Volume
Al	0.21	2.6	0.546
Fe	0.109	7.6	0.820
Cu	0.095	8.9	0.801
Zn	0.093	7.1	0.660
Brass	0.09	8.4	0.756
Sn	0.056	7.3	0.408
Pb	0.031	11.3	0.350

An inspection of the table will show that the rank of these metals in thermal capacity per unit volume is very different from their rank in specific heat. For example, aluminum, ranking first in specific heat, is fifth in rank in thermal capacity per unit volume. Brass is fifth in specific heat and third in thermal capacity per unit volume, while lead, tin, and zinc rank the

same in this group of metals when viewed from either standpoint.

THE MEASUREMENT OF HEAT

171. There are various ways in which a given quantity of heat may be measured. One of the most common methods is that in which **the quantity of heat to be measured is imparted to a known mass of water and the rise in temperature which takes place in the water is measured by means of a thermometer.** Knowing the mass of the water and the rise in temperature, the quantity of heat is calculated in the following manner: Since it requires 1 calorie to raise the temperature of 1 gram of water 1 degree, evidently it will require 10 calories to raise the temperature of 10 grams of water 1 degree, 100 calories to raise the temperature of 100 grams of water 1 degree, etc. Or, in general, if the mass of the water is M, then M calories of heat are required to raise the temperature of the water 1 degree. If the temperature rise is 2 degrees, $2M$ calories will be required; if the temperature rises 10 degrees, $10M$ calories; or, in general, if the temperature of the water rises from t_1 to t_2, the total heat required is

$$H = M(t_2 - t_1) \qquad (59)$$

THE METHOD OF MIXTURES

172. For determining the thermal capacity of a body or the specific heat of a substance the method known as the "method of mixtures" is employed. Let it be assumed that it is desired to measure the specific heat of lead. Given the quantity of lead, m grams, conveniently in a granular form, or in the form of shot, and heated to a temperature of $t_2°$. This is suddenly plunged into a suitable vessel containing, let us say, M grams of cold water at a temperature of $t_1°$. As a result of this "mixture" there will be an equalization of temperatures. The lead will fall in temperature, giving a part of its heat to the water, which in consequence will rise in temperature, the whole mixture coming finally to some intermediate temperature t.

On the assumption that no heat is received from, or given to, surrounding bodies, the total heat lost by the lead must be equal to the heat gained by the water. As determined above, the heat gained by the water $= M(t - t_1)$ (Equation 59), and in the same way it will be evident that the heat lost by the lead $= ms(t_2 - t)$ in which s is written for the specific heat of lead. We have, therefore,

$$M(t - t_1) = ms(t_2 - t)$$

$$s = \frac{M(t - t_1)}{m(t_2 - t)} \tag{60}$$

from which s is readily calculated. Figure 118 is a diagram for showing the temperature relations.

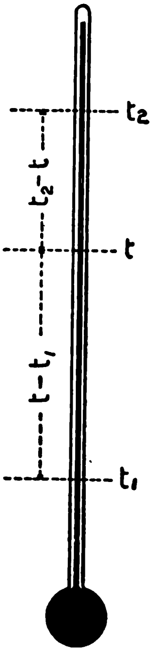


FIG. 118. — Showing Temperature Changes in the Method of Mixtures.

It is assumed in this discussion that the total heat given up by the lead is taken up by the water. This is never true. Evidently the vessel containing the water will rise in temperature when the water rises in temperature, and will therefore absorb a certain amount of heat, which must be taken account of, particularly in those cases in which the containing vessel is of appreciable thermal capacity. If the thermal capacity of the vessel is C, the absorption of heat by the water and containing vessel will be exactly the same as if the total mass of water were $M + C$ grams and no containing vessel used. Therefore, a more accurate expression for s, the specific heat of the lead in the discussion above given, would be

$$s = \frac{(M + C)(t - t_1)}{m(t_2 - t)} \tag{61}$$

The quantity C is sometimes called the "water equivalent" of the containing vessel. A vessel used in this manner for the measurement of heat is called a **calorimeter**.

THE HEAT OF FUSION

173. The heat of fusion of a substance is defined as the heat required to change one gram of the substance from the solid to the liquid state without a change of temperature. It is found, for example, that it requires 79.25 calories to change one gram of ice into water at the same temperature. Thus we say the heat of fusion of ice is 79.25 calories. Evidently a gram of water at 0° in changing to ice at the same temperature will give up 79.25 calories.

The heat required to convert a solid into a liquid without changing its temperature is somewhat analogous to the work required to pulverize a stone. After doing a certain amount of work upon a stone in pulverizing it, that which we have left is evidently stone. The only difference between it and the original stone is a difference in the separation of its parts. We may therefore think of the heat which is required to melt a gram of ice as being used up in a way, in tearing down the crystalline structure of the ice. The liquid may therefore be thought of as possessing a kind of potential energy which it gives up as it sinks back into the solid state.

HEAT OF VAPORIZATION

174. Just as a certain amount of heat is required to convert a gram of ice into water at the same temperature, so a certain amount of heat is required to convert a gram of water into steam at the same temperature. This quantity of heat is known as the "heat of vaporization." The heat of vaporization of water at 100° C. is 536.5 calories per gram. That is to say, it requires 536.5 calories to change a gram of water at 100° into steam at the same temperature. The steam may therefore be thought of as possessing a sort of potential energy which it gives up as it goes back into the liquid form, that is to say, as it condenses. The heat of vaporization of water, or any other liquid, depends upon the temperature at which the vaporization takes place. Thus the heat of vaporization of water at 200° C. is 464.3 calories.

THE ICE CALORIMETER

175. The ice calorimeter is a device for measuring quantity of heat, which depends for its indication upon the heat of fusion of ice. A simple form of ice calorimeter is represented in Figure 119. Let A represent a block of ice with a cup-shaped hollow in its upper surface. This hollow is covered by a second block of ice B. The calorimeter is used in the following manner. Let it be required, for example, to determine the specific heat of lead. A suitable mass, m, of the lead is heated to some convenient temperature t. It is then placed in the cup-shaped cavity in the block A, the ice cover B being quickly replaced. Care must be taken in the use of the apparatus that no water is present in the cavity before the lead is inserted. The lead will, under these circumstances, very quickly cool to the temperature of the ice, that is, 0° C. In so doing it will give up a certain quantity of heat which is all used in melting ice. Evidently the heat given up by the lead is mst calories. The heat taken up by the ice is $M \times 79.25$, in which M represents the number of grams of ice melted. We have, therefore,

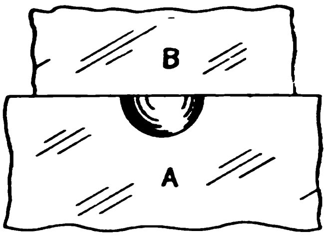


FIG. 119. — The Ice Calorimeter.

$$s = \frac{79.25\,M}{mt} \qquad (62)$$

The quantity of ice melted in such a calorimeter may be determined by taking up the water with a cold sponge which is weighed before and after it is dipped in the calorimeter. Thus, the total quantity of ice converted into water is determined. Figure 120 shows the temperature relations in the experiment.

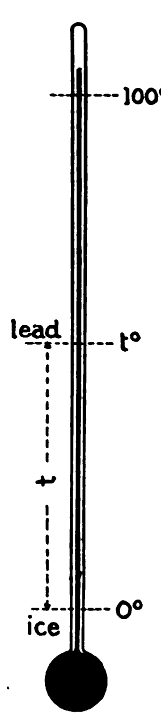


FIG. 120. — Showing Temperature Change in the Ice Calorimeter Experiment.

THE STEAM CALORIMETER

176. The steam calorimeter is similar in principle to the ice calorimeter. In this device the heat taken up, for example, by m grams of lead as its temperature rises through a given range is obtained from a mass of, say, M grams of steam as it condenses to the liquid form. Heat is imparted to the lead until its temperature is that of the steam, that is, until no more steam will condense upon the lead. We have, therefore, the heat taken up by the lead is equal to $ms(t_2 - t_1)$. The heat given up by the steam is, of course, $M \times 536.5$, assuming that the temperature of the steam is 100° C. Therefore we have

$$s = \frac{M \times 536.5}{m(t_2 - t_1)}$$

$$= \frac{536.5\,M}{m(100 - t_1)} \qquad (63)$$

The mass M of the steam which is condensed upon the lead as its temperature rises is usually determined by weighing the condensed water which drops away from the lead into a suitable receptacle. The temperature relations are shown in Figure 121.

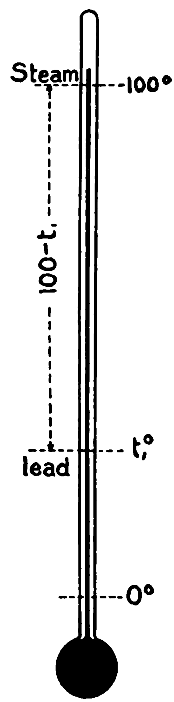


FIG. 121.—Showing Temperature Change in the Steam Calorimeter Experiment.

Problems

1. A copper calorimeter weighs 70 g. What is its thermal capacity?

2. How much heat is required to raise the temperature of 1000 g. of copper from 0° C. to 100° C.?

3. If 1000 calories are required to raise the temperature of 500 g. of a metal 50° C., what is the specific heat of the metal?

4. A copper beaker weighing 100 g. contains 400 g. of water at 15° C. If a piece of copper weighing 150 g. is heated to 100° C. and plunged into the water, what is the final temperature?

5. A copper calorimeter weighing 100 g. contains 300 g. of water at 10° C. 40 g. of lead and 60 g. of iron are heated to 100° C. and plunged into the water. What is the resulting temperature?

6. If 3 g. of shot heated to 100° C. is dropped into an ice (block) calorimeter, how much ice will it melt? Specific heat of lead = 0.0315.

7. 500 g. of a certain metal at 98° C. is dropped into 350 g. of water at 4° C. and raises its temperature to 10° C. Find the specific heat of the metal. Regard the specific heat of water as unity.

8. 300 g. of shot at 100° C. is dropped into a glass beaker weighing 30 g. and containing 60 g. of H_2O at 0° C. and 2 g. of ice. What will be the temperature of the mixture? Specific heat of glass is 0.2, lead 0.0315.

9. What will be the result of mixing a mass of snow at 0° C. with an equal mass of water at 50° C.?

10. How many B. T. U. would be required to melt 150 lb. of ice?

11. How much steam at 100° C. must be passed into 2000 g. of water at 0° C. containing 500 g. of ice to raise the whole to 50° C.?

12. How much heat would be required to change 20 g. of ice at – 10° C. to steam at 110° C.? Assume specific heat of steam at constant pressure = 0.5.

VAPORIZATION AND SOLIDIFICATION

CHAPTER XV

EVAPORATION AND EBULLITION

177. If water is placed in an open vessel in a room at ordinary temperature it will be observed to gradually disappear from the vessel. **This escape of the liquid by passing over into the vapor state, a change which takes place wholly at the surface of the liquid, is called *evaporation*.** If the water is heated until it comes to the temperature known as the boiling point, vapor will begin to form in bubbles within the body of the liquid itself. These bubbles of vapor rise to the top, expanding as they rise into regions of diminished pressure, bursting at the surface and setting free large quantities of the vapor. **This formation of vapor within the liquid is known as ebullition, or boiling.**

The boiling point of a liquid may therefore be defined as that temperature at which the bubbles of vapor are formed within the liquid. Evidently this formation of vapor is one kind of vaporization, but it is distinguished from the process of evaporation by the fact that vapor is formed within the liquid.

THE EFFECT OF PRESSURE UPON THE BOILING POINT OF A LIQUID

178. It will be remembered that in discussing what is known as the steam point of a thermometer (Section 156), it was specified that the water should be boiling under the pressure of a standard atmosphere. The reason for thus specifying the pressure under which the water was assumed to be boiling is that **the temperature of boiling water depends upon the pressure of the atmosphere which is acting upon the surface of the water as it boils.** It is very readily demonstrated that if the pressure

referred to is by any means diminished, the boiling point is lowered. If the pressure is increased, the boiling point is raised.

A very instructive experiment for demonstrating the dependence of the temperature of boiling upon the pressure acting upon the surface of the liquid is the following. Figure 122 represents a large glass bottle partly filled with water. The bottle, in an upright position, is placed over a suitable source of heat — say, a Bunsen burner — and the water made to boil vigorously. The steam which is developed as the water boils displaces a large part of the air in the upper part of the flask. The bottle is now stoppered and inverted, the neck of the bottle being placed in a second vessel containing water. This is to prevent the entrance of air into the bottle. If now cold water is poured upon the upper portion of the flask in its inverted position, the water will boil, **even after the water has cooled down to room temperature**, providing care has been taken to expel a large part of the air in the bottle before inverting it. The explanation of this phenomenon is as follows: The cold water which is poured upon the upper portion of the vessel chills, and therefore condenses a portion of the steam within the flask. This diminishes the pressure, since the condensed steam occupies a very much smaller volume than it did before it was condensed. The result is that while the water is at a temperature altogether too low to boil under atmospheric pressure, its temperature may yet be sufficient to cause it to boil under the diminished pressure within the flask.

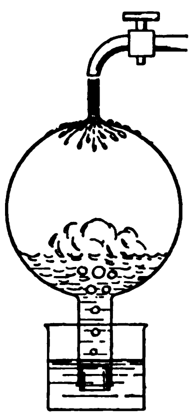


FIG. 122.—The Effect of Pressure upon the Boiling Point.

The effect of this lowering of the temperature of the boiling point with diminished pressure on the surface of the liquid is oftentimes quite noticeable in cooking and similar operations. We have seen that the pressure of the atmosphere diminishes as we go to higher and higher altitudes. We would very naturally expect, therefore, that water would boil at lower temperatures as we ascend to higher altitudes. This is found

to be the case. Water boils at Quito, South America, at 90° C.; on Pike's Peak, Colorado, at 86° C.; and on Mont Blanc at 84° C. Evidently all cooking operations in which boiling water is used would be retarded at these higher altitudes if the water were allowed to boil in open vessels, since such cooking operations depend upon the temperature of the boiling water.

In the same manner it is found that to increase the pressure upon the surface of the liquid in the vessel necessitates the raising of the water to a higher temperature before boiling ensues. In steam boilers which furnish steam for engines the space above the water is inclosed, and the vapor as it develops from the water fills the boiler and exerts a pressure which may be very much above that of the atmosphere. Under these circumstances water boils at temperatures which are above 100° C. For example, water boiling under a pressure of two atmospheres has a temperature of 120° C., and water under a pressure of six atmospheres must be raised to a temperature of 160° C. before it will boil.

THE FREEZING POINT OF A LIQUID

179. The freezing point of a liquid is the temperature at which the liquid freezes or changes to the solid state. Under normal conditions a substance changes from the liquid state to the solid state and *vice versa* at the same temperature, *e.g.* water freezes and ice fuses at 0° C.

The freezing point of a liquid depends upon the nature of the substance, upon the presence of other substances held in solution, and upon the pressure to which it is subjected.

The freezing point is sharply defined for crystalline substances, but is more or less indefinite for non-crystalline bodies. Thus wax, glass, iron, etc., gradually soften in passing from the solid to the liquid state and the fusing temperature is not sharply defined.

REGELATION

180. By the application of pressure it is possible to lower the freezing point of water below 0° C.; that is to say, if water is subjected to a very high pressure, it may be cooled below 0° without freezing, and if ice at a temperature below 0° is sub-

jected to pressure sufficiently great, it may be fused, even though its temperature remains below the freezing point. The effect of pressure upon the freezing point is, however, not nearly so marked as the effect of pressure upon the boiling point. It requires enormous pressures to lower the freezing point of water appreciably. For example, an increase of one atmosphere lowers the temperature of the freezing point of water by 0.0072 of a degree Centigrade. Mousson by enormous pressure lowered the freezing point of water to $-20°$ C. Since water expands upon freezing, and increased pressure would tend to prevent expansion, the thought would naturally occur to one that a substance which contracts upon freezing would have its freezing point raised by pressure. This is actually the case. Bunsen found that paraffine wax, which melted at 46.6° C. under atmospheric pressure melted at 49.9° C. under a pressure 100 times as great.

The general statement of the case is, therefore, that **increased pressure lowers the freezing point of solids which expand upon freezing and raises the freezing point of solids which contract upon freezing.**

This effect of the lowering of the melting point by the application of pressure to ice may be very simply demonstrated by pressing together two pieces of ice which are at the melting point, first taking care to dry them by means of blotting paper or a towel. It will be found that by firmly pressing the pieces of ice together they will freeze. The explanation of the phenomenon is as follows. Referring to Figure 123, let A and B represent the two pieces of ice, and C the point of contact between them. When A and B are firmly pressed together at this point, the ice, although perhaps slightly below 0° on the Centigrade scale, will melt because of the increased pressure at that point. The water will flow away from the region of high pressure right and left as indicated by the shaded portions, but as soon as it comes into this region of low

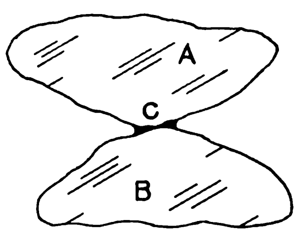


Fig. 123. — Illustrating the Effect of Pressure upon the Melting Point.

pressure it immediately refreezes. This refreezing of the water as it escapes from the region of high pressure is known as **regelation.** A very instructive and interesting example of regelation is that represented in Figure 124. *A* is a block of ice suitably supported. *B* and *C* are weights suspended by means of a fine steel or copper wire from the block of ice *A* as indicated. The weights *B* and *C* give rise to a high pressure beneath the slender wire where it lies in contact with the ice. The result is that the ice melts under this high pressure, the water flows around the wire and coming into the region of low pressure back of the wire, immediately refreezes. Thus the wire passes after a time completely through the block of ice, but leaves the ice behind it just as solid, or even more solid than at any other portion of the block.

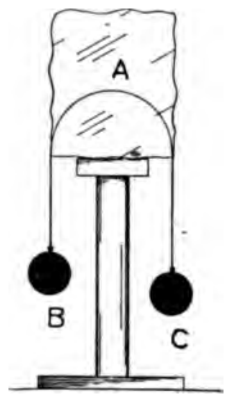


Fig. 124. — Regelation.

The phenomenon of regelation is used to explain, in part at least, the motion of glaciers down the sloping valleys of mountain ranges. It is found that these large masses of solid ice move slowly toward the lower levels notwithstanding that they are in the form of solid ice. It is supposed that those portions of the glacier which come in contact with bowlders and obstructions of various kinds are melted by the enormous pressures due to the weight of the glacier. The water then flowing around the obstruction refreezes, and the mass moves on past the obstruction in this manner.

EVAPORATION

181. Evaporation probably takes place in something like the following manner. Let *A*, Figure 125, represent a closed vessel partly filled with a liquid as indicated, the upper part of the vessel being exhausted so as to have in that portion of the vessel a complete vacuum. Let it be assumed that the temperature of the liquid is *T* (absolute). This temperature is nothing more than a specification of the average rate at which the molecular parts of the liquid are vibrating. (See

Section 163.) Now while the average vibratory condition of the molecular parts may be quite constant, it will be readily understood that some of the molecules will have velocities which are higher than the average and others will be moving more slowly than the average.

Let it be assumed that some of those particles which have the highest velocities come to the upper surface of the liquid. Then it is conceivable that if their velocities are sufficiently great, they may actually break away from the body of the liquid and escape into the region of the vacuum. That is to say, their velocities are so great when they come to the surface of the liquid that they are able to break away from the cohesive force actions of their neighbors and escape into the vacuum above. *B* represents the state of affairs which is reached after the lapse of a certain time during which a considerable number of particles has escaped into the upper portion of the vessel. The molecules present in the upper part of the vessel are represented by the dots. The arrows drawn in this figure indicate that the number of particles in the upper portion of the vessel has become so great that occasionally one is jostled back into the neighborhood of the surface of the liquid and comes within range of the molecular action of the surface particles and once more becomes a part of the liquid. The number of particles escaping from the liquid is, however, in this stage, assumed to be greater than the number returning. This is indicated by the arrows. *C* represents the third stage in the vaporization of the liquid. The number of molecules in the upper part of the vessel is now, comparatively speaking, large. They have indeed become so numerous that the number which goes back into the liquid in any given length of time is equal to the

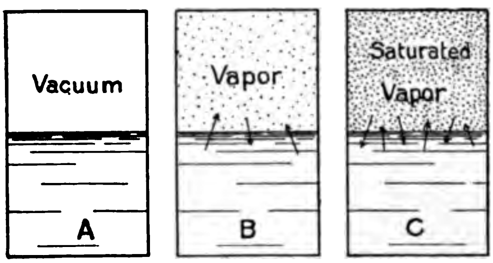


FIG. 125. — Illustrating the Successive Stages in the Formation of Saturated Vapor.

number which escapes from the liquid into the upper portion of the vessel. The upper portions of the vessels *B* and *C* are said to be filled with vapor. In the stage represented at *C* the vapor is said to be **saturated.**

PROPERTIES OF SATURATED VAPOR

182. It is impossible by change of volume to change the pressure of a saturated vapor so long as the temperature remains constant. In other words, the pressure of a saturated vapor depends solely upon the nature of the liquid and the temperature. For example, referring to *C*, Figure 125, let it be assumed that by suitable means the volume of the upper portion of the vessel is increased. The number of particles in the upper portion of the vessel now having greater freedom of motion because of the fact that they occupy larger space, would not return as rapidly to the liquid. Thus the number of particles escaping from the liquid in a given time would be greater than the number reaching the liquid, and very soon, under these conditions, the condition of saturation would again be reached in the increased volume. It is assumed in this discussion that the temperature of the liquid has been the same throughout. On the other hand, if the volume of the upper portion of the vessel is diminished, more molecules will return to the liquid in a given time than escape from it, that is to say, under these conditions a portion of the saturated vapor will be condensed, while in the case just referred to a portion of the liquid was evaporated. **Any change, therefore, in the volume of the upper portion of the vessel results in the vaporization of more of the liquid or the condensation of a part of the vapor without any change in the vapor pressure.**

If, however, the liquid be raised in temperature, the average velocity of its particles will be increased, therefore a greater number of particles will escape from the surface of the liquid in a given time; hence, it will be necessary to have a larger number of the vapor particles in the upper portion of the vessel; that is to say, it will be necessary to have a larger vapor pressure in the upper portion of the vessel, before equilibrium of vaporization and condensation is again established.

It is therefore evident that the pressure of a saturated vapor depends directly upon its temperature, and any change in the temperature of the vapor is followed by a change in its pressure.

THE COOLING EFFECT OF VAPORIZATION

183. Since a substance in changing from the liquid to the vapor state takes from its surrounding a certain amount of heat, known as the heat of vaporization, it will be evident that vaporization is a cooling process. If water is sprinkled upon the sidewalk on a hot summer's day, it quickly evaporates. Each gram of the liquid evaporated takes up a quantity of heat which is determined by the temperature at which it is changed into the vapor state. This heat taken up by the water in evaporating is abstracted from its surroundings, which are thereby cooled.

THE CRYOPHORUS

184. A very striking illustration of the cooling effect of vaporization is afforded by the cryophorus. This instrument consists of two bulbs joined together by a slender tube, the whole being of glass (see Figure 126). The cryophorus is prepared for use by partly filling it with water and causing the same to boil vigorously, a small opening in the upper bulb being left to allow the steam to escape. The escaping steam displaces the air in the upper part of the tube, and after the lapse of a given time the air is almost entirely removed from the bulbs and the connecting tube. When this condition has been secured the opening through which the steam is escaping is sealed. After this is done the tube will contain a small quantity of water, the remaining portion of the tube being filled with water vapor. The liquid is now transferred to the upper bulb, and the cryophorus is suspended in the position shown in Figure 126, the lower bulb being immersed in a freezing mixture. This freezing mixture causes the vapor in the lower

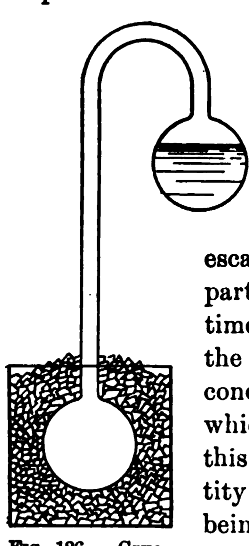


FIG. 126. — Cryophorus.

bulb to condense and freeze, thus diminishing the pressure in that part of the tube. Some of the vapor flows over from the upper tube toward this region of low pressure and is in turn condensed and frozen. This process goes forward more or less rapidly, **each portion of the water in the upper bulb which is vaporized in the operation taking its heat of vaporization from the liquid which remains.** The result is that the liquid in the upper bulb is gradually cooled and eventually frozen.

THE CO_2 EXPERIMENT

185. Carbon dioxide is used quite extensively for the purpose of charging soda fountains and the like, and is to be obtained upon the market in strong steel bottles into which the CO_2 is compressed when the bottle is filled. The ordinary bottle, when received from the manufacturers, is partly filled with liquid CO_2, the remaining portion of the vessel being, of course, filled with saturated CO_2 vapor. If such a bottle is placed in an inverted position, Figure 127, evidently the liquid will fill the lower part of the vessel, its density being greater than that of the vapor, and the upper portion of the bottle will be filled with the saturated vapor. If now the stopcock is opened, the liquid, which is subjected to a high pressure, will be forced out through the opening in a slender stream. CO_2 is normally a vapor at ordinary room temperatures and pressures. The tendency is, therefore, for this thin stream of liquid to pass quickly into the vapor state. In so doing it takes up its heat of vaporization from its surroundings, namely, from the air, from the bottle, and from that portion of the liquid which is not vaporized. These three sources of heat become chilled in this process, and so much so that a very low temperature is reached as the process of vaporization goes forward. Evidently the central portion of the stream is chilled in the same measure that the other surroundings are, and it will be found after a short time that this portion of the stream, instead of vaporizing,

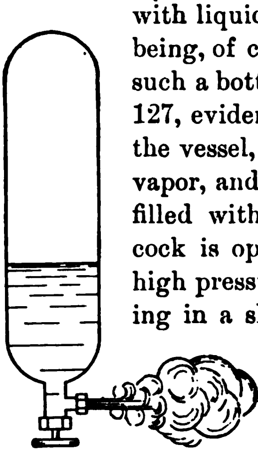


FIG. 127.—The Formation of CO_2 Snow.

is converted, because of the low temperature, into the solid form, that is to say, the liquid CO_2 at the center of the stream becomes frozen. The temperature of this CO_2 "snow" is about $-80°$ C., and the experiment affords a very good illustration of the marked cooling action produced by the vaporization of a liquid.

CRITICAL TEMPERATURE

186. There is a certain temperature known as the "critical temperature" for the vapor of every substance down to which the vapor must be cooled before it can be liquefied by the application of pressure. At or below the critical temperature the liquefication of a vapor is possible by the application of pressure. Above the critical temperature no amount of pressure will convert the vapor into the liquid state. The critical temperature for ammonia (NH_3) is 130° C. Ordinary temperatures are therefore below the critical temperature for this substance. The following table gives the critical temperatures for some of the more important substances:

TABLE OF CRITICAL TEMPERATURES

SUBSTANCE	CRITICAL TEMPERATURE	CRITICAL PRESSURE
H	$-240°$ C.	14
Air	$-140°$ C.	39
CO_2	31° C.	73
NH_3	130° C.	115
H_2O	365° C.	200

The critical pressure given in the third column is expressed in atmospheres, and gives the pressure required to liquefy the vapor at the critical temperature. If the temperature is below the critical temperature, smaller pressures will suffice to convert vapor into the liquid state.

PRESSURE-TEMPERATURE CURVE

187. If a curve is plotted between temperatures as abscissæ and corresponding pressures of a saturated vapor as ordinates, a curve like that shown in Figure 128 is obtained. This curve, which marks the boundary between the liquid and vapor states

of the given substance, is sometimes called the **steam line.** It will be evident that a given condition of the vapor, as to pressure and temperature, is completely represented in this diagram by a point. For example, the point A represents the condition of a certain mass of vapor when its pressure is p and its temperature t. This diagram is very useful in discussions involving the passing of a given substance from the vapor to the liquid state. Evidently, such transformation may take place in either of two distinct processes, that is, **by lowering the temperature, the pressure being held constant, or by increasing the pressure, the temperature being held constant.**

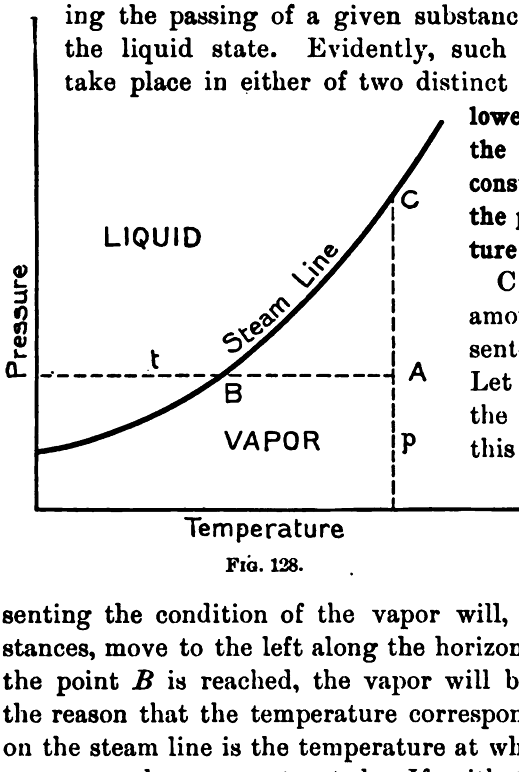


FIG. 128.

Consider a given amount of vapor represented by the point A. Let it be assumed that the pressure acting upon this vapor is kept constant and that the temperature is gradually lowered. The point representing the condition of the vapor will, under these circumstances, move to the left along the horizontal line, AB. When the point B is reached, the vapor will begin to condense for the reason that the temperature corresponding to the point B on the steam line is the temperature at which a vapor having a pressure p becomes saturated. If, with the vapor in the condition represented by B, an attempt is made to still further lower the temperature, the substance will all pass into the liquid state.

Again, let it be imagined that the vapor is in the condition indicated by a point A, and let it be assumed that the temperature is held constant while the pressure is increased. The point representing the condition of the vapor will, under these circumstances, move along the vertical line AC. When the

pressure corresponding to point C is reached, the vapor will be saturated, and any further increase of pressure will result in the condensation of the vapor. This must be evident, since the pressure corresponding to the point C on the steam line is that pressure at which the vapor at a temperature t becomes saturated.

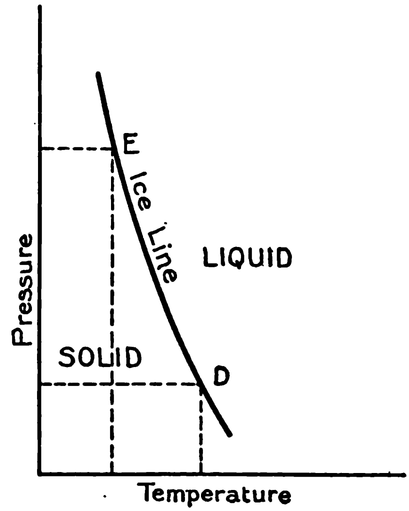


FIG. 129.

In Figure 129 is shown a pressure-temperature curve called the **ice line**. This curve gives the relation between the pressure and temperature of the melting point of a given substance. The curve is plotted as follows: Let the ordinate of the point D represent the pressure and its abscissa the corresponding temperature of the melting point of the substance in question. Let it be assumed that the substance is of such a nature that it expands upon freezing. The melting point of such a substance is lowered by pressure, as pointed out in a previous section. If, therefore, the substance is melted at a higher pressure, such as that corresponding to the point E, it will melt at a lower temperature, which means, of course, that the point E will lie nearer the axis of pressures than the point D. Thus, if successive values of melting point and corresponding pressure are plotted, the curve DE is obtained. Evidently, this curve

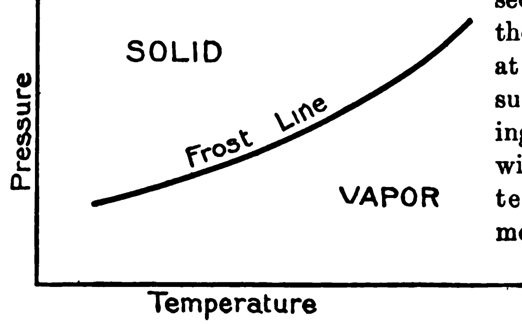


FIG. 130.

marks the boundary between the liquid state of the substance on the right and the solid state on the left. If the substance in question is of such nature that it contracts upon freezing, evidently the ice line will slope in the opposite direction; that is, E will lie farther from the axis of pressure than does the point D.

In the same way a pressure-temperature curve may be drawn to represent the boundary condition between the vapor and solid states. Such a curve is shown in Figure 130 and is called the **frost line.** When a substance passes directly from the solid to the vapor state (sublimation), it crosses this boundary line. The reverse of this process is familiar to every one in the formation of frost, from the water vapor of the atmosphere.

TRIPLE POINT

188. The three temperature-pressure curves discussed above may all be placed in the same diagram. When so placed, they

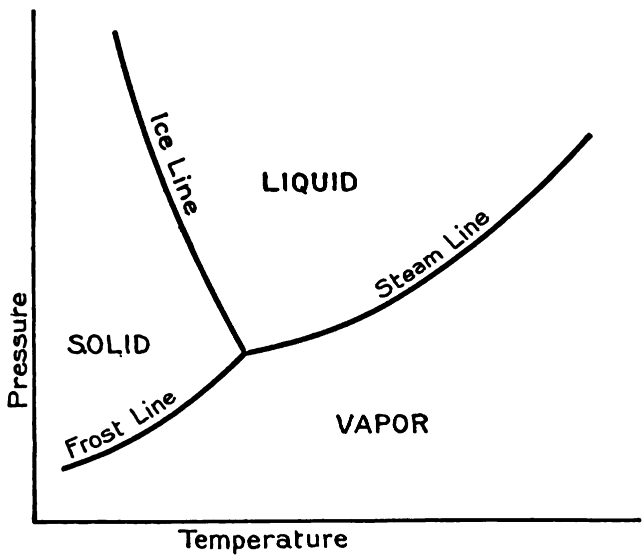


FIG. 131.—The Triple Point.

intersect in a common point, which is known as the **triple point** for the substance in question. For example, Figure 131 repre-

sents the triple point and corresponding temperature-pressure curves for water. The curves here shown are not drawn to scale and therefore can only be used to show general relations.

The **steam line** gives the conditions under which the vapor and the liquid may exist together in equilibrium, the **ice line** those under which the liquid and the solid may exist together, and the **frost line** those under which the solid and vapor may, exist simultaneously. It is obvious, therefore, that the **triple point** gives the pressure and temperature conditions under which all three, solid, liquid, and vapor, can exist together in equilibrium. The triple point for water corresponds to a pressure of 0.046 centimeter of mercury and a temperature slightly above 0° C.

PRESSURE-VOLUME CURVES OF A GAS

189. Another diagram much used in connection with discussions on the behavior of gases and vapors under varying pressure is the **pressure-volume** diagram. Let A, Figure 132, represent the condition of a given mass of gas having a volume v when subjected to pressure p. Let it be assumed that the temperature of the gas is held constant at, say, 20° C., and the pressure is steadily increased. Then the point representing the condition of the gas on this diagram will move along the curve AB. This curve is an equilateral hyperbola, the equation to which is

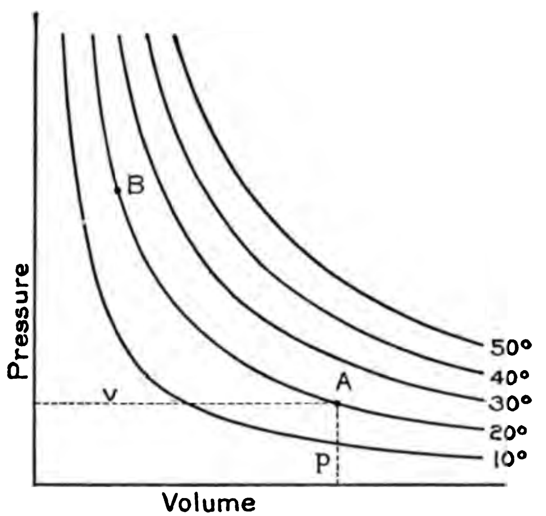


FIG. 132. — Isothermals of a Perfect Gas.

$$pv = \text{a constant (Boyle's Law)}$$

Since the temperature of the gas remains constant, as the pressure and volume change, as contemplated in this discussion, the curve *AB* is sometimes called an **isothermal.**

If the mass of gas considered in the above discussion is taken at some other initial temperature and is allowed to expand at constant temperature, the isothermal corresponding will lie above or below the curve *AB*, according to the temperature chosen. The isothermals shown in Figure 132 are separated by temperature intervals of 10° C.

ISOTHERMALS OF A VAPOR

190. In the above discussion it is assumed that the substance under consideration is a perfect gas. If, instead of such a gas, a vapor, near its saturation temperature, is considered, the isothermal is no longer an equilateral hyperbola, but assumes a form like that represented by *CDEF* in Figure 133. Let *C* represent the condition of the vapor at the outset. Let it be assumed that the temperature is held constant and the pressure is steadily increased, as before. The points representing the successive conditions of the vapor as to pressure and volume will lie along the curve *CD*. The point *D* is assumed to correspond to the saturation pressure of the vapor, so that when this point is reached, the vapor will begin to condense and the pressure will remain constant until all of the vapor passes into the liquid state. This change is represented by the horizontal line *DE*. The point *E* corresponds to the condition in which all of the vapor is liquefied. An increase of pressure from this point will be followed by a decrease in volume of the liquid, as indicated by the line *EF*. This portion of the curve is very steep, since the substance is much less compressible in the liquid state than it is in the vapor state.

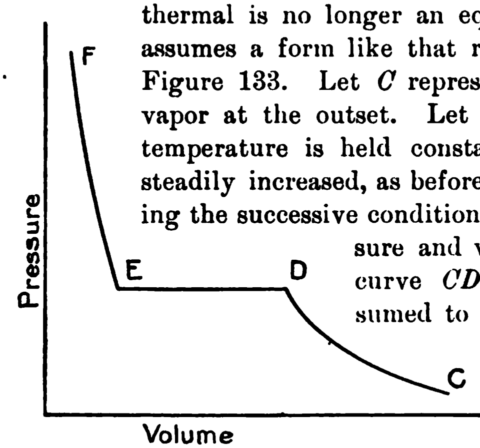


FIG. 133. — Isothermal of a Vapor.

In the above discussion, the relation between pressure and volume has been considered for one temperature only. Evidently, if the gas had been taken at a lower temperature or at a higher temperature, a curve similar to *CDEF* would have been developed, which would lie either above this curve or below, according as the initial temperature was higher or lower than that assumed above.

ISOTHERMAL AT THE CRITICAL TEMPERATURE

191. In Figure 134 are shown a series of isothermals for CO_2 vapor at different temperatures, as determined by actual experiment. It will be observed from this diagram that the horizontal portion, *DE*, of the isothermal corresponding to the condition of saturated vapor is shorter for the higher temperatures, and in the isothermal corresponding to a temperature of 31.6° C. is entirely wanting. The meaning of this is that when CO_2 vapor at a temperature of 31.6° C. is subjected to increasing pressure, its volume decreases, that is to say, the vapor becomes more dense, just as when compressed at lower temperature, but no matter how far the process is carried, the vapor does not pass through the saturated stage. That is to say, the vapor cannot be converted into a liquid by the application of pressure at this temperature. This temperature of 31.6° C. is, therefore, called the **critical tem-**

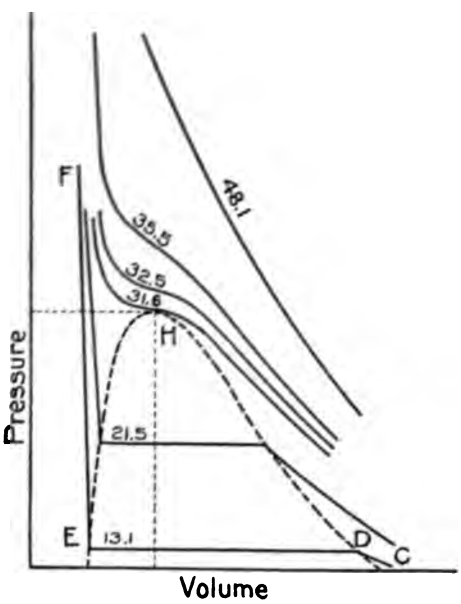


FIG. 134. — Isothermals of CO_2 at Various Temperatures.

perature of carbon dioxide. The pressure and volume corresponding to the point *H* in Figure 134 are called the **critical pressure** and **critical volume** of the given mass of the substance considered.

If CO_2 vapor is compressed isothermally at temperatures considerably above 31.6° C., the isothermals obtained are like those of a perfect gas, since for temperatures far above the critical temperature the vapor obeys Boyle's Law. If the temperature of the compressed gas is near the critical temperature, it departs to some extent from the law which is followed by a perfect gas. This is indicated in the diagram by the form of the curves. The higher curves in the diagram are like the isothermals of a perfect gas. For this reason a gas is sometimes distinguished from a vapor by saying that a gas is a vapor far removed from its critical temperature.

LIQUEFACTION OF GASES

192. It will be evident from the above discussions on critical temperature that in any attempt to liquefy a gas it is first necessary to secure a lowering of the temperature which will bring the gas below its critical temperature. When this condition has been secured, an increase of pressure will force the substance through the condition of saturated vapor into the liquid state. One of the earliest experiments in the liquefaction of gases was that performed by Faraday, who made use of a bent glass tube, something like that represented in Figure 135. In the long arm of the tube *A* is placed a chemical compound, from which the gas to be experimented upon is driven off by the application of heat. The other end

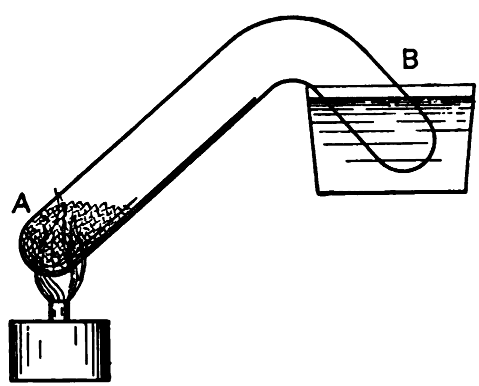


FIG. 135. — Faraday's Bent Glass Tube Experiment.

of the bent tube is placed in a freezing mixture which is capable of lowering the temperature of the gas within the tube below the critical point. When heat is applied at *A* and the gas is driven off, it, of course, spreads to all parts of the tube, and as more and more gas is evolved, the pressure increases to higher and higher values. The vapor in the *B* end of the tube, being held at the temperature of the freezing mixture, will undergo a change in condition, which is very well represented by one of the lower curves in Figure 134. That is to say, as the pressure rises, it will eventually reach the condition of saturation and liquid will begin to form in the bottom of the tube, at *B*. By means of this simple device, Faraday succeeded in liquefying chlorine, carbon dioxide, cyanogen, and ammonia, which, up to that time, had only been known in the gaseous state.

In liquefying such gases as hydrogen, oxygen, nitrogen, and air, a more elaborate apparatus is required, since the critical temperatures of these gases are so low as to be attained only with the greatest of difficulty. The means resorted to for reaching the extremely low temperatures required in the liquefaction of these gases is that of allowing the substance to cool itself by sudden expansion, after it has been placed under high pressure and cooled by other means as far as possible.

HYGROMETRY

CHAPTER XVI

HUMIDITY

193. Hygrometry is that branch of physics which deals with the condition of the atmosphere as regards the water vapor which it contains. The atmosphere is made up largely of oxygen and nitrogen in almost constant proportions. In addition there are other substances present in relatively small amounts. The most important constituent of the atmosphere aside from the two first mentioned is water vapor. The water vapor contained in the atmosphere is altogether variable, there being at times large quantities of water vapor, at other times relatively small amounts. The condition of the air as regards the quantity of water vapor contained is of the greatest importance in determining the weather or the climate of a given place. It is therefore necessary in all weather observations to determine carefully the condition of the atmosphere in this respect. There are various ways in which the condition of the atmosphere as regards the quantity of water vapor contained is determined. In one class of determinations the absolute humidity is measured. In the other class the relative humidity is determined.

The *absolute humidity* of the atmosphere is defined as the mass of water vapor contained per unit volume. It is commonly expressed in grams per cubic meter. **The *relative humidity* of the atmosphere is defined as the ratio of the quantity of water vapor present to that which would be necessary to bring about the condition of saturation** (Sections 181 and 182).

THE CHEMICAL HYGROMETER

194. A device used for determining the condition of the air as regards the quantity of water vapor present is called a hygrometer.

The principle upon which the **chemical hygrometer** operates is as follows: A known volume of the air in question is passed through a series of U-tubes *A*, *B*, *C* (Figure 136), filled with calcium chloride or other hygroscopic material. The tubes together with their contents are weighed before and after the experiment. The difference between the two weights is the amount of water which has been absorbed by the calcium chloride. This instrument measures, therefore, the absolute humidity of the atmosphere. A convenient means of passing a known volume of air through the tubes is that indicated in the figure. *D* is an air-tight vessel which is filled with water at the beginning of the experiment. When the stopcock at the bottom is turned, the water flows out and air flows in through the connection at the top. The volume of air which passes through the tubes is therefore equal to the volume of water which flows out of the vessel *D*.

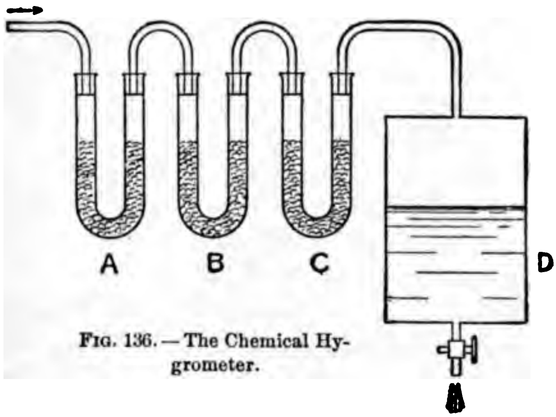


FIG. 136. — The Chemical Hygrometer.

THE DEW POINT HYGROMETER

195. The dew point hygrometer depends for its indications upon **the fact that air containing non-saturated water vapor may be brought to the saturation point by reducing its temperature.** At low temperatures it requires less water vapor to bring about the condition of saturation than is required under higher temperatures. If, therefore, a given quantity of air with the contained water vapor is sufficiently cooled, this water vapor will begin to condense. The temperature at which this takes place is called the **dew point**. The extent to which the air must be cooled before the dew point is reached depends upon the rela-

tive humidity of the air. Therefore, if the dew point can be determined, it is possible, by comparing this temperature with the actual temperature of the air, to estimate the amount of water vapor present. Tables have been made up from which the relative humidity may be obtained when the temperature and dew point of the air are known.

WET- AND DRY-BULB THERMOMETERS

196. Two thermometers which are as nearly as possible identical in construction, etc., are mounted side by side, the one being left exposed in the ordinary way, the other having wrapped about its bulb a bit of candle wicking which dips into a vessel of water below. By capillarity the candle wicking, and therefore the bulb of the thermometer about which the candle wicking is wrapped, will be kept wet, and this moisture in evaporating will produce a cooling effect upon the thermometer. This wet-bulb thermometer will therefore give a lower reading than the dry-bulb thermometer. **The difference in temperatures indicated by the wet- and dry-bulb thermometers depends upon the relative humidity of the atmosphere in which they are placed.** Evidently if the atmosphere is filled with saturated water vapor, the moisture on the wet bulb will not evaporate and there will be no lowering of temperature. Under these conditions the two thermometers will give the same reading. On the other hand, if the water vapor present in the air is far from saturation, there will be a rapid evaporation of moisture from the wet-bulb and the difference between the two thermometer readings will be correspondingly great. By comparing the indications of this instrument with the determinations of the chemical hygrometer tables have been made by means of which it is possible to interpret the indications of the wet- and dry-bulb thermometers in terms of relative humidity.

PRECIPITATION

197. When the dew point is reached, the moisture in the air begins to precipitate in one of several different ways, depending upon the actual temperature of the air and manner in which it is cooled. If the air is cooled in those layers only

which come into intimate contact with cold bodies, the result is the formation of **dew** or **frost** according as the temperature is above or below the freezing point of water. If the chilling takes place throughout the body of the air itself, precipitation takes place in the form of **fog** or **cloud**, from which **rain** may be formed by coalescence of the minute particles of liquid, or in the form of **ice clouds** in which the particles are minute crystals of ice instead of spheres of water. From this cloud, **snow** may be formed by the slow growth of ice crystals, or snowflakes. **Hail** is formed if rain drops pass through a cold layer of air and are sufficiently chilled to freeze as they fall.

KINETIC THEORY OF GASES

CHAPTER XVII

THE VIBRATORY MOTION OF GAS ATOMS

198. A body of gas is conceived to consist of a great number of distinct particles (atoms or molecules), minute in size and separated by distances which are large in comparison with the size of one of the particles. It is further assumed that these particles possess a rapid vibratory motion, rebounding with undiminished velocity when they strike the walls of the containing vessel. The particles are also assumed to exert no appreciable attraction for one another and to seldom collide.

This conception of the nature of a gas is almost universally accepted for the reason that it enables all the principle laws of gases to be readily explained and understood.

GAS PRESSURE

199. Consider a mass of gas M at a pressure p, volume v, and absolute temperature T, represented in section by Figure 137. The molecular kinetic energy of this body of gas is constant, since the particles are assumed to rebound from the walls with undiminished velocity and to seldom collide. Now the kinetic energy of a moving particle is $\frac{1}{2} m\omega^2$, m being the mass of the particle and ω its velocity. Hence, if the molecular kinetic energy of the body of gas remains the same, it follows that the **average value of ω^2** for all particles must be constant, since m is the same for all particles.

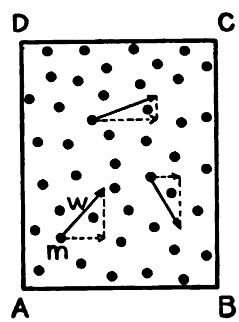


FIG. 137.

Now the pressure on any wall of the inclosing vessel is due to the bombardment of that wall by the moving particles.

When a particle strikes a given wall, the force with which it pushes against the wall depends upon the direction of its motion with respect to the wall, in other words, it is determined by that component of the velocity which is perpendicular to the wall. It is convenient, therefore, to consider the velocity of each moving particle as resolved into rectangular components x, y, and z. Assume that the x-component and y-component are measured in the directions AB and AD, Figure 137, and the z-component perpendicular to the plane of the paper. Evidently the pressure on the walls AD and BC is determined by the average value of the x-components of the velocities of the bombarding particles, the pressure on AB and CD by the average value of the y-components, and the pressure on the walls parallel to the plane of the paper by the average value of the z-components.

Now there are just as many particles per cubic centimeter moving in one direction as in another, and it therefore follows that the average value of the x-, y-, and z-components are equal. Let us call these components a, b, and c respectively.

If a, b, and c are the rectangular components of ω, therefore,

$$\omega^2 = a^2 + b^2 + c^2$$

and since

$$a = b = c$$

$$\therefore \omega^2 = 3\,a^2$$

or

$$a^2 = \tfrac{1}{3}\,\omega^2 \qquad (64)$$

Let the width of the containing vessel, that is, the distance between the walls AD and BC, be d. Then the time required by the average particle to travel from AD to BC or from BC to AD is $\frac{d}{a}$, hence it will strike $\frac{a}{d}$ times per second, and the number of times it will strike either wall will be $\frac{a}{2\,d}$. At each impact its velocity changes by the amount $2\,a$ (from $+\,a$ to $-\,a$). Its change in momentum is therefore $2\,am$ for each impact and in one second against one wall $2\,am \times \frac{a}{2\,d}$ or $\frac{a^2m}{d}$.

We have seen that the rate of change of momentum of a body is numerically equal to the force which causes that change,

and, since action is equal to reaction, its rate of change of momentum is equal to its reaction. The total force with which the gas acts (pushes) on the wall AD is therefore,

$$F = \frac{N \cdot a^2 m}{d}$$

in which N is the total number of particles in the given volume of gas.

Call the area of the wall S. Then the pressure on this wall is

$$p = \frac{F}{S} = \frac{Na^2m}{sd}$$

But $$sd = v \text{ and } a^2 = \tfrac{1}{3}\,\omega^2$$

$$\therefore p = \frac{1}{3} \cdot \frac{Nm\omega^2}{v} \tag{65}$$

Equation (65) gives the relation between the pressure and volume of a gas, the total number of particles, the mass, and the average square of the velocity of the individual particle. **This relation derived from purely theoretical considerations embodies and explains all of the fundamental laws of gases with which we have to deal in Physics.**

BOYLE'S LAW AND CHARLES' LAW

200. If we assume that the absolute temperature of a gas is proportional to the average kinetic energy per molecule, in other words, if we assume that T is proportional to $\frac{1}{2}m\omega^2$, or say $\frac{1}{2}m\omega^2 = K \cdot T$, in which K is a constant, then Equation (65) may be written

$$pv = R \cdot T \tag{57 bis}$$

in which R is written for $\frac{2}{3}N \cdot K$. This is the general law of a gas and includes the laws of Boyle and Charles (Section 162).

AVOGADRO'S PRINCIPLE

201. Avogadro's principle states that **under the same conditions as to pressure and temperature all gases have the same number of molecules per cubic centimeter.**

For N, Equation (65), we may substitute $n \cdot v$, v being the volume of the gas and n the number of molecules per unit volume. We have then

$$p = \frac{1}{3} n \cdot m \cdot \omega^2$$

Now consider two gases at the same temperature and pressure. Let n_1 be the number of molecules per cubic centimeter of the first gas, m_1 the mass of the individual molecule, and ω_1^2 the average square of the velocity of a molecule. For the other gas, let n_2, m_2, and ω_2^2 represent the corresponding quantities. Then since the pressures are equal, we have

$$\frac{1}{3} n_1 m_1 \omega_1^2 = \frac{1}{3} n_2 m_2 \omega_2^2$$

But since the temperatures are equal

$$m_1 \omega_1^2 = m_2 \omega_2^2$$

It therefore follows that

$$n_1 = n_2$$

which is the principle of Avogadro.

DALTON'S LAW

202. Dalton's Law states that **when two gases occupy the same space, each exerts the same pressure that it would exert if it occupied the space alone.** In other words, the pressure exerted by each gas is independent of that exerted by the other, and the total pressure on the walls of the containing vessel is the sum of the pressures exerted by the individual gases.

On the assumption that the gas particles are so small that they do not interfere with one another in their motions, this condition of the non-interference of one gas with another occupying the same space is exactly that which would be expected under the kinetic theory.

THE POROUS PLUG EXPERIMENT

203. Joule and Thomson carried out a series of experiments on the expansion of a gas from a region of high pressure to a region of low pressure through a porous plug. A general idea of the experiment may be gained from Figure

138. *A* and *B* are two cylinders connected by a narrow orifice *P* (a porous plug in Joule and Thomson's experiment), through which the gas passes from the region of high pressure p_1 to the region of low pressure p_2. The pistons are assumed to move without friction, and are weighted with W_1 and W_2, which **serve to maintain the pressure and therefore the density of the gas in each cylinder constant.** It will be understood that the above is but a schematic diagram of the apparatus.

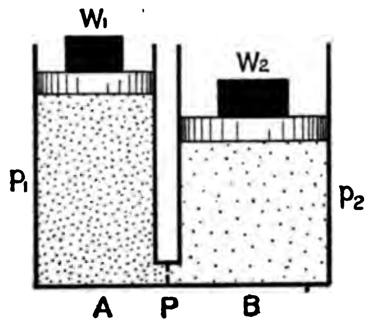


FIG. 138. — The Porous Plug Experiment.

If we assume that there is an attraction between particles, then as each particle of gas passes through the orifice it is pulled forward by particles near the orifice in *B* and held back by particles near the orifice in *A*. But the gas in *A* is more dense than that in *B*, therefore there are more particles pulling back than forward on the escaping particle and **it will lose in velocity as it passes the opening.** This corresponds to a fall in temperature.

On the assumption that the particles repel one another, it is evident **a particle would gain in velocity as it passed the opening.** This corresponds to a rise in temperature.

In the experiments of Joule and Thomson, sensitive thermometers were placed on either side of the porous plug. In experiments upon oxygen, hydrogen, and nitrogen, under ordinary pressures and temperatures only very small effects were observed. This would indicate that **under ordinary conditions as to pressure and temperature such gases are almost entirely free from intermolecular force actions.**

Oxygen and nitrogen were as a matter of fact slightly cooled in the experiment, a result which would indicate a slight attraction between the molecules of these gases under the given conditions. On the other hand, hydrogen, was slightly warmed, a result which would indicate that under the given pressures and temperatures, hydrogen molecules repel one another slightly.

If gases having complex molecules are used in the experiment, a more pronounced cooling effect results. Gases at low temperature and under high pressures (molecules relatively close together) show a more pronounced cooling under free expansion. This is exhibited in Linde's liquid air machine.

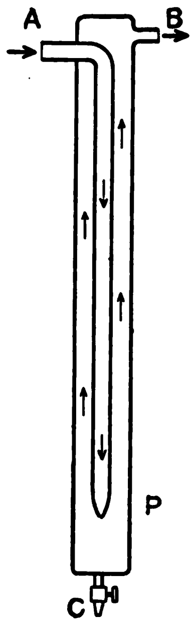


FIG. 139. — Principle of Linde's Liquid Air Machine.

LINDE'S LIQUID AIR MACHINE

204. In Linde's method of liquefying air, dry air at ordinary temperatures and under a pressure of about 200 atmospheres is led into a system of tubes, the essential features of which are shown in Figure 139. The air at high pressure enters at A and, passing through the inner tube, expands through the narrow orifice P into the outer tube, a region of lower pressure. The air is cooled by the Joule-Thomson effect, as it passes the orifice P, and flowing upward about the inner tube causes a lowering of its temperature. Thus each succeeding volume of escaping air is colder than that which preceded it. The escaping air becomes colder and colder as the operation continues, and is eventually liquefied. The liquid air accumulates in the bottom of the outer tube and may be drawn off at C.

THE EQUATION OF VAN DER WAALS

205. The equation $pv = RT$ is not rigidly exact. It would seem that the two assumptions made in the kinetic theory (1) that the size of the molecules may be neglected, (2) that the molecules exert no mutual attraction, are not quite justified.

It was suggested by Van der Waals that the actual behavior of a gas could be more accurately represented by writing the general law in the following form.

$$\left(p + \frac{a}{v^2}\right)(v - b) = RT. \tag{66}$$

By writing $v - b$ for v, allowance is made for the effect of the size of the molecules. If the molecules have appreciable size, they strike the walls of the containing vessel before their centers of mass reach the walls and collisions occur more frequently than they otherwise would. This amounts to a reduction of the effective volume of the containing vessel. The quantity b is a constant which depends upon the amount and nature of the gas.

By writing $p + \frac{a}{v^2}$ for p, allowance is made for the effect of mutual attraction between the molecules. This attraction tends to slow down the motion of the particles as they approach the walls of the containing vessel and, therefore, tends to reduce the pressure. This reduction of pressure can be shown to be inversely proportional to the square of the volume of the gas. The quantity a depends upon the amount and nature of the gas.

THE TRANSMISSION OF HEAT

CHAPTER XVIII

CONVECTION

206. There are three distinct ways in which heat is transmitted from one point to another, namely, by **convection**, **conduction**, and by **radiation**. In the first process heat is transferred by the motion of the heated substance. The motion of the substance in this process is due to the change in density which takes place in the heated portions. For example, when a vessel of water is placed upon the stove, those portions of the liquid in the bottom of the vessel become heated. Their densities are thereby decreased and they tend to rise among the heavier, colder portions, so that convection currents are set up in the water, the warmer portions rising and the colder portions falling. If the vessel, instead of being placed upon the stove, is heated by the flame of a spirit lamp so that the heat is applied to a limited portion of the bottom of the vessel, these convection currents will be distinct and easily followed by the eye. The general manner in which the circulation takes place is indicated in Figure 140 at *A*. Evidently the reverse of this effect may be brought about by chilling that portion of the water which is near the center of the vessel, for example, by placing a body of ice on the surface of the water as indicated in Figure 140 at *B*. The convection currents in this case are exactly like those represented at *A*, except that they are reversed in direction.

The convection currents in a pond or other exposed body of water, as the cold weather of winter comes on, are of interest. Evidently those portions of the liquid which are first cooled are the surface layers. These portions tend to sink, as in the experiment illustrated in Figure 140, *B*, so that the convection currents will persist so long as the chilled portions at the top

are of greater density than the warm portions below. **Now at a temperature of 4° C. water has its maximum density.** If we imagine, therefore, that the convection currents described above have persisted until the whole pond is chilled to 4° C., it will be evident that a further cooling of the surface layers will tend to arrest this cooling process, that is to say, **as the surface layers become cooled below 4° C. they become lighter and therefore tend to remain at the surface.** The result is that **the body of water in the pond will be cooled by the process described, down to the temperature of 4° C. Any chilling effect below this will be confined to the surface layers.** It is altogether probable that this fact makes it possible for certain forms of animal life to

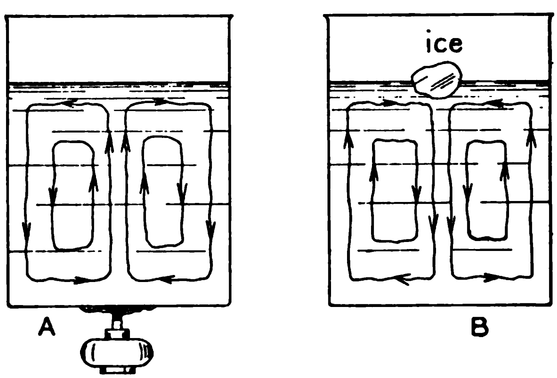


FIG. 140.—Convection Currents.

continue through the winter, which would not be possible if the chilling action due to the convection currents continued down to the freezing point, since in this case the pond would tend to become frozen from bottom to top.

The trade winds, which are winds experienced in regions a few degrees north and south of the equator, and which persist for long periods of time almost without change in direction, are convection currents on a large scale. They are due to the heating of the earth's atmosphere in the equatorial regions, which causes those portions of the atmosphere to rise and air to flow in from north and south to take the place of the ascending volumes. These currents coming from north and south

constitute the trade winds. If the earth were stationary and perfectly smooth on its surface, these trade winds would come in from the north and the south. Owing to the influence of the earth's rotation, however, these incoming currents are deflected toward the west, so that the trade winds south of the equator come from the southeast, and those north of the equator come from the northeast.

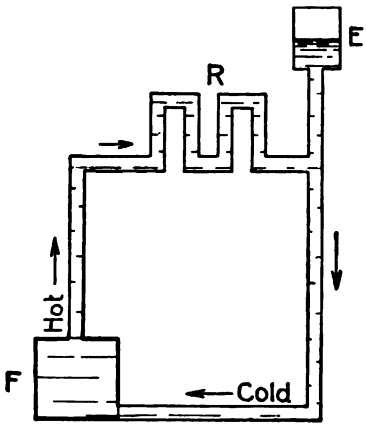


Fig. 140 *a*. — Hot-water Heating System.

A hot-water heating system, like that employed in dwellings, affords an example of the application of convection currents. Such a system is represented in Figure 140 *a*. *F* is the boiler in which the water is heated, *R* a radiator, and *E* the "expansion tank." The expansion tank allows for the increase in volume of the water when it is heated. The heated water rises through the pipe connected to the top of the boiler, passes through the radiators, where it gives up a portion of its heat, and then returns through the pipe connected to the bottom of the boiler to be again heated. The convection current in the apparatus is indicated by the arrows.

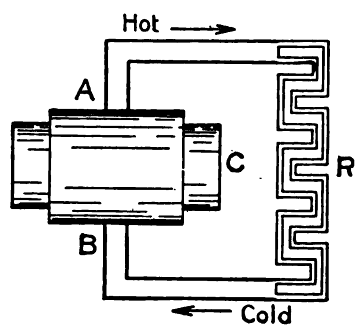


Fig. 140 *b*. — "Thermo-siphon" for cooling Cylinder of Gasoline Engine.

Another application of the convection principle is found in the "water-cooled" gasoline engine. Unless some means is taken to prevent it, the cylinder of a gasoline engine when in operation becomes excessively heated. In the water-cooled type the cylinder is cooled by means of a "water jacket." This consists of a hollow chamber filled with water which surrounds the cylinder. Figure 140 *b* illustrates

the arrangement in its simplest form. *AB* is the water jacket. The water, heated by contact with the hot cylinder, rises through a pipe at the top of the cylinder, and is replaced by cold water which flows in through a pipe at the bottom. In the automobile the hot water is made to pass through a radiator *R*, at the front of the machine, where it is cooled by the air which flows among the thin-walled pipes which makes up this part of the apparatus. After being cooled in this manner it returns to the water jacket and again passes around the circuit. In some machines the convection current alone is depended upon to maintain the circulation, in others the water is circulated by means of a pump placed in the circulating system and operated by the engine.

CONDUCTION

207. It will be evident from the very nature of the process that the transfer of heat by convection is possible only in liquids and in gases. The process of heat transference known as **conduction** takes place most readily in solids, although it is possible in liquids and in gases. In this process heat is handed on from particle to particle in the substance heated in the following manner: The first layer of the substance which is in contact with the source of heat becomes heated, that is, according to the kinetic theory its molecular parts are thrown into a state of rapid vibratory motion. Since, however, this layer is bound by the forces of cohesion to the adjacent layer it will be impossible for its parts to vibrate to any extent without dragging the adjacent layer also into motion; and this layer, because of the bonds which bind it to the next layer, will impart some of its motion to that layer; and so on, until every layer of the body participates in the motion. It is evident that in this process as well as the convection process the entire medium through which the heat has been transferred becomes heated.

Some substances possess the property of transmitting heat in this manner much more readily than others. These substances are spoken of as **good conductors** of heat. Those which do not transmit heat by this process so readily are spoken of as **bad conductors.**

THERMAL CONDUCTIVITY

208. Experiment shows that the quantity of heat conducted through a layer of any substance in a given time is proportional (*a*) to the area across which the heat flows, (*b*) to the difference in temperature between the two surfaces, (*c*) to the time, and (*d*) varies inversely as the thickness of the layer. Consider the case represented in Figure 141. *AB* is a vessel of water standing on a stove *CD*. The water is assumed to be boiling and remains therefore at constant temperature, call it t_2. Let the temperature of the stove be t_1. Call the area of the bottom of the vessel *A*, and its thickness *d*. Let *Q* represent the quantity of heat conducted from the stove to the water through the bottom of the vessel. We have, therefore, from the above general statement, —

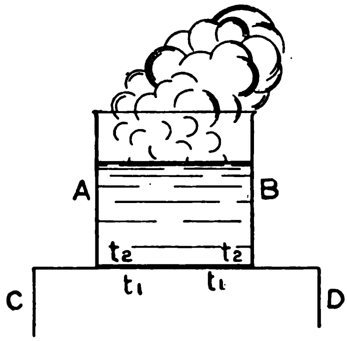


FIG. 141. — Flow of Heat from a Stove to a Vessel of Water.

$$Q \propto \frac{A(t_1 - t_2)}{d} \times \text{time}$$

or,
$$Q = \frac{K \cdot A(t_1 - t_2)}{d} \cdot \tau$$

If it is desired to find **the rate at which heat is conducted from the stove to the water**, we have, —

$$\frac{Q}{\tau} = KA \cdot \frac{(t_1 - t_2)}{d} \tag{67}$$

The quantity *K* is called the **thermal conductivity** of the substance. It is evidently equal to the heat transferred in one second through a layer of the substance one centimeter thick, the area of each face being one square centimeter, the difference of temperature between these faces being one degree on the Centigrade scale.

THERMAL CONDUCTIVITIES

SUBSTANCE	TEMPERATURE (CENTIGRADE)	THERMAL CONDUCTIVITIES
Silver	0°	1.096
Copper	15°	0.713
Aluminum	0° to 100°	0.343
Tin	0° to 100°	0.152
Iron	15°	0.149
Lead	0° to 100°	0.083
Alcohol	13°	0.00054
Water	18°	0.00124
Hydrogen	7.56	0.00033
Air	7.56	0.000051

THE MEASUREMENT OF CONDUCTIVITY

209. The thermal conductivity of a given substance is usually determined by some form of experiment involving the use of the relation given in Equation (67). The thermal conductivities of metals may be roughly compared by the following experiment: *a*, *b*, *c*, and *d*, Figure 142, represent wires of copper, zinc, iron, and lead. If the junction point *A* of these several wires is heated by means of a Bunsen burner, heat will be conducted along each of the wires. It will be conducted quite readily along the copper wire, less readily along the zinc, and so on, so that after the lapse of a given time the extremities of the wires *a*, *b*, *c*, and *d* are heated to an extent which depends upon the thermal conductivities of the metals of which they are composed. If an unlighted match is applied to the end of each wire and moved slowly along toward the source, as soon as a point is reached for which the temperature is equal to that of ignition, the match will be lighted. If this

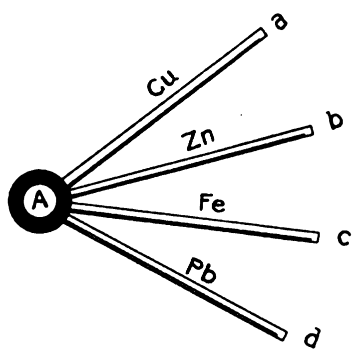


FIG. 142.—Conductivity Experiment.

point is found to be far from A, the conclusion is that the substance of the wire in question is a good conductor, that is, its thermal conductivity is high. If the point of ignition is found to be near the source A, the thermal conductivity is low.

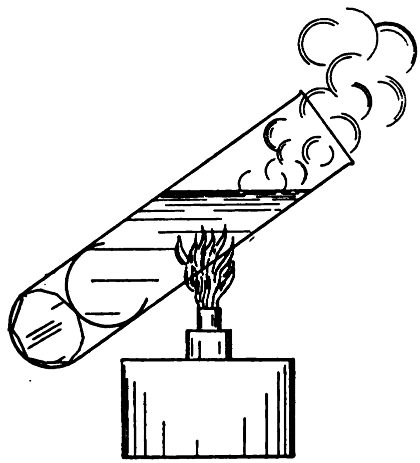


FIG. 143.—Illustrating the Low Conductivity of Water.

The determination of the conductivity of liquids and gases is a matter of extreme difficulty for the reason that it is almost impossible to arrange an experiment in such manner as to eliminate the effects of convection and radiation. In fact, in the most satisfactory determinations which have been made, these effects have not been eliminated, but have been corrected for. Thus the statement might be made that it is not possible to measure the conductivity of liquids and gases directly. That liquids, for example, have low thermal conductivities in general may be demonstrated by experiments like the following.

If a test tube is partly filled with ice, the ice being held in the lower part of the test tube, and the test tube is then filled with water, it may be placed in the flame of a Bunsen burner or alcohol lamp in the manner indicated in Figure 143, and the water will boil in the upper part of the test tube without melting the ice below. This, of course, indicates that but small amounts of heat are transmitted by conduction through the water.

In Figure 144 is shown another simple experiment for demonstrating the low value of the thermal conductivity of liquids, for example, water. A is an air thermometer, the bulb of which is placed within a suitable quantity of water contained in the vessel B as indicated. Upon the surface of the water in B a small metallic vessel C is caused to float. This vessel con-

tains a small quantity of alcohol. If the alcohol is ignited it will burn at high temperature and develop large quantities of heat. The upper layers of the water in *B* will thus be subjected to a high temperature. If any appreciable amount of heat were conducted by the upper layers of the liquid in *B*, evidently the effect would be apparent by the falling of the column in the air thermometer. It will be found, however, even after the lapse of considerable time, that the column in the air thermometer remains immovable, thus indicating that practically no heat has been received by the bulb of the air thermometer, although it is located but a short distance from the burning alcohol.

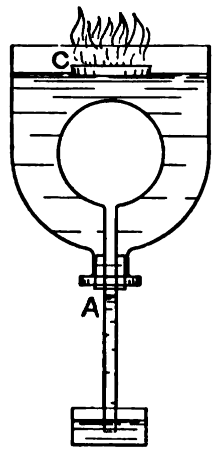


FIG. 144. — Another experiment showing that Water is a Poor Conductor of Heat.

THE TEMPERATURE GRADIENT

210. Consider a wall one side of which is at a temperature T_1 and the other at a temperature T_2. If $T_1 > T_2$ there will be a flow of heat from the side having a temperature T_1 to the other. It is interesting to inquire how the temperature varies from point to point **within the wall.** In the case represented in Figure 145, the point *a* has a temperature T_1 and *f* a temperature T_2. The intermediate points *b*, *c*, *d*, *e* are at different temperatures between the limits T_1 and T_2. It must be evident that the temperature of *b* is lower than that of *a*, but higher than that of *c*. The temperature of *c* is lower than that of *b* and higher than that of *d*, etc. In other words, **there is a fall of temperature from point to point in the direction of the flow of heat.** That such is the case follows at once from the principle that heat can flow only from a higher to a lower temperature. The ratio of

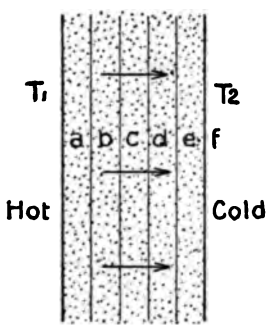


FIG. 145. — Temperature Gradient.

the difference in temperature between two points to the distance between them is called the **temperature gradient**.

RADIATION

211. If a thermometer is held a short distance below a hot body, it will receive heat from the hot body. The heat received in this manner is evidently not to be accounted for by convection, since convection currents would tend to convey the heat in an upward direction from the hot body, as we have seen. Furthermore, air being a poor conductor, the amount of heat received by the thermometer in such an experiment cannot be accounted for by conduction. It is still more evident that the heat transferred from the hot body to the thermometer has taken place by neither of these processes when we determine by further experiment that this heat transfer takes place even though the hot body and thermometer are placed in a vacuum. The heat received by the thermometer in this experiment is said to be **radiated** from the hot body, and the process of heat transfer involved is known as **radiation**.

Attention has been called to the fact that in both the convection and conduction processes the medium of transfer is a material substance. Radiation is distinguished from conduction and convection by the fact that it may take place in a vacuum. In radiation the medium of transfer is the **ether**, a medium which is supposed to extend throughout all space and to fill those portions of space which are occupied by ordinary matter as well as those which are vacuous. Under the study of light and electricity reference will be made to this medium, which is supposed to transmit light and electric disturbances as well as heat, and a more complete discussion of its properties will be given when those subjects are taken up. It is sufficient for our present purpose to refer to the existence of this medium and to show that upon the assumption of the existence of such a medium the phenomena of radiation are easily explained.

Another fact which distinguishes radiation from the other processes of heat transfer is that the medium through which radiation takes place is not heated in the process. The earth receives vast quantities of heat from the sun, although the

space which separates the earth from the sun remains very cold.

The transfer of heat by this process is supposed to take place by means of a wave motion in the ether. A hot body is capable of setting up a wave motion in the ether which is in contact with it, its ability to do this depending, of course, upon the vibratory motion of its molecular parts. This wave motion spreads through the ether, and when it falls upon a material substance is able to impart vibratory motion to the molecular parts of that substance, that is to say, it is able to raise the temperature of that substance. Thus the transfer of heat from one body to another by the process of radiation is to be thought of as a double process: first, the conversion of the heat energy of the hot body into wave motion of the ether; second, a reconversion of the ether wave motion into heat in the body warmed.

PREVOST'S THEORY OF EXCHANGES

212. The various phenomena of rise and fall of temperature in bodies due to this process of radiation are best explained by Prevost's theory of exchanges. Briefly stated, this theory is as follows: That **all bodies, cold or hot, radiate heat.** Other things being equal, **hot bodies radiate heat more rapidly than cold ones; but whatever the temperature of the body and whatever its surroundings, it is at all times radiating heat. All bodies are also to be thought of as receiving heat or absorbing heat which has been radiated from surrounding bodies.** Therefore, the condition of a body as to temperature is determined by the ratio of the heat which that body radiates to the heat which it absorbs. If the heat radiated by a body is just equal to that absorbed, its temperature will remain constant. If it radiates more heat than it absorbs, its temperature will fall. If it absorbs more than it radiates, its temperature will rise.

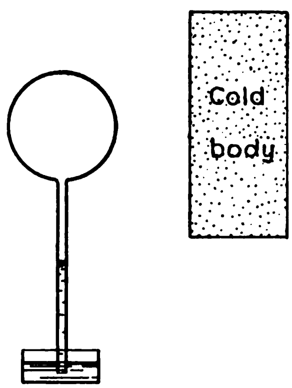


FIG. 146. — Illustrating Prevost's Theory of Exchanges.

This theory affords an explanation for what appears to be the radiation of cold. For example, if an air thermometer is placed near a cold body, as indicated in Figure 146, the thermometer will immediately indicate a fall of temperature, as if "cold" had been radiated from the cold body to the bulb of the thermometer. Under Prevost's theory the explanation of the fall in temperature is as follows: The air thermometer is both radiating and absorbing heat. Since it is warmer than the cold body near it, it is radiating heat more rapidly than the cold body. Therefore the quantity of heat radiated by the thermometer to the cold body is greater than that radiated to the thermometer from the cold body. This results in a net loss of heat in the air thermometer and therefore a fall in temperature.

DEPENDENCE OF RADIATION UPON THE CHARACTER OF THE SURFACE OF THE RADIATING BODY

213. It is found by experiment that **the amount of heat radiated by a body depends** first of all, as indicated above, upon its temperature; second, **upon the character of its surface.** Certain surfaces seem to facilitate the process of radiation, while others are not so well adapted to the process. Generally speaking, a rough black surface radiates well. Lampblack, for example, is an excellent radiating surface as compared with other substances. In fact, in comparisons of this kind it is customary to consider lampblack as a perfect radiator, and the amount of heat radiated from a lampblack surface, other things being equal, is taken as 100 per cent. It should not be thought in this connection that the color black is especially significant, for it can readily be shown by experiment that certain white substances are almost as good radiators as lampblack, for example, ordinary white unglazed paper radiates almost as well as lampblack.

Polished surfaces in general are poor radiators.

RADIATION AND ABSORPTION

214. The facility with which a given surface absorbs heat is found to be in every case proportional to the facility with which it radiates. That is to say, **a good radiator of heat is a**

good absorber, and a surface which radiates heat slowly will absorb it slowly, other things being equal. The equality of radiation and absorption as depending upon the character of the surface is shown by the experiment illustrated in Figure 147. *ABCD* represents a hot body. The surface *AB* is coated with lampblack. Opposite this face is placed an air thermometer *E* having, for convenience, a flat bulb. The side of the thermometer which is turned toward the hot body is of polished metal. The side *DC* of the hot body is of polished metal like that used in the thermometer *E*. Opposite this face stands an air thermometer *F* similar to *E* except that the face which this thermometer presents to the hot body is coated with lampblack. If the distances which separate the thermometers *E* and *F* from the hot body are the same, the rise in temperature indicated by the two thermometers will be the same. The lampblack surface *AB* radiates more heat than the polished metal surface *DC*. On the other hand, the polished metal surface of the thermometer *E* is capable of taking up but a small fraction of the large quantity of heat radiated to it from the lampblack surface, while the lampblack surface of the thermometer *F* absorbs most of the heat which falls upon it from the polished metal surface *DC*. Evidently if the rise in temperature indicated by the two thermometers is the same, this experiment affords a proof that the radiating and absorbing properties of a given surface are equal.

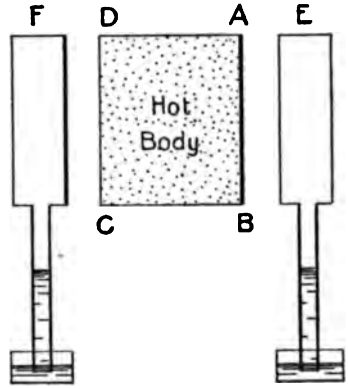


FIG. 147. — Apparatus for demonstrating Equality of Radiation and Absorption.

THE TRANSMISSION OF RADIATED HEAT BY MATERIAL SUBSTANCES

215. The wave motion in the ether which constitutes radiation is found by experiment to be able to pass through certain material substances with more or less facility, and to be quite

completely intercepted by thin layers of other substances. We have, therefore, to distinguish between those bodies which allow this wave motion to pass through them and which are therefore "transparent to radiated heat," and those which are in this sense opaque.

The readiness with which this wave motion passes through a given substance depends upon the wave length of the disturbance. We have already referred to this radiation as being made up of waves. It should also be borne in mind that the waves given out by a radiating body are not all of the same wave length; in fact, a radiating body is to be thought of as giving off short waves and long waves and intermediate waves of various lengths. If the heat waves given off in this manner are of a certain wave length (about the 50,000th part of an inch), they affect the retina of the eye and are called light waves. If they are too long to affect the optic nerve, they are called dark heat waves. A given material like glass is found to transmit radiation in the form of short waves (light waves) and at the same time to be quite opaque to the long, dark heat waves. This explains how a "hotbed" which is covered with glass is warmed in the early spring. The energy which is passed into the hotbed in the form of light is absorbed in part by the surface of the soil upon which it falls. The surface of the soil gradually becomes heated, and would tend to cool by radiation, but **the heat waves given off by the soil under these circumstances are long waves, that is, dark heat waves.** These are intercepted by the glass, and radiation of this kind of heat from the soil is prevented. Thus the heat is, as it were, entrapped and the result is a rise in temperature of the soil.

REFLECTION OF RADIATED HEAT

216. The wave motion of the ether which constitutes radiation is reflected more or less completely by certain surfaces in exactly the same manner that light is reflected. The law of reflection, namely, that the angle of reflection is equal to the angle of incidence, which applies in the case of the reflection of light, applies to the reflection of heat. In general, the amount of this wave motion which is reflected depends upon

the angle of incidence and upon the character of the surface. Polished surfaces of course reflect better than rougher surfaces.

In general, the radiated heat which falls upon the surface of a body is divided into three parts:

(*a*) That part which is absorbed and tends to raise the temperature of the body upon the surface of which it falls.

(*b*) That part which is transmitted, that is to say, which passes through the body.

(*c*) That part which is reflected.

Evidently $(a)+(b)+(c)$ must be equal to the total heat which falls upon the surface of the body in question. In certain cases the part (*a*) will be large, and (*b*) and (*c*) relatively small. In other cases (*b*) may be large, or (*b*) and (*c*) relatively large and (*a*) small, and so on.

Problems

1. A sheet of copper has an area of 100 sq. cm. and a thickness of 6 mm. The temperature of one side is 100° C., that of the other, 0° C. How much heat is conducted through the plate per second? Thermal conductivity of copper = 0.713 c. g. s. unit.

2. Water is boiled at atmospheric pressure in an iron vessel 6 mm. thick. The heating area of the vessel is 1 sq. m. If the surface exposed to the fire is kept at 250° C., how much water will be evaporated per hour? Thermal conductivity of iron = 0.119 c. g. s. unit.

3. A man is clothed in a fabric 3 mm. thick, the thermal conductivity of which is 0.000122. If the temperature of his body is 30° C. and that of the air is 0° C., how much heat does he lose from 100 sq. cm. of the surface of his body per hour?

4. A circular tank of water 2 m. in diameter is covered with ice 4 cm. thick. The thermal conductivity of ice is 0.0023 c. g. s. unit. If the air is at a temperature of − 20° C., how much heat is transmitted through the ice per hour?

5. A wall is built of a material having a thermal conductivity of 0.0072 c. g. s. unit. If 360 calories are conducted through the wall per square meter per second, what is the temperature gradient in the wall?

6. When the temperature gradient in a metal is 20 degrees/cm. it conducts 840 calories per square centimeter per minute. What is the thermal conductivity of the metal?

THERMODYNAMICS

CHAPTER XIX

CONVERSION OF WORK INTO HEAT

217. **Thermodynamics** is that branch of physics which **treats of the transformation of mechanical energy into heat and the transformation of heat into mechanical energy.**

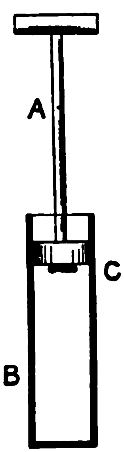


FIG. 148. — The Fire Syringe.

That mechanical energy may be transformed into heat is demonstrated by the familiar phenomena of friction. When a metal button is rubbed on cloth or wood, it becomes heated. The journals or bearings of a car are sometimes strongly heated by friction, causing a "hot box." Certain tribes of savages start fires by rubbing sticks of wood together. A simple experiment for illustrating the transformation of mechanical energy into heat is the following: In Figure 148, *B* represents a hollow cylinder having a tight-fitting piston *A*. If the piston is forced into the cylinder, the air contained in the cylinder will be compressed and heated, the work done in moving the piston being transformed into heat. If the cylinder is filled with air at ordinary room temperature and pressure and the piston is very quickly forced into the cylinder, the temperature attained by the compressed air may be high enough to ignite a bit of inflammable material *C* (tinder) attached to the piston, which will continue to burn after the piston is withdrawn.

An interesting illustration of the transformation of heat into mechanical energy is afforded by the simple form of Hero's steam engine, illustrated in Figure 149.

B is a small boiler suspended from a suitable support by means of two cords, as shown. This boiler is provided with two small

tubes extending radially from opposite sides, their outer ends being bent horizontally at right angles and in opposite directions.

A small amount of water having been placed in the boiler, steam is generated by placing a Bunsen burner or alcohol lamp beneath it. The reaction of the steam escaping from the side pipes gives the well-known Hero engine effect and the boiler is set into rapid rotation about a vertical axis. This rotation twists the suspending cords, and the boiler is steadily lifted from the flame. When it has reached a certain elevation, steam will no longer be generated and the motion will cease. The boiler will now descend under the action of gravity and the cords in untwisting will cause the boiler to revolve in the reverse sense. Descending in this manner toward the flame, the boiler will eventually reach a position in which steam will again be generated. The escaping steam by its reaction will stop the backward rotation of the boiler, reverse its motion, and cause it to revolve as in the first instance. Once more it will "climb" out of reach of the flame, and the action will be repeated.

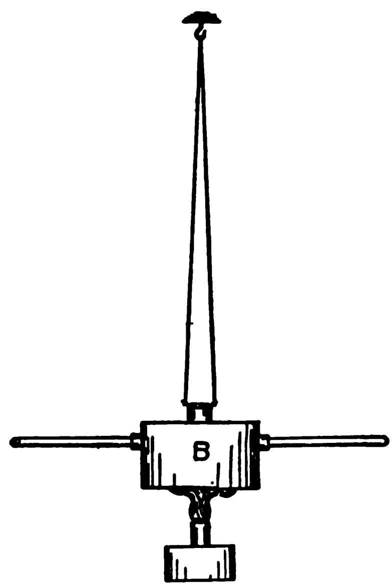


FIG. 149.— Hero's Engine.

The energy transformations are as follows: The chemical potential energy of the gas or alcohol vapor in the presence of the oxygen of the air is, by combustion, transformed into heat. This heat is transformed into the potential energy of the hot steam, the energy of the steam being again transformed into the kinetic energy of the rotating boiler. Finally this kinetic energy is transformed into the gravitational potential energy of the lifted boiler.

As the boiler descends, its potential energy is partly trans-

formed into kinetic energy of rotation and a part is used in doing work against the resisting forces caused by the reaction of the escaping jets of steam as the boiler slows down to its position of momentary rest just above the flame.

Ordinarily, the oscillations of the boiler as it rises and falls will become less and less until it comes to rest at such elevation above the flame that the reaction of the escaping steam will just balance the torque action of the twisted string.

THE FIRST LAW OF THERMODYNAMICS

218. Mechanical energy may be transformed into heat and heat may be transformed into mechanical energy, and in every case of a transformation of this character the ratio of the quantity of heat to the quantity of mechanical energy involved is constant. That is,

$$W = J \cdot H \tag{68}$$

This is known as the first law of thermodynamics.

It has been found by experiment that whenever mechanical energy is wholly converted into heat, for every 4.187 joules of mechanical energy that disappears one calorie of heat is developed; that is,

$$1 \text{ calorie} = 4.187 \text{ joules}$$

i.e.

$$J = 4.187$$

This number 4.187 (the factor J, Equation 68) is called the **mechanical equivalent of heat.**

In the f. p. s. system of units the mechanical equivalent of heat is 778, which means that, —

$$1 \text{ B. T. U.} = 778 \text{ foot-pounds.}$$

The method used in determining the mechanical equivalent of heat was to churn a given quantity of water by means of paddle wheel rotating in a suitable vessel. The amount of work done in turning the paddle wheel was measured, and the rise in temperature of the water and containing vessel, which served in this experiment as the calorimeter, was noted. The amount of heat developed was therefore known; and by comparing this quantity of heat with the work expended in turning

the paddle wheel, proper allowance being made for radiation and other sources of error, the above ratio was determined.

THE SECOND LAW OF THERMODYNAMICS

219. It is impossible for heat of itself to pass from a cold to a hot body. This is known as the second law of thermodynamics.

Having in mind the analogy which was employed at the beginning of this subject, namely, that heat flows from regions of high temperature to regions of low temperature in much the same manner that water flows from a high level to a low one, the significance of the second law becomes at once apparent. It is conceivable, of course, that heat **may be made to pass** from a cold body to a hot one, just as we may pump water from a lower to a higher level. Thus, in the operation of the ammonia refrigerating machine, heat is continually being abstracted from the cold brine in the brine tank and transferred to the relatively hot water which fills the cooling tank, so that the apparatus constitutes a **heat pump** which transfers heat from the cold brine to the warmer cooling tank.

It should be carefully noted, however, that this transfer of heat from the cold brine tank to the warm cooling tank goes forward only so long as mechanical energy is supplied from some outside source to operate the pump. As soon as the pump stops, as soon as the supply of mechanical energy from the outside is cut off, heat will begin to pass in the opposite direction and by conduction, convection, and radiation will pass from the warmer to the cooler parts of the apparatus.

THE STEAM ENGINE

220. The simplest form of steam engine cylinder is that shown in Figure 150. *A* and *B* are two pipes connected to the cylinder *C*, each of which serves alternately as inlet and outlet for the steam. When the piston *P* is moving in the direction of

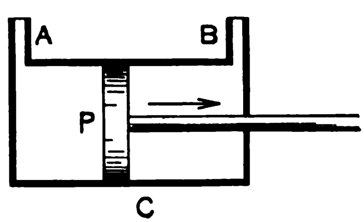


FIG. 150. — Simple Steam Engine Cylinder.

the arrow, the steam is entering at *A* and the "cold steam," which on the preceding stroke pushed the piston to the upper end of the cylinder, is "exhausting" (flowing out) at *B*. When the piston has reached the bottom of the cylinder, steam is admitted at *B*, and *A* is connected to the exhaust. Thus the hot steam from the "boiler" is admitted alternately at *A* and *B* and pushes the piston to and fro in the cylinder.

The steam may be allowed to flow throughout the entire stroke, but in that case the exhaust steam is nearly as hot as the steam in the boiler. The engine working in this way is inefficient because a great deal of heat energy is carried away in the exhaust steam and is lost. To increase the efficiency of the engine, steam is admitted during a part of the stroke only, the stroke being completed by the **expansion of the steam within the cylinder.** During this expansion the steam cools, **that is, continues to give up heat energy.** A larger part of the heat energy of the steam is thus made available and the efficiency of the engine increased. It is advantageous also to close the exhaust ahead of the piston before it quite reaches the end of its stroke. By this means the pressure and temperature in the cylinder is raised before steam is admitted to nearly that of the boiler.

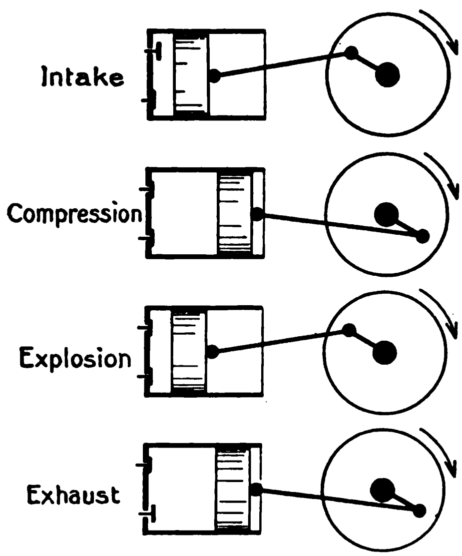


FIG. 151. — Diagram showing the Successive Operations in a Gasoline Engine Cylinder.

THE GASOLINE ENGINE

221. The source of energy in the gasoline engine is the heat evolved in the combustion of a mixture of gasoline vapor and air. During the combustion, which is made to take place within the cylinder of the engine, a gas at high pressure is

evolved. This gas in expanding does work upon the piston very much as the steam does in a steam engine. In the single acting engine there is one explosion for two revolutions of the engine or four strokes of the piston. During the first stroke the explosive mixture is drawn into the cylinder. In the second stroke the mixture is compressed. The explosion occurs on the third stroke, and during the fourth stroke the products of combustion are forced out of the cylinder. These successive operations are represented in order in Figure 151.

MECHANICAL REFRIGERATION

222. The process of mechanical refrigeration is essentially the reverse of that employed in the steam or gasoline engine. In the steam engine the steam passes from the boiler at high temperature to the cylinder, there **giving up a portion of its heat energy as it does work on the piston.** In the refrigerating apparatus the working substance is drawn into the cylinder of a **compressor** at low temperature and **is heated as work is done upon it by the piston.** After giving up its excess of heat it is allowed to expand, and by the cooling effects of expansion and vaporization it reaches a refrigerating temperature. One of the common forms of refrigeration apparatus is that in which ammonia is used, the cooling effect being secured by the vaporization of the liquid ammonia. A simple device of this character is represented diagrammatically in Figure 152. *ABC* is a force pump or compressor. The pipe connections at *A* and *B* are provided with valves, that at *A* opening into the cylinder, that at *B* opening out from the cylinder so that on the upstroke of the plunger the pump acts as an air pump, *A* being the intake. On the downstroke it acts as a force pump, *B* being the outlet. In the operation of the pump ammonia gas is drawn in at *A*, compressed in the cylinder, and forced under relatively high pressure (about 10 atmospheres) into the coil of pipe represented at *I*. This compression results in a strong heating of the compressed ammonia which is partly in the liquid state and partly in the form of vapor as it enters the coil *I*. The coil *I* is surrounded by cold water, the tank *EF* being supplied by a constant stream of

water. This cools the ammonia in the coil *I* to, let us say, a few degrees below ordinary room temperature. When cool the ammonia, being still under high pressure, is allowed to escape through a regulating valve *D* into the coil *J*. This coil *J* is continually being exhausted by the pump *ABC* so that within this coil there is low pressure. The liquid therefore, as it passes the valve *D* in the liquid state from the region of high pressure to the region of low pressure vaporizes in much the same way that the CO_2 does in the experiment described in Section 185. The result is that the ammonia vapor in the coil

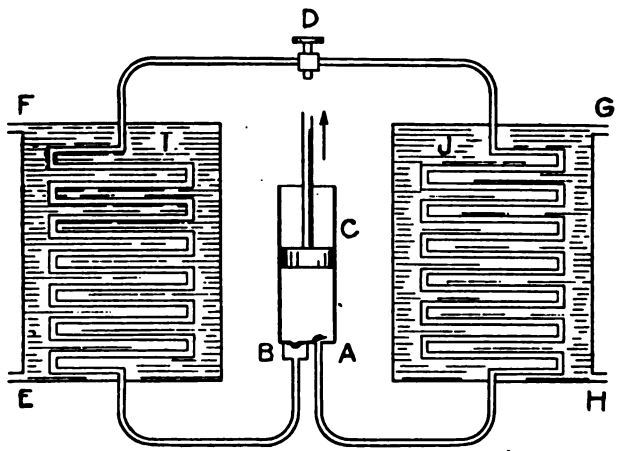


FIG. 152.—Refrigerating Machine.

J together with the coil and its surroundings are lowered in temperature. This cooling effect is sufficient under the circumstances described to reduce the temperature of the coil *J* and its surroundings considerably below the freezing point of water. The vapor which is thus formed in the coil *J* is again taken up by the pump and is made to pass once more through the cycle as described, and so on. For the convenient utilization of this low temperature the coil *J* is usually immersed in a tank of brine which, being in contact with the coil *J*, is cooled down to a temperature below the freezing point of water. This cold brine is then pumped into coils arranged much the same as radiators are arranged in a steam heating plant. In

this manner the rooms in which the cooling coils are placed are cooled.

Ammonia lends itself with advantage to the refrigeration process described above because of the fact that it can be converted from the vapor state to the liquid state at ordinary temperatures by the application of pressure alone. If an attempt were made to use a gas like air, oxygen, or hydrogen in place of the ammonia, it would not be found possible to secure the same result; since, no matter how much pressure is applied to oxygen or hydrogen at ordinary temperatures, it is impossible to change them over into the liquid form (Section 186). However, a similar effect in smaller degree may be secured by using an ordinary gas in the refrigerating apparatus described. In this case the lowering of the temperature of the tank *J* is due to the **cooling effect of expansion in a gas.** One of the disadvantages of the ammonia process is, that ammonia gas is dangerous to life in case it escapes from the apparatus.

THE PRODUCTION OF "ARTIFICIAL ICE."

223. In the production of artificial ice, brine from the brine tank (*J*, Figure 152) of a refrigerating plant is caused to circulate about pans filled with the water to be frozen. The water gives up heat to the cold brine with which the pans are in contact, and thereby becomes lowered in temperature until it freezes.

WATT'S DIAGRAM

224. It has already been pointed out that the physical state of a gas is completely represented by a point in the pressure-volume diagram. In the same way the successive states through which the gas passes, because of changes in its pressure, or its volume, or both, are represented by the successive points on a curve. The volumes represented in such a diagram may be either total volumes of a given mass of gas or volumes per unit mass. When such a diagram is drawn to represent the relations between the pressure and total volume of the gas, it is called Watt's diagram.

AREA IN WATT'S DIAGRAM REPRESENTS WORK

225. In Watt's diagram the area **under a process curve, that** is, the area bounded by the curve, its end ordinates, and the axis, represents the work done on the gas, or by the gas, during the change of its physical state represented by that curve. This will be readily understood from the following illustrations: In Figure 153 let CC represent a cylinder fitted with a frictionless piston and containing a given mass of gas at pressure p and volume v_1. The state of the gas may be represented by the point A in Watt's diagram in the upper part of the figure. Let it be imagined that the gas in the cylinder expands without change of pressure until its volume is increased to v_2. Its condition will now be represented by the point B.

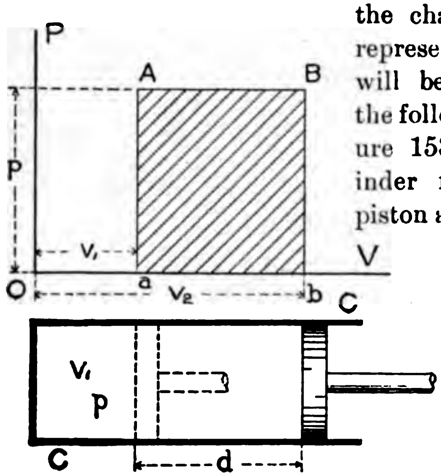


FIG. 153. — Watt's Diagram, Constant Pressure.

The work done by the gas during its expansion is determined as follows: Call the area of the piston s. The total force with which the gas pushes upon the piston is then ps. This force moves the piston through the distance d as shown in the figure. The work done is therefore

$$W = F \cdot d$$
$$= psd$$

But sd is the change (increase) in volume of the gas. It follows therefore that **the work done by the expanding gas is numerically equal to the product of the pressure and change in volume.** That is

$$W = p(v_2 - v_1) \qquad (69)$$

Now, – Ob represents v_2 to scale and Oa represents v_1. Thus ab represents $(v_2 - v_1)$. Also aA represents to scale the value of the pressure.

$$\therefore \; W \propto aA \times ab.$$

But $aA \times ab$ is the area of the rectangle $aABb$. Therefore, the work done by the expanding gas during the process described is represented by (is proportional to) the area under the corresponding curve AB.

If the process is one in which both pressure and volume change, the same relation holds, since evidently in this case the work done is equal to the product of the change in volume and the average pressure during the volume change, while the area under the curve is given by the product of ab and the average ordinate of the curve AB, Figure 154.

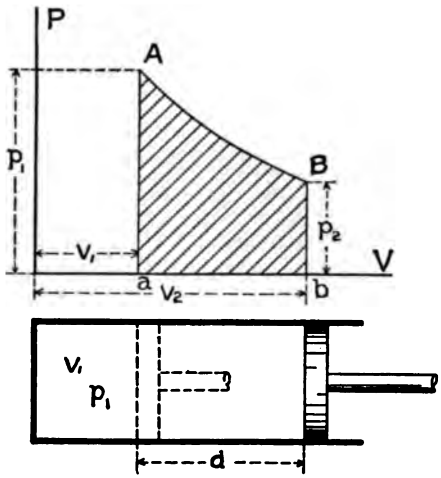


FIG. 154. — Watt's Diagram, Pressure Variable.

In a process like that represented by the curve AB, Figure 153 or Figure 154, the volume of the gas increases and the gas does work. If the process is reversed, work must be done on the gas. Thus the area $aABb$, Figure 153, represents the work which would have to be done upon the gas to reduce its volume from v_2 to v_1 without change of pressure.

In Figure 154 $aABb$ represents the work which would have to be done upon v_2 cubic centimeters of gas at a pressure p_2 dynes per square centimeter to reduce its volume to v_1 cubic centimeters, its pressure rising during the change of condition to p_1 dynes per square centimeter.

ISOTHERMAL AND ADIABATIC PROCESSES

226. There are two distinct processes by which a gas may expand to an increased volume and diminished pressure, (1) by **isothermal expansion**, (2) by **adiabatic expansion**.

An isothermal process is one in which the temperature of gas remains constant. In order that a given body of gas may

undergo such a process, heat must be imparted to it (if the gas expands) or abstracted from it (if the gas is compressed). Consider the gas inclosed in the cylinder of any form of heat engine. As the gas expands it pushes the piston back and does work. The source of this work is the heat energy of the gas. But if the gas gives up some of its heat energy, it cools. **If, therefore, the gas is to expand without cooling, it must be supplied with heat during the process of expansion.** Similarly, if the gas in such a cylinder is compressed, its temperature will rise, that is, its heat energy will increase. The source of this increase of heat energy is the work done in compressing the gas. **If, therefore, the gas is to be compressed without rise of temperature, heat must be abstracted from it during the process of compression.**

An adiabatic process is one in which there is no interchange of heat between the gas and its surroundings. Such a process is always accompanied by a change in the temperature of the gas. Consider the gas inclosed in the cylinder of any form of heat engine. As the gas expands, it pushes the piston back and does work. The source of this work is the heat energy of the gas. Hence, **if the gas receives no heat from its surroundings during the process, its temperature will fall as it gives up the heat energy which is transformed into work.** Similarly, the work done in compressing the gas in such a cylinder is transformed into heat, and **if the gas loses no heat to its surroundings during the process, its temperature will rise.**

These processes may be approximately realized in a cylinder filled with compressed gas. (1) Imagine the gas to expand very slowly. This will be an **isothermal** process, since by heat conduction from the walls of the cylinder the temperature of the gas will be kept constant. (2) Imagine the gas to expand very quickly. This will be an **adiabatic** process, since no appreciable amount of heat can flow from the cylinder walls to the gas during the expansion.

CARNOT'S CYCLE

227. In order that work may be obtained by repeated expansion and compression of a given body of gas, it will be evident that the expansion and compression processes must be different.

Consider the process represented by the curve *AB*, Figure 154. When the gas expands from the volume v_1 to the volume v_2, an amount of work, represented by the area *aABb*, is done **by the gas.** If now the process is reversed and the gas is compressed from the volume v_2 to the volume v_1, an equal amount of work, represented again by the area *aABb*, is done **on the gas.** Evidently an engine working in this way could do no external work, since all of the work done by the gas during the expansion stroke would be required to compress the gas during the compression stroke. It follows, therefore, that **the expansion and compression processes**, through which the gas in the cylinder of a heat engine is carried, **must be different if the engine is to be capable of doing external useful work.**

When the gas in a heat engine is carried through a number of processes and returned to its initial condition, it is said to pass through **a cycle** of operations.

An ideal cycle for the heat engine was suggested by Carnot. Carnot's cycle consists of four processes as follows:

(1) Isothermal expansion	(temp. T_1)
(2) Adiabatic expansion	(from temp. T_1 to temp. T_2)
(3) Isothermal compression	(temp. T_2)
(4) Adiabatic compression	(from temp. T_2 to temp. T_1)

This cycle is represented in Figure 155. *AB* represents (1) isothermal expansion at the temperature T_1. *BC* represents (2) adiabatic expansion, during which the temperature of the gas falls from T_1 to T_2. *CD* represents (3) isothermal compression at the temperature T_2. *DA* represents (4) adiabatic compression, during which the temperature of the gas rises from T_2 to T_1. The work done by the gas in process (1) is represented by the area *aABb*, and in process (2) by *bBCc*. The total work done **by the gas** is therefore represented by *aABCc*, that is, the area under the line *ABC*. The work done on the gas in process (3) is represented by the area *cCDd* and in process (4) by *dDAa*. The total work done **on the gas** is therefore represented by *cCDAa*, that is, the area under the line *CDA*. The work done by the gas exceeds the work done on the gas by an amount represented by the area *ABCD*. Call this work *W*.

During the process (1), a certain amount of heat must be supplied to the gas (Section 226). Call this heat H_1. During process (3) a certain amount of heat is rejected by the gas. Call this heat H_2. Then the difference between the heat received and the heat rejected by the gas during the cycle is $H_1 - H_2$. The important results of the cycle are, therefore, as follows:

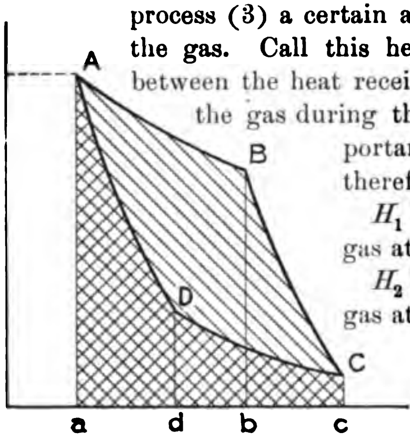


FIG. 155. — Carnot's Cycle.

H_1 heat units are taken up by the gas at the temperature T_1.

H_2 heat units are rejected by the gas at the temperature T_2.

W units of work have been done by the gas.

In other words, $H_1 - H_2$ units of heat have disappeared in the operation, and W units of mechanical energy have made their appearance. It follows, therefore, from the first law of thermodynamics, that,

$$W = J \cdot (H_1 - H_2)$$

EFFICIENCY OF AN IDEAL HEAT ENGINE

228. Carnot imagined an engine in which this theoretical cycle might be realized. This he called the ideal heat engine. The study of Carnot's theoretical cycle and ideal engine leads to a number of important principles of thermodynamics.

The efficiency of a heat engine is defined as the ratio of the heat transformed by the engine into work to the total heat received by the engine. For Carnot's ideal engine, we have, therefore,

$$E = \frac{H_1 - H_2}{H_1} \tag{70}$$

in which E is the efficiency.

The most important property of Carnot's cycle is that it may be reversed. That is, the ideal engine may take a quantity of heat H_2 from a source at a temperature T_2 and deliver a quantity of heat H_1 at a higher temperature T_1, providing an amount of work represented by the area $ABCD$, Figure 155, is

done on the gas during the cycle. Carnot's cycle is therefore called a **reversible cycle.**

The more important principles deduced from a study of Carnot's cycle are as follows:

An engine having a reversible cycle has the greatest possible efficiency;

All engines having reversible cycles, whatever the nature of the gas or working substance, have the same efficiency; **and**

The efficiency of a reversible engine depends only upon the temperatures T_1 and T_2 between which the engine works. As a matter of fact it may be shown that the expression for the efficiency of a reversible engine given above is equivalent to

$$E = \frac{T_1 - T_2}{T_1} \tag{71}$$

This relation leads to the conception of a new scale of temperatures, depending only upon Carnot's cycle and **independent of the nature or properties of any particular kind of matter.** Lord Kelvin devised such a scale, called the thermodynamic scale, and found that it did not differ materially from that of the hydrogen thermometer.

THE INDICATOR CARD

229. The indicator card is a Watt diagram extensively employed by engineers for determining the conditions under which a steam engine is operating. A device is attached to the engine cylinder whereby the diagram is automatically drawn by the moving piston and the varying pressure of the steam.

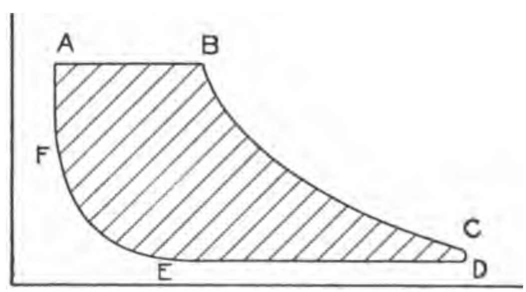
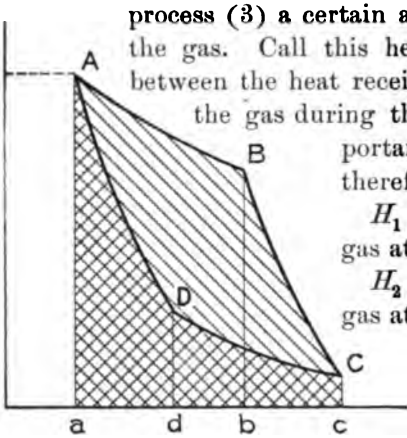


Fig. 156. — The Indicator Card.

In Figure 156 a diagram of this kind is shown. The ordinate of the point A represents the pressure at which steam is admitted

During the process (1), a certain amount of heat must be supplied to the gas (Section 226). Call this heat H_1. During process (3) a certain amount of heat is rejected by the gas. Call this heat H_2. Then the difference between the heat received and the heat rejected by the gas during the cycle is $H_1 - H_2$. The important results of the cycle are, therefore, as follows:



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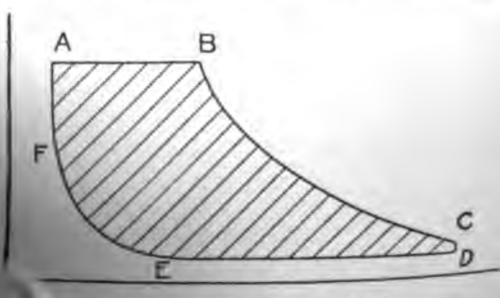

FIG. 156.—The Indicator Card.

[illegible] nd is shown. The ordinate [illegible] t which steam is admitted

to the cylinder. During a portion of the stroke corresponding to AB steam flows into the cylinder at boiler pressure. At B communication with the boiler is cut off. During the rest of the stroke the steam expands (adiabatically, nearly). At the end of the stroke a valve (exhaust port) is opened and the pressure falls to that of the outside air (or condenser). During the return stroke the pressure remains constant from D to E. At E the exhaust port is closed and the steam remaining in the cylinder is compressed. At F the valve admitting steam from the boiler is opened and the pressure at once rises to that of the boiler.

The area of the diagram *ABCDEF* is proportional to the heat energy transformed into work during one stroke of the engine. If, therefore, the length of stroke of the engine and the boiler pressure are known, the work done per stroke may be calculated from the measured area of "the card."

Problems

1. How much heat can be developed by a weight of 1 kg. in falling 5 m.? Assume the transformation to be complete.

2. If 5000 ft.-lb. of work are expended in stirring a half pound of water, what will be the rise in temperature of the water? Assume no heat is lost in the operation.

3. What is the theoretical efficiency of a steam engine taking steam at a temperature of 160° C. and exhausting into a condenser at 40° C.?

4. The temperature in the cylinder of a gasoline engine at the moment of explosion is 1800° C. and at the moment of exhaust is 800° C. What is the theoretical efficiency of the engine?

5. Assuming a refrigerating machine to be a perfect engine, how much work is required to take 1000 calories of heat from a room at − 10° C. and deliver it to the cooling pipes at 60° C.?

PART III

ELECTRICITY AND MAGNETISM

undergo such a process, heat must be imparted to it (if the gas expands) or abstracted from it (if the gas is compressed). Consider the gas inclosed in the cylinder of any form of heat engine. As the gas expands, it pushes the piston back and does work. The source of this work is the heat energy of the gas. But if the gas gives up some of its heat energy, it cools. **If, therefore, the gas is to expand without cooling, it must be supplied with heat during the process of expansion.** Similarly, if the gas in such a cylinder is compressed, its temperature will rise, that is, its heat energy will increase. The source of this increase of heat energy is the work done in compressing the gas. **If, therefore, the gas is to be compressed without rise of temperature, heat must be abstracted from it during the process of compression.**

An adiabatic process is one in which there is no interchange of heat between the gas and its surroundings. Such a process is always accompanied by a change in the temperature of the gas. Consider the gas inclosed in the cylinder of any form of heat engine. As the gas expands, it pushes the piston back and does work. The source of this work is the heat energy of the gas. Hence, **if the gas receives no heat from its surroundings during the process, its temperature will fall as it gives up the heat energy which is transformed into work.** Similarly, the work done in compressing the gas in such a cylinder is transformed into heat, and **if the gas loses no heat to its surroundings during the process, its temperature will rise.**

These processes may be approximately realized in a cylinder filled with compressed gas. (1) Imagine the gas to expand very slowly. This will be an **isothermal** process, since by heat conduction from the walls of the cylinder the temperature of the gas will be kept constant. (2) Imagine the gas to expand very quickly. This will be an **adiabatic** process, since no appreciable amount of heat can flow from the cylinder walls to the gas during the expansion.

CARNOT'S CYCLE

227. In order that work may be obtained by repeated expansion and compression of a given body of gas, it will be evident that the expansion and compression processes must be different.

Consider the process represented by the curve *AB*, Figure 154. When the gas expands from the volume v_1 to the volume v_2, an amount of work, represented by the area *aABb*, is done **by the gas.** If now the process is reversed and the gas is compressed from the volume v_2 to the volume v_1, an equal amount of work, represented again by the area *aABb*, is done **on the gas.** Evidently an engine working in this way could do no external work, since all of the work done by the gas during the expansion stroke would be required to compress the gas during the compression stroke. It follows, therefore, that **the expansion and compression processes**, through which the gas in the cylinder of a heat engine is carried, **must be different if the engine is to be capable of doing external useful work.**

When the gas in a heat engine is carried through a number of processes and returned to its initial condition, it is said to pass through **a cycle** of operations.

An ideal cycle for the heat engine was suggested by Carnot. Carnot's cycle consists of four processes as follows:

(1) Isothermal expansion (temp. T_1)
(2) Adiabatic expansion (from temp. T_1 to temp. T_2)
(3) Isothermal compression (temp. T_2)
(4) Adiabatic compression (from temp. T_2 to temp. T_1)

This cycle is represented in Figure 155. *AB* represents (1) isothermal expansion at the temperature T_1. *BC* represents (2) adiabatic expansion, during which the temperature of the gas falls from T_1 to T_2. *CD* represents (3) isothermal compression at the temperature T_2. *DA* represents (4) adiabatic compression, during which the temperature of the gas rises from T_2 to T_1. The work done by the gas in process (1) is represented by the area *aABb*, and in process (2) by *bBCc*. The total work done **by the gas** is therefore represented by *aABCc*, that is, the area under the line *ABC*. The work done on the gas in process (3) is represented by the area *cCDd* and in process (4) by *dDAa*. The total work done **on the gas** is therefore represented by *cCDAa*, that is, the area under the line *CDA*. The work done by the gas exceeds the work done on the gas by an amount represented by the area *ABCD*. Call this work *W*.

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During the process (1), a certain amount of heat must be supplied to the gas (Section 226). Call this heat H_1. During process (3) a certain amount of heat is rejected by the gas. Call this heat H_2. Then the difference between the heat received and the heat rejected by the gas during the cycle is $H_1 - H_2$. The important results of the cycle are, therefore, as follows:

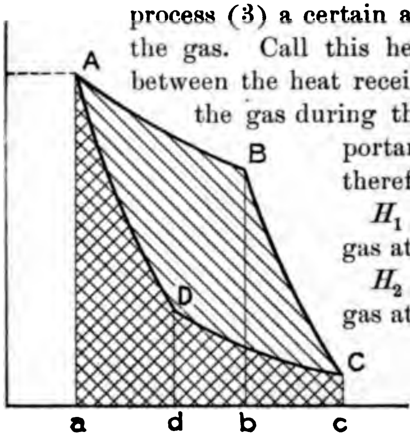

Fig. 155. — Carnot's Cycle.

H_1 heat units are taken up by the gas at the temperature T_1.

H_2 heat units are rejected by the gas at the temperature T_2.

W units of work have been done by the gas.

In other words, $H_1 - H_2$ units of heat have disappeared in the operation, and W units of mechanical energy have made their appearance. It follows, therefore, from the first law of thermodynamics, that,

$$W = J \cdot (H_1 - H_2)$$

EFFICIENCY OF AN IDEAL HEAT ENGINE

228. Carnot imagined an engine in which this theoretical cycle might be realized. This he called the ideal heat engine. The study of Carnot's theoretical cycle and ideal engine leads to a number of important principles of thermodynamics.

The efficiency of a heat engine is defined as the ratio of the heat transformed by the engine into work to the total heat received by the engine. For Carnot's ideal engine, we have, therefore,

$$E = \frac{H_1 - H_2}{H_1} \tag{70}$$

in which E is the efficiency.

The most important property of Carnot's cycle is that it may be reversed. That is, the ideal engine may take a quantity of heat H_2 from a source at a temperature T_2 and deliver a quantity of heat H_1 at a higher temperature T_1, providing an amount of work represented by the area $ABCD$, Figure 155, is

done on the gas during the cycle. Carnot's cycle is therefore called a **reversible cycle.**

The more important principles deduced from a study of Carnot's cycle are as follows:

An engine having a reversible cycle has the greatest possible efficiency;

All engines having reversible cycles, whatever the nature of the gas or working substance, have the same efficiency; and

The efficiency of a reversible engine depends only upon the temperatures T_1 and T_2 between which the engine works. As a matter of fact it may be shown that the expression for the efficiency of a reversible engine given above is equivalent to

$$E = \frac{T_1 - T_2}{T_1} \tag{71}$$

This relation leads to the conception of a new scale of temperatures, depending only upon Carnot's cycle and **independent of the nature or properties of any particular kind of matter.** Lord Kelvin devised such a scale, called the thermodynamic scale, and found that it did not differ materially from that of the hydrogen thermometer.

THE INDICATOR CARD

229. The indicator card is a Watt diagram extensively employed by engineers for determining the conditions under which a steam engine is operating. A device is attached to the engine cylinder whereby the diagram is automatically drawn by the moving piston and the varying pressure of the steam.

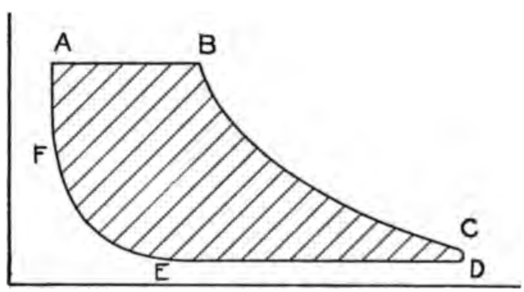

FIG. 156.—The Indicator Card.

In Figure 156 a diagram of this kind is shown. The ordinate of the point A represents the pressure at which steam is admitted

to the cylinder. During a portion of the stroke corresponding to *AB* steam flows into the cylinder at boiler pressure. At *B* communication with the boiler is cut off. During the rest of the stroke the steam expands (adiabatically, nearly). At the end of the stroke a valve (exhaust port) is opened and the pressure falls to that of the outside air (or condenser). During the return stroke the pressure remains constant from *D* to *E*. At *E* the exhaust port is closed and the steam remaining in the cylinder is compressed. At *F* the valve admitting steam from the boiler is opened and the pressure at once rises to that of the boiler.

The area of the diagram *ABCDEF* is proportional to the heat energy transformed into work during one stroke of the engine. If, therefore, the length of stroke of the engine and the boiler pressure are known, the work done per stroke may be calculated from the measured area of "the card."

Problems

1. How much heat can be developed by a weight of 1 kg. in falling 5 m.? Assume the transformation to be complete.

2. If 5000 ft.-lb. of work are expended in stirring a half pound of water, what will be the rise in temperature of the water? Assume no heat is lost in the operation.

3. What is the theoretical efficiency of a steam engine taking steam at a temperature of 160° C. and exhausting into a condenser at 40° C.?

4. The temperature in the cylinder of a gasoline engine at the moment of explosion is 1800° C. and at the moment of exhaust is 800° C. What is the theoretical efficiency of the engine?

5. Assuming a refrigerating machine to be a perfect engine, how much work is required to take 1000 calories of heat from a room at − 10° C. and deliver it to the cooling pipes at 60° C.?

PART III

ELECTRICITY AND MAGNETISM

ELECTRICITY AND MAGNETISM

CHAPTER XX

ELECTROSTATICS

230. It was discovered about 2500 years ago that a piece of amber rubbed with silk acquires the property of attracting to itself small, light bodies, for example, bits of paper, chaff, etc. This condition of the amber, after being excited by frictional contact with the silk, is known as electrification. The amber while in this condition was said to be **electrified**. Electrification was for centuries considered to be peculiar to amber. It was only about 300 years ago that the discovery was made that other bodies may be electrified. It is now known that **any substance may be electrified by frictional contact with a dissimilar substance.**

That branch of physics which deals with electrified bodies and the force actions between them is called **electrostatics.**

POSITIVE AND NEGATIVE ELECTRICITY

231. An electrified body is said to possess **a charge of electricity**. Experiment shows that there are two kinds of electricity, which are distinguished as positive and negative.

If a dry rubber rod is stroked with cat's fur, it becomes strongly electrified. A dry glass rod rubbed with silk also acquires this property in a marked degree. The rubber rod and the glass rod under these circumstances both behave like amber in the experiment referred to, in that they exhibit marked attraction for small, light bodies. Examination will show, however, that the electric charge possessed by the glass rod is in some important respects different from that possessed by the rubber rod. For example, if the rubber rod, after being electrified, is hung in a stirrup which is suspended by a thread,

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as shown in Figure 157, so as to be free to turn, and the glass rod is brought near as shown in the figure, the charge on the glass rod exhibits strong attraction for the charge on the rubber rod. If now, in place of the glass rod, **a second rubber rod be used, the suspended rubber rod, instead of being attracted, will be repelled**, thus showing that there is a difference in the nature of the electric charges on the glass and rubber rods. It should be carefully noted that the force actions referred to are between the electric charges possessed by the glass and rubber rods and not between the rods themselves.

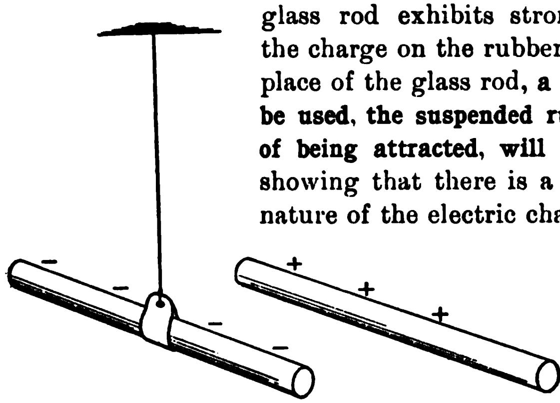

FIG. 157. — Illustrating the Attraction of Unlike Charges.

The charge on the glass rod is called a positive (+) charge, and **a body is said to be charged positively when it has a charge like that which appears on a glass rod when it is rubbed with silk.**

The rubber rod in the above experiment is said to possess a negative (–) charge and **a body is said to be charged negatively when it possesses a charge like that which appears upon a rubber rod when it is rubbed with cat's fur.**

The above experiment may be repeated, placing the charged glass rod in the stirrup. Under these circumstances the suspended rod is attracted by the charged rubber rod, but is repelled by a similarly charged glass rod. As a result of these experiments we are led to the conclusion that **like charges repel, while unlike charges attract.**

THE SINGLE FLUID THEORY

232. In the discussion above given no reference has been made to the nature of electricity, and the experiments referred to, together with all of those with which we have to deal in the present discussion, may be made without reference to the

real nature of that which we call electricity. Nevertheless various attempts have been made to explain its nature and to formulate a theory which will account for the various phenomena of electrostatics. One of the theories advanced is the so-called **single fluid theory. This theory assumes that electricity is a fluid, that all substances have a certain affinity for this fluid, and when normal in this respect, possess a certain amount of electricity which renders them neutral as to electric force actions on other bodies in similar condition.** A negatively charged body, under this theory, is one possessing less than the normal amount of the electric fluid. A positively charged body is one which possesses an excess of the fluid. This theory was advocated by Benjamin Franklin.

THE TWO FLUID THEORY

233. The two fluid theory assumes that there are two kinds of electric fluid, the positive and the negative. A positively charged body is one which possesses an excess of the positive fluid. A negatively charged body is one which possesses an excess of the negative fluid. An uncharged (neutral) body is one which possesses equal amounts of the positive and negative fluids. This is the theory which is commonly adopted in explaining the various phenomena of electrostatics. The two fluid theory affords the simplest explanation of these phenomena, and providing it is borne in mind that we make use of it simply as a means of facilitating discussions of this character, it may be used without hesitation.

THE DIELECTRIC THEORY

234. Another theory of electrostatics is known as the dielectric theory and has been championed by such noted physicists as Faraday and Maxwell. **This theory assumes that electric charges are simply manifestations of a certain kind of strain in the ether** (Section 211). Under this theory to charge a body is to strain the ether near the body and to discharge a body is to relieve existing ether strain in its neighborhood.

THE ELECTRON THEORY

235. The most modern theory of electrification is the **electron theory.** This is really a fluid theory and is somewhat analogous to the single fluid theory of Franklin (Section 232). It differs from the old single fluid theory in that **it assumes that the fluid is negative.** It maintains that electricity has atomic structure, and that small particles called **electrons** are associated with the atoms of matter. These electrons may, under certain conditions, be separated from the atoms with which they are normally associated. When a number of electrons have been removed from a body in normal (neutral) condition, the body is left "positively charged." When a body possesses more than its normal amount of electrons, it is negatively charged.

THE CLASSIFICATION OF BODIES WITH RESPECT TO THE CHARGES WHICH APPEAR UPON THEM

236. It is found that certain substances acquire positive charges under almost all circumstances of frictional contact with other bodies. Certain other bodies appear to take on a negative charge under the same circumstances. There are again other bodies which acquire sometimes positive and sometimes negative charges, depending upon the nature of the body with which they are brought into contact. Generally speaking, it is possible to tabulate the various substances in such manner that if a substance at the top of the list is brought into frictional contact with one lower in the table, the upper one acquires a positive charge and the lower one a negative charge. Such a table is given below:

Cat's fur
Polished glass
Woolen stuffs
Feathers
Wood
Paper
Silk

Thus it is possible by stroking feathers with cat's fur to give them a negative charge or by stroking them with silk to give

them a positive charge. It will be evident, therefore, that the **charge acquired by a body** when brought into frictional contact with a second body **depends, not only upon the nature of the body itself, but also upon the nature of the body with which it comes into contact.**

CONDUCTORS

237. If an electric charge is imparted to one end of a long wire, a portion of this charge immediately spreads to the more remote extremity of the wire. The wire, under these circumstances, is said to **conduct** the electricity from the nearer to the farther end. **A substance which is capable of doing this is called a conductor of electricity.** It is found that certain substances conduct electricity with great readiness, others less readily, and certain substances with the greatest difficulty. Hence the various substances are divided in a general way into two classes: good conductors or simply **conductors**, and very poor conductors or **insulators.** Below are given tables of the more common conductors and insulators:

Conductors	Insulators
All metals	Shellac
Charcoal	Amber
Plumbago	Resins
Concentrated acids	Glass
Metallic ores	Mica
Water	Ebonite
Moist earth	Silk
	Dry paper
	Porcelain

The electron theory explains conduction by assuming that in conductors the electrons have considerable freedom of motion, while in insulators they have little or none at all.

ELECTROSCOPES

238. An electroscope is a device for detecting the presence of an electric charge. There are several kinds of electroscopes, of which the following are the most convenient for such studies as are undertaken in this course.

(*a*) **The pith ball electroscope** consists of a very light ball, conveniently of pith, suspended by a silk thread, as shown in Figure 158. If this pith ball is given a positive charge, it will be attracted by a negatively charged body or repelled by a positively charged body. It can therefore be used, not only to detect the presence of charged bodies, but will also distinguish a positive from a negative charge.

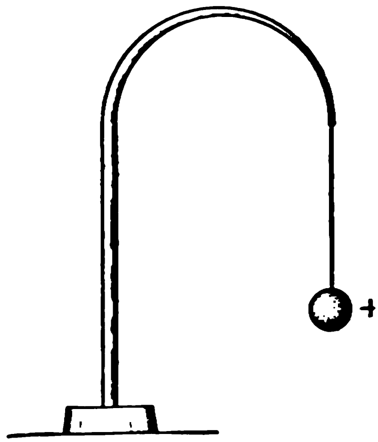

FIG. 158. — Pith Ball Electroscope.

(*b*) **The stirrup and charged rod.** The arrangement represented in Figure 157 constitutes an electroscope by means of which the presence of a charge upon any body may be readily detected and identified.

(*c*) **The gold leaf electroscope** consists of two slender strips of gold foil suspended from a metal rod which terminates at the top in a knob, as represented in Figure 159. For convenience the instrument is mounted in a glass vessel, as shown in the figure. When so mounted, it is protected from outside disturbances such as air currents. Care should be taken to insulate the rod where it passes through the stopper of the glass vessel. This is conveniently done by surrounding the rod at this point by shellac or amber or some similar insulating material. The indications of this instrument depend upon the fact that like charges repel, so that two bodies which carry like charges tend to separate. Suppose, for example, the knob in the electroscope represented in Figure 159 is stroked with cat's fur.

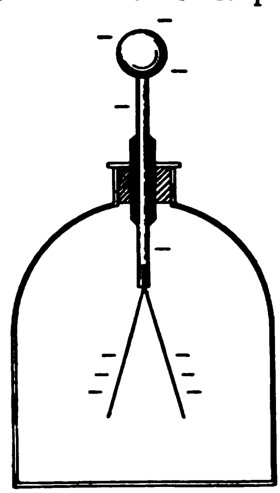

FIG. 159. — Gold Leaf Electroscope.

The rod together with the leaves will acquire a negative charge. Those portions of the charge which reside on the leaves repel one another with the result that the slender gold leaves separate by a certain amount. This separation of the leaves is an indication of a charged condition in the knob and the attached gold leaves.

THE EQUALITY OF THE POSITIVE AND NEGATIVE CHARGES DEVELOPED BY FRICTIONAL CONTACT

239. Experiment shows that **in any case of the development of electricity by the frictional contact of two bodies equal amounts of positive and negative electricity are developed.** Thus when a rubber rod is stroked with cat's fur a certain amount of negative electricity appears upon the rubber rod. An examination of the cat's fur will reveal the fact that an equal amount of positive electricity has been developed upon it. In the case of the glass rod rubbed with silk, the silk acquires an amount of negative electricity equal to the positive which appears upon the glass rod. A simple experiment for demonstrating this fact is the following: A, Figure 160, represents a rod of sealing wax. Over the upper end B is fitted a cap of flannel to the top of which is tied a silk thread C. If now the rod A is rotated in the cap B, both the rod and the cap will become charged, the rod with negative and the cap with positive electricity. So long as the cap remains on the rod, however, these charges will be unable to manifest their presence upon outside bodies, since the force action due to the charge on B is neutralized by the force action due to the charge on A, the one being positive and the other negative. If for example, the rod A with the cap B is presented to the suspended rubber rod represented in Figure 157, no force action will be apparent. As soon as the cap is removed, which is conveniently done by means of the silk thread

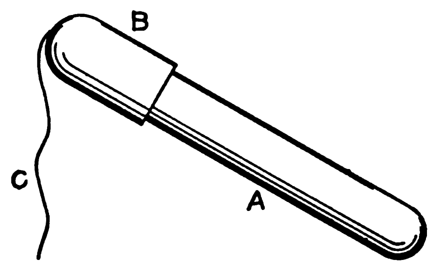

FIG. 160.

C, it is found that both the rod *A* and the cap *B* are in condition to influence the electroscope, thus showing that they are both charged. They influence the electroscope oppositely, thus showing that their charges are unlike. This and similar experiments lead to the conclusion that in every case of frictional contact equal amounts of positive and negative electricity are developed.

Under the electron theory this result follows as a matter of course, all electrons removed from the cap are added to the rod.

INDUCTION

240. Since the electrons in a conductor have a certain freedom of motion, it follows that a neutral conductor when brought into the presence of a charged body will show charge, since the electrons, responding to the influence of the charged body, will be attracted or repelled according to the nature of the charge, and the neutral condition of the conductor will be disturbed. For example, in Figure 161, let *B* represent an uncharged conductor. Let the body *A*, charged positively, be brought into the presence of *B*. Then the two electricities which, before *A* was brought up, neutralized each other at all points on *B*, will now be separated through the influence of the charge on *A* in the manner indicated in the figure. The electrons

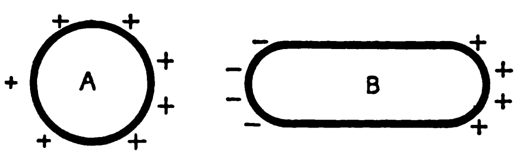

FIG. 161. — Charging by Induction.

in *B* will be drawn in large measure to the nearer end of *B*, thus giving that end a negative charge, and the farther end, because of the deficit of electrons, will exhibit positive charge. Evidently this condition of charge on *B* is a temporary one; and if the body *A* is removed, the electrons on *B* will distribute themselves over the entire body, thus reducing it to the neutral condition in which it was assumed to be at the beginning of the experiment.

The body *B* may be given a permanent charge by induction in the following manner: While *B* is in the presence of *A* and the charges upon it are separated as indicated in Figure 161, let

B be placed in communication with the earth by means of a wire, or by touching it with the finger. **Under these circumstances a number of electrons will flow from the earth to the body *B*** in response to the attraction of the positive charge on *A*. Evidently the group of electrons on *B* near *A* will have no tendency to flow to ground, since it is held or "bound" by the attractive influence of *A*. Since there is now an excess of electrons on *B*, it is evident that when the connection between *B* and the earth is broken, *B* will have a permanent negative charge. This process is known as charging by induction. It will be observed that the charge which the body *B* acquires in this process is opposite in sign to that of the inducing charge upon *A*. Had *A* possessed a negative charge, then upon connecting *B* to the earth a number of electrons would have been repelled, by the charge on *A*, to the ground, and *B* would have acquired a permanent positive charge (deficit of electrons).

An instructive method of showing that the charges upon *B* are separated as indicated in Figure 161 is that illustrated in Figure 162. *B* and *C* are two conductors which are placed in contact with one another as shown. In this position they constitute a single conductor. The inducing charge upon *A* is now brought up as in the former experiment, and the separation of electricities upon the body *CB* takes place as indicated. If now *C* is separated from *B* and then the body *A* with the inducing charge is removed, *C* will have a permanent positive charge and *B* a permanent negative charge. The charges upon *C* and *B* may be identified in the usual way by causing them to approach the charged electroscope.

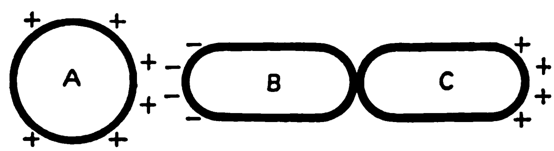

FIG. 162. — Method of securing Permanent Charges by Induction.

THE ICE PAIL EXPERIMENT

241. A very instructive experiment in induction is the following: Consider a hollow conductor *A*, Figure 163, to which is attached an electroscope *B*. Let a charged body *C* be slowly

lowered into the hollow conductor. Evidently the conductor *AB*, consisting of the hollow vessel with attached electroscope, will become charged by induction upon the approach of the body *C*. The negative electricity will be attracted to the inside of the hollow vessel *A*, and the positive repelled to the outside of *A* and to the electroscope. If the body *C* is removed without coming in contact with the body *A*, the temporary charged condition of *A* and *B* will disappear exactly as in the former experiment in induction. **If, however, before being removed, the body *C* is brought in contact with the body *A* on the inside and then removed, the system *AB* will retain a permanent charge.** In carrying out this experiment the following important observations are made:

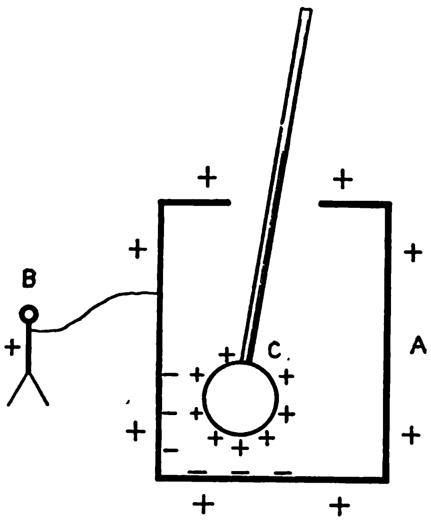

FIG. 163. — Ice Pail Experiment.

(1) That once the body *C* is well within the body *A* it may be moved about from point to point without altering the charge on *B*.

(2) It may be brought in contact with the inner wall of *A* without affecting the charge on *B*.

(3) When the body *C* is taken away, it is found to be entirely discharged.

(4) If the uncharged body *C* be again lowered into the vessel *A* and brought in contact with it, it will come away a second time without charge; whereas if it is brought in contact with the outside of the vessel *A*, it will take away a portion of the charge upon *A*.

These facts tend to show, first, that the induced charges upon the system *AB* are exactly equal in amount to the inducing charge upon *C*. If this were not so, then upon bringing the

body C in contact with A there would be a little more positive charge on C than is necessary to neutralize the negative charge on the inside of A, and this surplus would either come away with the body C, which experiment shows is not the case, or it would flow to the outside of A, thus increasing the positive charge upon the system AB. This would be evidenced by wider separation of the electroscope leaves. No such increase in the charge on B is shown; therefore the negative charge induced on the body A must be exactly equal to the inducing charge upon the body C. Second, a free charge on any body is confined to the surface of that body. This is evident from the fact that in the operation referred to as No. 4 above, when the uncharged body C is lowered the second time into the hollow vessel and brought in contact with its inner surface, it comes away without charge. If there were any charge within the hollow conductor, it would be shared by the body C.

The location of an electric charge upon the surface of a conductor follows from the fact that the different parts of a charge tend to separate from one another as widely as possible, and this separation is affected in the largest degree, of course, by the spreading of the charge over the surface of the body.

This experiment on induction may be repeated, making use of several hollow conductors, one within the other, each carefully insulated from the others. Under these circumstances, when the body C carrying the inducing charge is lowered into the inside vessel, all of the hollow vessels become charged by induction, — negative charge appearing upon the inside of each, positive charge appearing upon the outside of each and on the electroscope attached to the outside vessel. It is found that once the inducing charge C is well within the inner vessel, it may be moved about from point to point or brought in contact with the inner wall of this vessel without altering the charges upon the other vessels. Furthermore, the vessels may be brought in contact with one another without altering the charge upon the outside of the outside vessel. These facts tend to show that the induced and the inducing charge in the process of charging by induction are equal.

It is here assumed that the influence of the inducing charge

C does not extend beyond the vessel into which it is lowered. In other words, its entire influence is confined to the vessel *A*. This is true only when the vessel *A* quite completely surrounds the body *C*. Thus in the process of charging by induction represented in Figure 161, the charges induced upon *B* are always less in amount than that upon the body *A*, since a part of the influence of *A* extends right and left to other bodies, and its influence is not limited to the body *B* as is true in the ice pail experiment in which it is assumed that the body *A* quite completely surrounds the body *C*.

THE ELECTROSTATIC FIELD

242. The electrostatic field in the neighborhood of a charged body is that region of space into which the influence of the charge extends. We have seen that when one charged body is brought into the presence of a second charged body, there is a force action between them. This force action, whether it be of attraction or repulsion, becomes greater the nearer the two bodies are brought together, and becomes smaller as the bodies are more widely separated. While this force action between the two bodies falls off rapidly as the two bodies are carried farther and farther apart, it becomes zero theoretically only when the bodies are separated by an infinite distance. Theoretically, therefore, the electrostatic field which surrounds a charged body extends to infinity, assuming that there is but the one charged body. Practically, however, the field about a charged body is quite limited in extent.

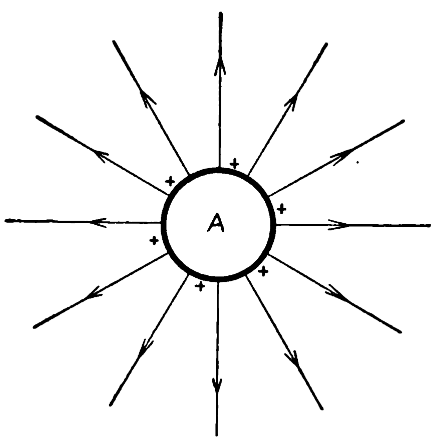

FIG. 164. — Field surrounding an Isolated + Charge.

ELECTROSTATIC LINES OF FORCE

243. It is convenient for many purposes to represent the electrostatic field about

a charged body by a series of lines. **These lines are so drawn that they represent at each point the direction of the force action which a small charged body would experience if placed at that point, or the direction in which a small charge would tend to move if placed at the point in question.** It is customary to place arrowheads upon these lines pointing in the direction in which a small positive charge would tend to move. The radial lines drawn in Figure 164 afford a picture of the electrostatic field which surrounds the isolated positive charge on the sphere A. The lines drawn between the bodies A and B in Figure 165 represent the electrostatic field surrounding the two charged bodies A and B.

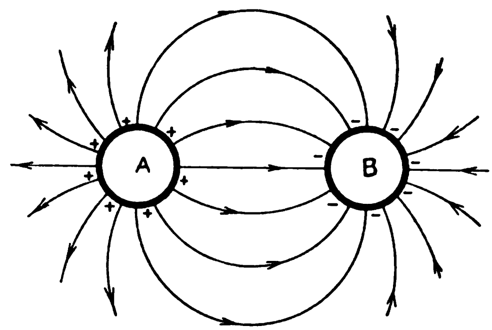

FIG. 165.—Field surrounding Two Unlike Charges.

These lines, which we may imagine to extend from any charge, are called electrostatic **lines of force.**

THE FORCE ACTION BETWEEN TWO CHARGES

244. It may be shown very readily by experiment that the force action between two electrostatic charges depends upon the magnitude of the charges, and upon the distance which separates them. It is easily shown that when a rubber rod, for example, is strongly charged, *i.e.* when it carries a large charge, it will exert a larger force action upon a second charge than when it carries but a small charge; and in the same way the force action may be shown to depend upon the magnitude of the second charge. Experiment also shows that the force varies inversely as the square of the distance between the charges. This dependence of the force action between two charges upon the magnitudes of the charges themselves and upon the distance which separates them, as determined by experiment, may be expressed algebraically as follows:

$$F = \frac{Q \cdot q}{d^2} \qquad (72)$$

in which Q and q represent the magnitudes of the two charges, and d the distance which separates them.

The force action between two charges depends upon the medium which fills the space between the charges. Equation (72) gives the force when the charges are in a vacuum. The force action between two charges in air is **practically** the same as in a vacuum.

THE ELECTROSTATIC UNIT OF CHARGE

245. The electrostatic unit of charge is defined from Equation (72) as follows: Let it be assumed that two equal charges are chosen of such magnitude that when they are placed one centimeter apart in a vacuum, the force action between them is one dyne. Then these charges are said to be unit charges. In other words, **the c. g. s. electrostatic unit of charge is that charge which placed at a distance of one centimeter from a similar charge will experience a force action of one dyne.**

ELECTROSTATIC FIELD INTENSITY

246. The electrostatic field intensity at any point is the force action per unit charge placed at that point. This may be stated in another way. Referring to Figure 166, and having in mind the point p at a distance d from the charged body A, it is found that if we bring to the point p different charges one after another, **the forces which these charges experience while at the point p are proportional to the magnitudes of the charges,** that is to say,

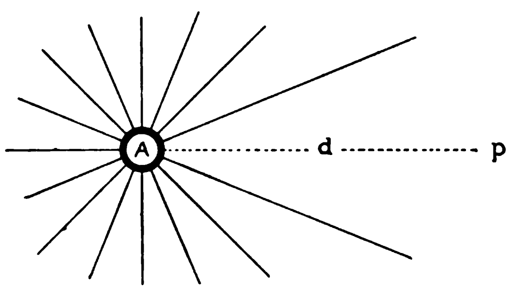

FIG. 166.

$$F \propto q$$

in which F is the force experienced by the charge q when placed at the point p. This may be written as follows:

$$F = f \cdot q \tag{73}$$

in which f, the proportionality factor, is the field intensity at the point p. From Equation (73), $f = \frac{F}{q}$, *i.e.* the field intensity equals the force action per unit charge.

Comparing Equations (72) and (73), and remembering that Equation (72) is the general expression for the force action between any two charged bodies and that it will therefore be applicable to the case under discussion, evidently,

$$f = \frac{Q}{d^2} \tag{74}$$

That is, the field intensity in the neighborhood of a charge Q and at a distance d therefrom is equal to the magnitude of the charge divided by the square of the distance of the point in question from that charge.

In the above discussions on the force actions between charged bodies it has been assumed that the bodies upon which the charges are supposed to rest are very small as compared with the distance d involved in the expressions.

THE SCREENING EFFECT OF A HOLLOW CONDUCTOR

247. It will be evident from the foregoing discussions that **lines of force terminate upon charges.** A line of force may be thought of as beginning upon the surface of a positively charged body and extending to the surface of a negatively charged body. For example, the lines of force involved in the ice pail experiment described in Section 241, before the body C is brought in contact with the inner wall of the vessel, would be something like those represented in Figure 167. As the body C is caused

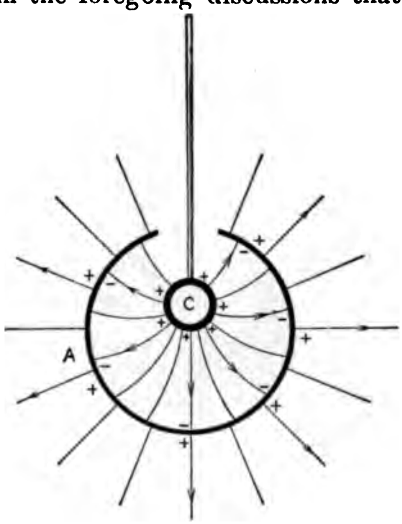

FIG. 167.

to approach the inner wall of the vessel more closely, the conditions would be more like that represented in Figure 168, in which the lines of force extending between A and C are now grouped together in the narrow space which separates these bodies while the lines of force without the vessel A remain unchanged. Finally, when the body C comes into contact with the body A, the lines of force within A disappear entirely, the lines of force on the outside of A still remaining undisturbed; **that is, there is no electrostatic field within** A after C is brought in contact with its inner wall, Figure 169.

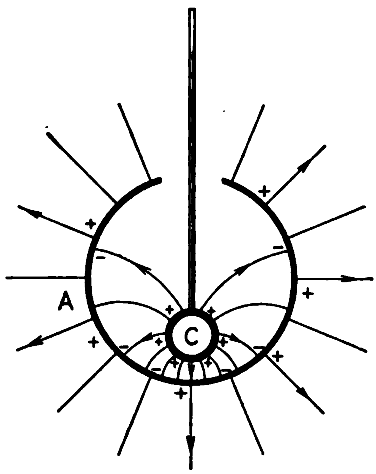

FIG. 168.

This effect does not depend upon the magnitude of the charge upon the outside of A. Therefore the conditions represented will be true for any value of charge upon the outer walls of the vessel A. This discussion shows that **the effect of the charge upon the surface of a hollow conductor is limited to those regions which lie without the conductor and does not extend to the space inclosed by it.** In other words, it is possible to screen a given region from electrostatic effects by surrounding it with a metallic conductor.

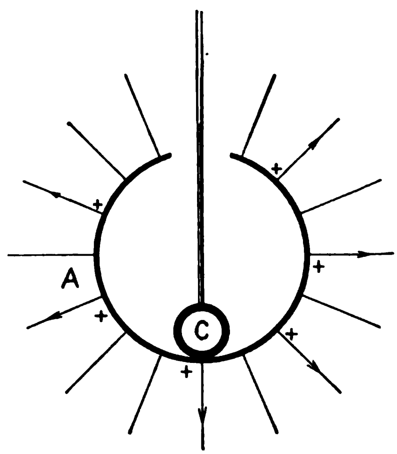

FIG. 169.

LINES OF FORCE ARE PERPENDICULAR TO THE SURFACE OF A CHARGED CONDUCTOR

248. It will be noticed that in all of the electrostatic fields which have been represented in the preceding figures, the lines of force drawn in each case leave the conductor at right angles to its surface. That is, the direction of the electrostatic field in the neighborhood of a charged body is represented as being normal to its surface. That this is true follows at once from the nature of the conductor. Let it be assumed that the field near the surface of a charged conductor is not perpendicular to the surface. There will then be a component of this field parallel to the surface. Under the influence of this component of the field the charge on the surface of the conductor will tend to move along the surface. This motion of a charge on the surface of the conductor will continue until the electrostatic field is everywhere at right angles to the surface of the charged body, that is, until the field component parallel to the surface disappears.

THE DISTRIBUTION OF CHARGE ON A CONDUCTOR

249. Except in one or two special cases the electrostatic charge upon a body is not distributed uniformly over its surface. This will be evident from the fact that the different

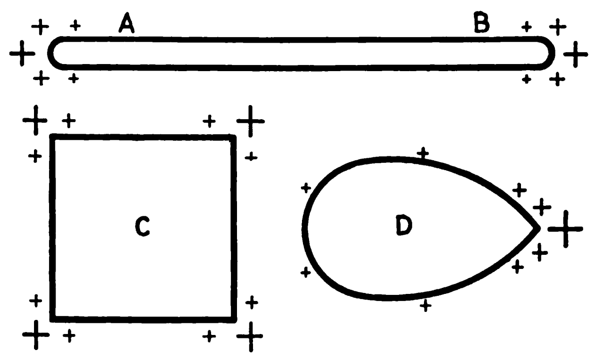

FIG. 170. — Distribution of Charge.

portions of a charge repel one another, and that a charge when placed upon a conductor will so distribute itself that this ten-

dency of the different portions to separate is satisfied as far as possible under the circumstances. It is easy to see, therefore, that upon such a body as is represented in *AB*, Figure 170, a greater portion of the charge will be distributed over the ends of the long conductor *AB* than is to be found on its central portions. In the same way there will be a heaping up or a concentration of the charge upon the corners of a square conductor *C*, and in the case of an egg-shaped body *D*, the concentration of charge will be greatest at the point. It is assumed, of course, that each of the bodies here discussed is free from the influence of charges on other bodies.

In general there is a concentration of the charge carried by any body about its angles and corners or those portions of its surface which are of sharp curvature.

SURFACE DENSITY OF CHARGE

250. The concentration or heaping up of charge at the corners and angles of an irregular charged body is usually expressed by saying that the surface density of the charge on these portions of the body is great.

***The surface density* of an electrostatic charge is the quantity of electricity per unit area of the surface.**

THE DISCHARGING ACTION OF A POINT

251. As indicated above, the surface density of a charge upon a conductor is greatest where the curvature of the surface is greatest. It will be easily understood, therefore, that any sharp point upon an electrical conductor will be a region of great surface density of charge.

The tendency of any charged body to lose its charge, that is to say, to be discharged by giving up a portion of its charge to the air which comes in contact with it, and the particles of dust, etc., which are carried in the air, **depends upon the surface density of the charge.** The action of a point upon a charged conductor is therefore to facilitate the escape of the charge which is upon it. In other words, **a point on a charged body tends to discharge it.**

Problems

1. What is the magnitude and direction of the force acting on a charge of 15 c. g. s. units (+) when placed at a distance of 20 cm. from a charge of 25 c. g. s. units (−)?

2. The force between two charges is F. What will be the force between them if both charges are doubled?

3. If the force between two charges is F when they are separated a distance d, what will be the value of the force when the distance is increased to $5\,d$?

4. Three charges, $Q_1 = +\,20$, $Q_2 = +\,30$, and $Q_3 = -\,40$ are placed at the corners of an equilateral triangle, each side of which measures 20 cm. What is the magnitude and direction of the resultant force acting on each charge?

5. What would be the magnitude and direction of the force acting on a charge of + 10 placed at the center of the triangle of problem 4?

6. A charge is placed at each corner of a hexagon. The charges taken in order around the figure are, + 10, − 20, + 30, − 40, + 50, and − 60. A charge of + 25 is placed at the center of the hexagon. Side of hexagon = 10 cm. What is the magnitude of the force on this charge?

7. What is the field intensity at a distance of 20 cm. from a concentrated charge of 500 c. g. s. units?

8. Two charges, + 40 and − 50, are separated by a distance of 30 cm. What is the field intensity at a point midway between them?

9. What is the field intensity at the center of the triangle of problem 4 due to the charges at the corners?

10. What is the field intensity at the center of the hexagon of problem 6 due to the charges at the corners?

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ELECTROSTATIC MACHINES

CHAPTER XXI

THE FRICTION MACHINE

252. Various devices are employed for the development of electrostatic charges rapidly and in large quantities. One of the earlier forms of electrostatic machine is the friction machine. This device, which is represented in Figure 171, is a machine for developing electrostatic charge by friction. The essential parts of the apparatus are shown in the figure. *A* is a disk of glass which is caused to revolve on the axis *O*. At *B* is a clamp fitted with chamois skin pads which presses upon both sides of the glass disk as it revolves. The friction between *A* and *B* develops electrostatic charges on these bodies. The glass plate, revolving in the direction indicated, passes on from the body *B* bearing on both of its faces positive charge, while *B* is charged negatively. At *C* is placed a metallic comb which presents a number of sharp points to the face of the disk. The comb is a part of a large conductor *D* as indicated in the figure. It is upon *D* that the positive electricity is accumulated. The manner in which *D* becomes charged is as follows: When the positive charge upon the glass plate is brought into the presence of the comb *C* the conductor *CD* becomes charged by induction, positive charge appearing at the farther end of *D*, the negative

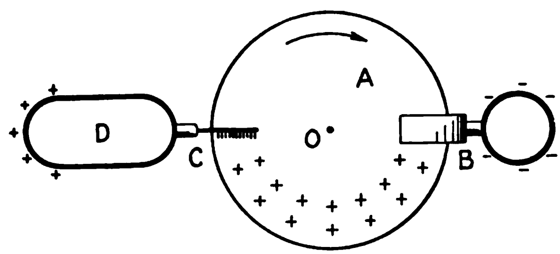

FIG. 171.—The Friction Machine.

electricity being drawn into the comb. The discharging action of the points coming into play, this negative electricity is discharged upon the face of the glass plate, to which it is attracted by the positive charge which that body carries. The positive charge upon the glass plate is thus neutralized and the plate passes toward *B* where it is again charged by frictional contact with that body. The friction machine is very inefficient. Most of the work put into the machine is transformed into heat.

THE ELECTROPHORUS

253. The electrophorus is a device for the rapid accumulation of charge which depends for its action upon the principle of induction. It consists essentially of a cake of sealing wax or resinous material *A*, Figure 172, and a disk *B* of conducting material, provided with an insulating handle. The electrophorus is used in the following manner: The cake of sealing wax *A* is first rubbed with cat's fur. In this operation it becomes charged negatively. The disk *B* is now brought into the presence of the charge on *A* and becomes charged by induction, positive charge appearing upon the lower side of the disk and negative upon the upper side of the disk as indicated in the figure. If now the disk is "grounded," that is brought into communication with the earth, the repelled negative charge upon it will pass off to the earth. The ground connection being removed, there remains upon the disk *B* a permanent positive charge which may be carried upon the disk *B* and made use of as desired. This operation may be repeated again and again without any diminution of the original charge upon *A*.

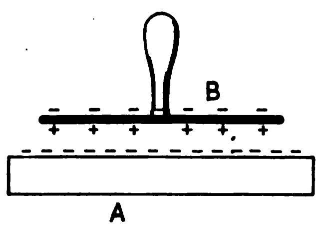

FIG. 172. — Electrophorus.

THE TOEPLER-HOLTZ MACHINE

254. The Toepler-Holtz machine, like the electrophorus, depends for its action upon the principle of induction. It consists essentially of two glass plates, the one stationary and the other

arranged to revolve in close proximity to the first plate. These plates are represented by *A* and *B*, Figure 173. Upon the stationary plate *A* are placed two "armatures," *C* and *D*, of paper and tin foil, represented in outline by dotted lines in the figure, the stationary plate being behind the moving plate *B*. Upon the moving plate *B* are six conductors (metal buttons) represented by the circles *E*, *F*, etc., in the diagram. There is a rod *RR* known as the neutralizing rod, which is provided at

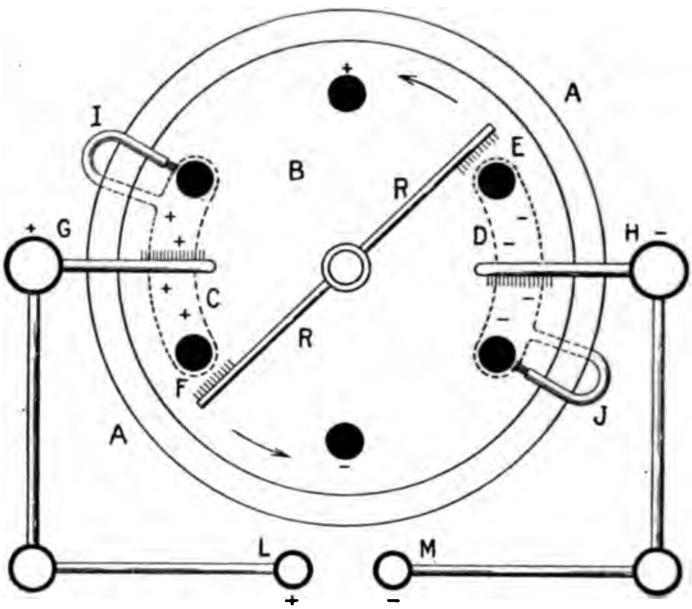

Fig. 173. — The Toepler-Holtz Machine.

each end with a metal brush and sharp points, the brush being arranged to come momentarily in contact with the buttons as the plate *B* is revolved. The terminals of the machine *GH* are large metal conductors provided with sharp-pointed metal combs which stand close in front of the moving plate *B* in the positions shown in the diagram.

The action of the machine is as follows: Let it be assumed that to begin with there is a small positive charge upon the armature *C*. This positive charge, acting inductively upon the

neutralizing rod and the two buttons *E*, *F*, which are momentarily in contact with its brushes, gives rise to a negative charge upon the button *F* and a positive charge upon the button *E*. The plate being revolved, the buttons *E* and *F* go forward, each carrying a free charge. The next pair of buttons coming into momentary contact with the brushes of the neutralizing rod become charged in a similar manner. The rotation being in the direction indicated by the arrow, it will be seen that the buttons as they pass across the top of the machine carry free positive charges toward the left, while those passing across the bottom of the machine carry free negative charges toward the right. A button charged positively, coming into the presence of *C*, will share its charge with *C*, since it is caused to come in contact with the brush *I* which communicates with *C*. It then passes on with its residual charge into the presence of the conductor *G*. This conductor *G* now becomes charged by induction, the positive electricity being repelled and the negative attracted by the positive charge on the button. The negative charge streams off from the metal comb attached to *G* and neutralizes the positive charge upon the button.

Meanwhile, a button charged negatively and passing toward the right into the presence of the armature *D* gives up, by touching momentarily the brush *J*, a portion of its charge to that armature. Passing on into the presence of the conductor *H*, it charges that conductor by induction, repelling the negative and attracting the positive, which flows off the points of the metal comb and neutralizes the inducing charge on the button. These two buttons, being now without charge, pass on into the presence of the neutralizing rod and are again charged by the action already described. The action of the machine is therefore continuous and cumulative, the charges upon the armatures *C* and *D* growing steadily larger, the charges upon the terminals of the machine *G* and *H* being likewise steadily increased. When sufficient quantities of positive and negative electricities have been accumulated upon *G* and *H*, a spark will pass between the knobs *L* and *M*. Charges will again be built up on the terminals until a second spark passes, and so on.

In addition to the charges which are carried by the metal

buttons as described above, there is a distributed charge on the glass plate itself. It will be easily understood that those portions of the glass plate which pass under the metal combs at the extremities of the neutralizing rod will become charged by electricity which streams off the points of the comb. This charge distributed on the glass augments the action of those charges carried by the buttons. The charging of the conductors *G* and *H* is further augmented by the inductive action of the charges on the armatures *C* and *D*, as will be apparent upon inspection of the figure.

THE REVERSIBILITY OF THE TOEPLER-HOLTZ MACHINE

255. A careful analysis of the operation of the Toepler-Holtz machine will show that **while it is generating electricity, work is being done in dragging apart the metal buttons and the armatures** in opposition to the electrostatic force actions which tend to hold them together. For example, the button *E* in Figure 173 is assumed to be charged positively, and all buttons coming to this position receive a positive charge from the comb of the neutralizing rod. Now the armature *D* is charged negatively. There will be at all times, therefore, a force of attraction between the metal buttons which come to the position *E* and the armature *D*. In order that the plate may be rotated in the direction indicated by the arrow the force of attraction between these two charges must be overcome. In other words, they must be dragged apart in opposition to the electrostatic forces. The same thing is taking place at the other side of the machine. The conductors which come into the position *F* are charged negatively and are then drawn away from the region of the armature *C* in opposition to the force of attraction between the charges on *C* and *F*. It would occur to one very naturally that it might be possible to reverse the Toepler-Holtz machine and make of it a device for transforming electric energy into mechanical energy instead of using it in the manner in which it is commonly employed, that is, to transform mechanical energy into the energy of electric charge. This is found to be possible, and experiment shows that if the terminals of the Toepler-Holtz machine are connected to a second machine from which it is

allowed to draw a supply of energy in the form of electric charges, it will tend to revolve as a sort of electric motor, running backward, that is to say, in the direction opposite to that in which it must be turned to operate as a generator.

POTENTIAL

256. In moving a charge about in an electrostatic field it is evident that whenever the motion is parallel to the lines of force work is being done, since there is present a force action upon the body being moved, the direction of which is that of the lines of force. **Work is the product of force and the distance through which the body moves under the influence of that force in the direction of the force.** It therefore follows that if the charge is caused to move in the direction of the lines of force, work is done. The work done upon the charge is positive when the body is moving in opposition to the electrostatic forces and negative when the body moves in the direction of the electrostatic forces. Referring to Figure 166, Section 246, imagine a small charge Q to be brought from an infinite distance, or from a region into which the influence of the charge on A does not extend, up to the point p. From the foregoing discussion it is evident a certain amount of work must be done in the operation. **Thus we come to associate with a charge Q, at the point p, a definite amount of work.** If Q is unit positive charge, the work done is called the potential of the point p. That is, **the *potential* of a point is the work which must be done upon unit positive charge to bring it from infinity up to that point.**

The **difference of potential** between two points is the work which must be done to move unit charge from one of the points to the other. **The potential difference between two points is unity** (c. g. s. unit) if one erg of work is required to carry unit charge (c. g. s. unit) from one point to the other.

THE POTENTIAL OF A POINT DUE TO A CHARGE Q AT A DISTANCE r

257. Consider a point A, Figure 174, at a distance r_1 from a charge Q. Let it be required to find the potential of the point A due to the charge Q. This potential according to the above

definition is the work required to bring unit positive charge from infinity to the point A. Let us divide this work into small parts. Consider first the work required to carry the charge from B to A. This is equal to the **average force** acting on the charge as it moved from B to A multiplied by the distance BA. The force which acts upon the charge when it is in the position B is $\frac{Q}{r_2^2}$, (Equation 72). The force acting upon it when at A is $\frac{Q}{r_1^2}$. Now the average force acting for all points between B and A is less than $\frac{Q}{r_1^2}$ and greater than $\frac{Q}{r_2^2}$. Let it be assumed that A and B are **very close together.** We may then write

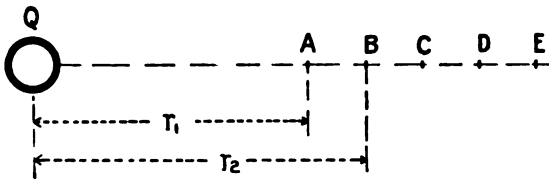

FIG. 174. — Potential due to a Charge Q at a distance r.

$$\text{average force} = \frac{Q}{r_1 r_2}$$

since

$$\frac{Q}{r_1^2} > \frac{Q}{r_1 r_2} > \frac{Q}{r_2^2}$$

and when r_1 and r_2 are nearly equal the average of r_1^2 and r_2^2 is $r_1 r_2$.

We have then: work done in moving unit charge from B to A = average force multiplied by distance AB, *i.e.*,

$$w = \frac{Q}{r_1 r_2}(r_2 - r_1)$$

$$= Q\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

This is the difference of potential between the points A and B. The difference of potential between the point A and infinity is,

$$W = Q\left(\frac{1}{r_1} - \frac{1}{\infty}\right) \text{ (add potential differences } A \text{ to } B,\ B \text{ to } C,\ C \text{ to } D, \text{ etc.)}$$

or,

$$W = \frac{Q}{r_1} \tag{75}$$

That is, the potential of the point A, due to the charge Q, is equal to the charge Q divided by its distance from A.

EQUIPOTENTIAL LINES

258. As indicated in the above discussion on the subject of potential, work is done in moving a charged body in an electrostatic field when the motion is wholly or partly in the direction of the field. If the body is moved at right angles to the lines of force, evidently no work will be done against the electrostatic forces. To move a charge through an electrostatic field in this manner would be analogous to sliding a weight along a horizontal plane. In the latter case no work is done against the gravitational forces. In the former case no work is done against the electrostatic forces.

A line drawn in an electrostatic field in such manner that it is everywhere at right angles to the lines of force is called an *equipotential* line. The equipotential lines about the charged body A, Figure 164, would evidently be circles concentric with the sphere A upon which the charge is supposed to be placed. The equipotential lines in Figure 165 are curves, more or less nearly circular, drawn about the individual charges A and B. The lines and curves referred to in the discussions above represent surfaces called equipotential surfaces which surround the actual charges represented in the figures. Thus the equipotential surfaces about the charge on an isolated sphere are spherical surfaces concentric with the charged sphere.

THE WORK DONE IN MOVING A CHARGE FROM ONE EQUIPOTENTIAL SURFACE TO ANOTHER IS INDEPENDENT OF THE PATH

259. The work done in moving a charged body from one equipotential surface to another is independent of the path along which it is moved, since if the work done were different along different paths, we might move the charge against the electrostatic forces along the easier path and allow it to slide back along the more difficult one. The negative work in this cycle would therefore exceed the positive work, and for each

cycle completed in this direction we could get out of the system a little more work than was put in. We would have, in such an arrangement, a device which would not only operate automatically, but which would be an inexhaustible source of energy. This is, of course, absurd. Therefore the work done in moving a charge from one equipotential surface to another is independent of the path along which it is moved.

Problems

1. What is the potential at a distance of 20 cm. from a concentrated charge of 500 c. g. s. units?

2. What is the potential of a point midway between two charges of + 100 separated by a distance 25 cm.?

3. What would be the potential of a point midway between the charges of problem 2 if the charges were of opposite sign?

4. Two charges of + 50 and − 40 are separated by a distance of 20 cm. At what point between them is the potential zero?

5. What is the potential at the center of the hexagon of problem 6, p. 265?

6. How much work would be required to move a charge of + 5 from the center of the hexagon of problem 5 to each of the corners?

7. How much work would be required to move a charge of − 5 around the hexagon of problem 5 from the charge + 10 to the charge − 40?

8. Does the work done in problem 7 depend upon the path? Is it the same if the charge is carried straight across?

9. Sketch roughly the lines of force and equipotential lines about the charges of problem 2.

10. Sketch the lines of force and equipotential lines about the charges of problem 4.

ELECTROSTATIC CAPACITY

CHAPTER XXII

GENERAL DEFINITION

260. When an uncharged conductor is brought into contact with a charged conductor, it receives from the charged conductor a part of its charge. The amount of charge given up by the charged body to the uncharged body depends upon the relative **capacities** of the two bodies in question. In a general way large conductors have large capacities, small conductors have small capacities; that is, a relatively large amount of electricity may be placed upon a large conductor, whereas in attempting to place the same amount of electricity upon a small conductor much greater difficulty is encountered. This will give a general idea of what is meant by electrostatic capacity. A more rigid definition of capacity is given below.

THE CONDENSER

261. The capacity of a conductor depends upon the presence of other conductors in its immediate neighborhood. This is readily demonstrated in the following manner: Let *A*, Figure 175, represent a plate of metal supported on a convenient insulating stand. Let this plate be connected by means of a wire to an electroscope *C*. The divergence of the leaves of this electroscope would measure in a rough way the condition of *A* with respect to free charge upon its surface. Let the plate *A* be charged, say, by bringing in contact with it the metal disk of the electrophorus. The plate *A* will acquire in this operation a strong charge which will result in a large divergence of the leaves of the electroscope *C*. Let the plate *A* be discharged, and, after bringing near it a second similar plate *B*,

which communicates with the ground by means of a wire, let the plate *A* be once more charged in the same manner as before by bringing into contact with it the metal disk of the electrophorus. It will be found that under these circumstances the divergence of the leaves of the electroscope *C* is much less marked than in the former case, indicating that while the plate *A* has been given the same electrostatic charge as in the former case, the charge does not manifest its presence in the same degree as in the first experiment. In other words, to charge *A* to the same degree (apparent) as before, more electricity must be added. Hence, **the capacity of *A* is increased by the presence of *B*.**

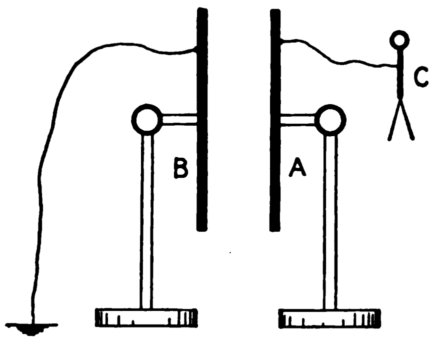

FIG. 175. — The Presence of *B* increases the Capacity of *A*.

A more logical statement of the case is the following: The potential of a point on or near the plate *A* is diminished by bringing up the plate *B*, since *B* becomes charged by induction, and its negative charge operates to lower the potential of all points in its neighborhood. Hence to raise the potential of the chosen point to its original value, more charge (+) must be added to *A*.

A combination of two plates (conductors) separated by a layer of non-conducting material (insulator) is called a *condenser*.

The combination shown in Figure 175 constitutes a simple condenser. In its most common form the condenser is made of layers of "tin foil" connected alternately, separated by sheets of waxed paper or mica, Figure 176. Experiment shows that the capacity of a condenser is proportional to the size of its plates and inversely proportional to the distance between them.


FIG. 176.

SPECIFIC INDUCTIVE CAPACITIES

262. The specific inductive capacity of any material is the ratio of the capacity of a condenser when its plates are separated by a layer of this material and the capacity of the same condenser having its plates separated by dry air. Thus, for example, if a given condenser has the space between its plates filled with plate glass, its capacity will be found to be six times as great as if the space between the plates were filled with air. Hence, we say, the specific inductive capacity of plate glass is six.

The specific inductive capacities of some of the more common dielectrics are given in the accompanying table:

SPECIFIC INDUCTIVE CAPACITIES

Glass	3–10	Mica	4–8
Vulcanite	2.5	Shellac	2.95–3.6
Paraffine	1.68–2.3	Turpentine	2.15–2.43
Beeswax	1.86	Petroleum	2.04–2.42

Equation 72 assumes that the specific inductive capacity of the medium separating the charges Q and q is unity. In case the charges are separated by a medium whose specific inductive capacity is other than unity, the equation for the force action between two charges is

$$F = \frac{Q \cdot q}{K \cdot d^2} \tag{76}$$

in which K is written for the specific inductive capacity of the medium surrounding the two charges.

THE LEYDEN JAR

263. The Leyden jar consists essentially of a wide-mouthed glass bottle coated part way from bottom to top on both inside and outside with a thin layer of tin foil. For convenience in establishing connection with the inside coating, a metallic knob is fastened to the stopper of the bottle and made to communicate with the inside coating of the jar by means of a chain. Thus, the knob on the stopper of the bottle is to be regarded as one of the terminals of the condenser, the outside coating of

the bottle as the other. This form of condenser is used on electrostatic machines.

SEAT OF THE CHARGE

264. An interesting experiment is performed with the dissectible Leyden jar. This is simply a condenser from which it is possible to remove one or both plates while the condenser is charged. If a dissectible condenser is charged and then carefully insulated and its plates removed one after the other, they will be found to be almost without charge. If they are now replaced, the condenser will be found to be charged. **The conclusion to be drawn from this experiment is that the dielectric is the true seat of the charge.**

Experiment shows that **the dielectric between two charged bodies is strained.** It has been shown, for example, that the linear dimensions of a Leyden jar change when it is charged. The change in size is small but measurable. It is also well known that if a condenser is too heavily charged, *i.e.* if the dielectric is subjected to too great a strain, the dielectric will be ruptured. If a Leyden jar is charged more and more strongly, there will come a time when this strain in the glass separating the plates is so great that the material of the glass will no longer be able to withstand it and is broken down.

THE RESIDUAL CHARGE

265. If a Leyden jar is heavily charged and then insulated, and then discharged by bringing the outside and inside coatings into communication, and then left to stand for a few minutes, it will be found to have upon it a small charge. Since the two coatings were brought into metallic contact it is evident that this residual charge could not have been left upon the coatings. We must, therefore, in this experiment, as in the experiment with the dissectible jar, look to the dielectric for the explanation of the charge. The residual charge is explained as follows: When the jar is first charged, the dielectric is under severe strain. When the jar is discharged, this strained condition relieves itself largely but not completely. This residue of strain in the dielectric gradually relieves itself, or better still, distrib-

utes itself to the surfaces of the dielectric against which are the coatings. **It is the residue of this strain in the dielectric which accounts for the residual charge in the condenser.** The residual charge is not exhibited by condensers having gaseous dielectrics. It would seem in such cases that the strain is completely relieved at the moment of the discharge of the condenser.

THE NUMERICAL SPECIFICATION OF CAPACITY

266. Consider a condenser without charge. Let it be imagined that a small positive charge is taken from one of the plates and transferred to the other. This will bring about a potential difference between the plates. The greater the charge transferred, the greater the potential difference thus established. Experiment shows that the charge Q, which must be transferred in this manner in order to bring about a potential difference E, is proportional to E, that is, $Q \propto E$. This may be expressed as follows:

$$Q = CE \tag{77}$$

The factor C is the capacity of the condenser. Therefore **the capacity of a condenser is that factor which multiplied by the potential difference between the plates will give the charge on one of them.**

CAPACITY OF AN ISOLATED SPHERE

267. The potential of a point at a distance r_1 from a charge Q is

$$E = \frac{Q}{r_1} \tag{75 bis}$$

This expression holds for any point outside of the charged body. It therefore applies to a point near to or upon the surface of the charged body. Therefore the potential of an isolated charged sphere of radius r is

$$E = \frac{Q}{r}$$

Now from Equation (77) the potential of a charged body of capacity, C, is,

$$E = \frac{Q}{C}$$

Comparing these expressions for E, evidently,

$$C = r \tag{78}$$

That is, **the capacity of an isolated charged sphere is equal to its radius.**

THE CAPACITY OF CONDENSERS IN PARALLEL

268. Two condensers are said to be connected in "parallel" when they are connected side by side with their corresponding plates in communication with one another. The condensers CC, Figure 177, are connected in parallel.

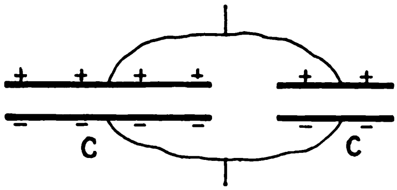

FIG. 177. — Condensers in Parallel.

The capacity of two condensers connected in parallel is the sum of the capacities of the individual condensers. If the condensers are similar in character, it will be evident that by connecting them in this manner we have, in effect, simply increased the size of the plates of one condenser by the size of the plates in the other.

THE CAPACITY OF CONDENSERS CONNECTED IN SERIES

269. Two condensers are said to be connected in series when the second plate of the first condenser is joined to the first plate of the second. C_1 and C_2, Figure 178, represent two condensers connected in series.

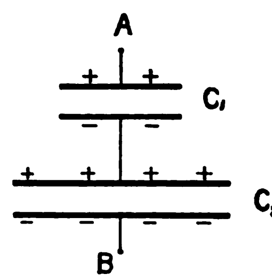

FIG. 178. — Condensers in Series.

The capacity of two condensers connected in series is obtained in the following manner. Let the difference of potential between the points A and B be represented by E. Let the potential difference between the plates of C_1 be E_1 and the potential difference between the plates of C_2 be E_2. Then evidently $E = E_1 + E_2$. But $E_1 = \frac{Q_1}{C_1}$ (Equation 77), where Q_1 is the

charge upon one of the plates of C_1 and $E_2 = \frac{Q_2}{C_2}$. Therefore,

$$E = \frac{Q_1}{C_1} + \frac{Q_2}{C_2}$$

Now there is evidently a condenser of such capacity that if placed by itself between the points A and B it will completely take the place of the combination C_1 and C_2 as represented in the figure. The capacity of this condenser will be equal to the capacity of the combination C_1 and C_2. Call this capacity C. If this third condenser is substituted for the combination shown in the figure, we may write,

$$E = \frac{Q_1}{C}$$

Combining these two equations for E,

$$\frac{Q_1}{C} = \frac{Q_1}{C_1} + \frac{Q_2}{C_2}$$

But Q_2 is equal to Q_1 from the following considerations. When the positive charge Q_1 appears upon the upper plate of C_1, an equal negative charge appears upon the lower plate of C_1. According to the electron theory this negative charge on the lower plate of C_1 has been brought about by the transfer of a number of electrons from the upper plate of C_2. Therefore the deficit of electrons on the upper plate of C_2 is equal to the excess of electrons on the lower plate of C_2. That is, $Q_2 = Q_1$. We have, therefore,

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

in which C is the equivalent capacity of the combination of C_1 and C_2 connected in series. Equation (47) may be written in the following form,

$$C = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} \tag{79}$$

i.e. **the capacity of two condensers connected in series is equal to the reciprocal of the sum of the reciprocals of their individual capacities.**

charge now accumulates upon J until it again reaches the value q, when a second discharge occurs between knobs ab. The charge on C is now $2q$. When the third spark passes between ab the charge on C is $3q$, etc. The process is continued until a spark passes between AD. Suppose that when this occurs three sparks have passed between ab. Another condenser C_1 is now substituted for C, all other connections remaining exactly the same, and the operation is repeated. Suppose that in the case of this second condenser, four sparks pass between a and b before the discharge occurs between A and D, then the capacity of C is to the capacity of C_1 as 3 is to 4.

THE ENERGY OF A CHARGE

271. Let it be assumed that a condenser is charged by transferring a number of unit charges one after another from one plate of a condenser to the other. Evidently the work done upon the first unit charge will be zero, since the initial potential difference is assumed to be zero. A very small amount of work will have to be done upon the second unit charge, a little more work upon the third unit charge, and so on. If the final potential difference is E, then it is evident that the average work done upon one of these unit charges is $\frac{1}{2}E\left(=\frac{E+0}{2}\right)$. This follows from the definition of potential and from the fact that the potential difference established is at each instant proportional to the number of unit charges which have been transferred. But **the average work done upon unit charge multiplied by the number of unit charges brought up will give the total work done in charging the body.** Thus we have,

$$W=\tfrac{1}{2}QE \tag{80}$$

or substituting for E its value in terms of the charge Q and the capacity of the condenser (Equation 77) we have,

$$W=\frac{1}{2}\frac{Q^2}{C}. \tag{81}$$

THE OSCILLATORY DISCHARGE

272. If a highly charged condenser is suddenly discharged, there will, in general, be an oscillation of the condition of electrification between the two plates. It is as if the negative charge on one plate, rushing over to neutralize the positive charge on the other, overshot the mark, so that after the first rush a certain excess of negative charge existed upon the second plate and an excess of positive upon the first plate. In other words, the electricity at the moment of discharge behaves as if it had a sort of inertia. As soon as the first reversal of the charge has taken place, the condenser will discharge again, and again the two opposite charges in rushing together will overshoot; that is, this inertia effect will again come into play, and the charge upon each plate of the condenser will be once more reversed in sign. The successive charges grow rapidly less in amount and quickly die away to zero value. This sort of discharge is known as the oscillatory discharge.

LIGHTNING

273. A lightning flash is a disruptive electric discharge, sometimes oscillatory in character, which takes place between two charged clouds or between a cloud and the earth.

The true character of the lightning flash was first proved by Benjamin Franklin in his classical kite experiment. In this experiment he "drew" electrostatic charge from passing clouds by means of a silk kite having a hempen string. The kite was provided with a sharp point, and the charged cloud in passing the kite electrified the kite and string by induction, the attracted charge streaming off the sharp point at the kite, the repelled charge appearing at the lower end of the string. In addition to this experiment, which demonstrated the fact that clouds, during a thunderstorm, are charged with static electricity, Franklin proceeded to identify the lightning flash with the electric discharge by comparing the several different effects of each. He found them, for example, to be identical in their heating effects, lighting effects, in the production of sound, in their mechanical effects and physiological effects.

THE SOURCE OF THE HIGH POTENTIALS OF THUNDERSTORMS

274. The manner in which clouds receive their initial charges of electricity is not clearly understood. Various attempts have been made to account for these initial charges, but no entirely satisfactory theory has yet been evolved. Assuming the presence of small initial charges, however, it is easy to explain the development of the enormous potentials which must evidently be present in order to cause discharges through the great distances through which lightning is known to "strike." It would seem that lightning flashes are oftentimes one half or three quarters of a mile in length, or even more. The potentials represented by these flashes are of course very great. The development of these high potentials is explained as follows:

Let it be assumed that the cloud is made up of minute particles of water vapor, each one bearing an infinitesimal charge of electricity. When condensation sets in and these small drops coalesce to form larger ones, the charges carried by the small drops are combined into larger charges upon the larger drops. Now it has been demonstrated (Section 267) that the capacity of a sphere is numerically equal to its radius. Any change in the size of a charged drop of water will therefore be accompanied by a change in its potential, since, as we have seen, the potential of a body varies inversely as the capacity, providing the charge remains the same. Now the volume of a sphere is proportional to the cube of its radius, therefore, eight small drops would combine to form one drop of twice the radius of the smaller drops. Furthermore, this large drop will have upon it eight times as much charge as the small drop, but since the capacity of the large drop is but twice as great as the capacity of one of the small drops, it will be evident that the potential to which the large drop is charged by these eight small charges combined upon its surface will be four times as great as the potential of the small drops when they exist singly. Thus, as condensation goes forward and the size of the drops increases, the potential of the cloud rapidly rises. Positive and negative charges are developed in nearly equal amounts among the clouds, and the large majority of lightning flashes are therefore from

cloud to cloud. The number of flashes which reach the earth is relatively small.

THE LIGHTNING ROD

275. The manner in which a lightning rod operates to prevent lightning discharge between a cloud and a building is as follows:

Let *A*, Figure 180, represent a charged cloud; *B*, a building provided with a lightning rod. The positive charge on the cloud charges the surface of the earth in the neighborhood of the cloud by induction as indicated in the figure, so that the surface of the ground immediately beneath the cloud, and the building *B* as well, are strongly electrified. This negative charge upon the building *B* is discharged by the sharp point of the rod, thus relieving the strain in the air in the neighborhood of the building.

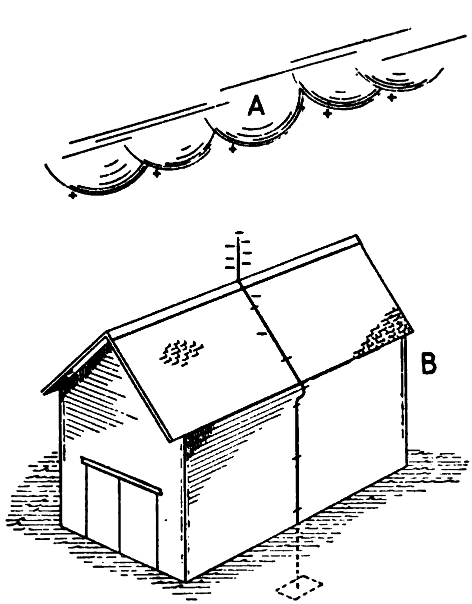

FIG. 180. — Action of the Lightning Rod.

Evidently, as this action goes forward the potential difference between *A* and *B* grows steadily less and the danger of a disruptive discharge is diminished. In the absence of the sharp point with its discharging action the dielectric strain between *A* and *B* might become greater and greater until it was relieved by a disruptive discharge between *A* and *B*, that is to say, until the building was "struck by lightning."

PROTECTION AFFORDED BY A LIGHTNING ROD

276. The lightning rod unquestionably protects the building upon which it is placed providing the dielectric strain which is

developed in the neighborhood of the building is developed slowly. When the dielectric strain is suddenly developed, a lightning rod seems to afford little or no protection against a disruptive discharge. It is customary to distinguish two kinds of dielectric strain as developed in the thunderstorm. The first is known as the condition of **steady strain.** The second condition is known as the condition of **sudden strain.**

The case of the **steady strain** may be illustrated in the following manner: Let A and B, Figure 181, represent the terminals of an electric machine. C is a condenser with its plates connected between the terminals as indicated. D and E are two conductors providing a spark gap for the discharge of the condenser C. Upon the conductor D is placed two terminals, one having a sharp point and the other terminating in a round metal knob. If, with the connections as represented, the machine is put into operation, the potential difference between the plates of C will tend to increase to higher and higher values. It will be found, however, that no spark will pass between the terminals D and E. The reason is that the discharging action of the point comes into play and the dielectric strain is steadily and continuously relieved. It will be noted that the strain which comes upon the dielectric between D and E is gradually developed.

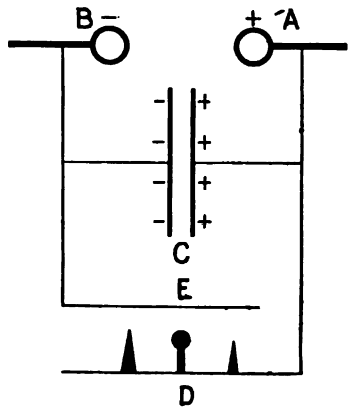

FIG. 181.—The Condition of "Steady Strain."

The case of **sudden strain** is illustrated in Figure 182. A and B are the terminals of the electric machine. C_1 and C_2 are condensers. The first plate of each condenser is connected to a terminal of the machine. The second plates of C_1 and C_2 communicate with one another by means of the conductors D and E and the spark gap between them. As the machine is put into operation, charges accumulate upon the condensers as indicated in the figure, the separation of electricities upon the

THE ESSENTIALS OF A GOOD LIGHTNING ROD

277. The essentials of a good lightning rod are:

(*a*) Continuity.

(*b*) Sharp points.

(*c*) Good ground connections.

A lightning rod which is discontinuous or has bad joints in it is a source of danger, since whatever discharge passes the rod may be of the nature of a disruptive discharge at the poor connection. A heating effect is therefore produced which may result in setting the building on fire. Evidently in order that the discharging action referred to in Section 275 may take place readily, it is necessary that the rod be provided with a sharp point, or points, and that the connection between it and the moist earth upon which the negative charge is induced be a good one.

Problems

1. Two spheres of equal size are given charges of +50 and −30 respectively. They are brought into contact and then separated. How are the charges altered?

2. How are the potentials altered in problem 1?

3. A condenser of capacity 80 and charge 400 is connected by a poor conductor to earth. When the energy of the charged condenser is reduced to one sixteenth its initial value, what charge remains on the condenser?

4. How is the potential altered in problem 3?

5. A condenser with air between its plates has a capacity of 500. When glass is substituted for the air, the capacity of the condenser is 3200. What is the specific inductive capacity of the glass?

6. The force action between two charged plates separated by a distance d in air is F. What would be the force if the space between the plates were filled with a liquid having a specific inductive capacity of 2.4?

7. One plate of a condenser is connected to earth. A charge of 500 c. g. s. units on the other plate will raise its potential to 100. What is the capacity of the condenser?

8. What energy would the charged condenser of problem 7 possess?

9. Two condensers in series are charged to a potential difference of 1000 c. g. s. units. The capacities of the condensers are 5 and 15 c. g. s. units. What energy is stored in the charged condensers?

10. To what potential difference are the condensers of problem 9 individually subjected?

THE UNIT JAR

270. The unit jar is a Leyden jar provided with an adjustable spark gap. The charge which may be placed upon this condenser depends upon the distance which separates the knobs of the spark gap. When such a condenser is connected to the terminals of the Holtz machine, the potential difference between the plates and the knobs of the spark gap rises until it is sufficient to break down the air between the knobs. At this point a discharge occurs, and the potential difference again builds up until a second spark passes, and so on. Evidently the quantity stored in the condenser at the moment the spark passes is each time the same so long as the distance between the knobs remains unchanged.

The unit jar may be used for making a rough comparison of the capacities of condensers in the following manner: In Figure 179 let C represent a condenser whose capacity is to be measured. It is connected as shown, in series with the unit jar J between the knobs AB of an electric machine. ab are the knobs of the adjustable spark gap of the unit jar. AD is a spark gap across which the condenser C discharges when the potential difference between its plates rises to a sufficient value. When the machine is operated, the following action takes place: Positive and negative charges accumulate upon the plates of the two condensers as indicated in the figure. When a sufficient quantity, say q, has accumulated on the condenser J, a spark will pass between the knobs ab. At this same instant there is a charge q on the condenser C also. When the unit jar discharges, it loses its charge, of course, but the charge q remains upon C. A new

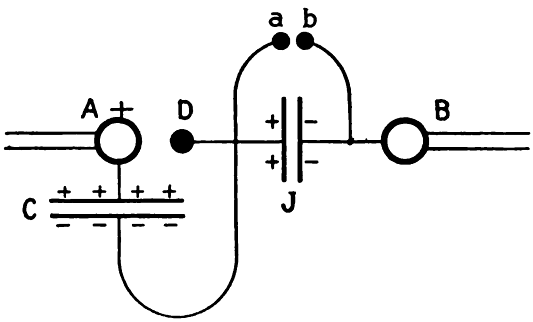

FIG. 179. — Showing how the Unit Jar is Used.

charge now accumulates upon J until it again reaches the value q, when a second discharge occurs between knobs ab. The charge on C is now $2q$. When the third spark passes between ab the charge on C is $3q$, etc. The process is continued until a spark passes between AD. Suppose that when this occurs three sparks have passed between ab. Another condenser C_1 is now substituted for C, all other connections remaining exactly the same, and the operation is repeated. Suppose that in the case of this second condenser, four sparks pass between a and b before the discharge occurs between A and D, then the capacity of C is to the capacity of C_1 as 3 is to 4.

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THE SOURCE OF THE HIGH POTENTIALS OF THUNDERSTORMS

274. The manner in which clouds receive their initial charges of electricity is not clearly understood. Various attempts have been made to account for these initial charges, but no entirely satisfactory theory has yet been evolved. Assuming the presence of small initial charges, however, it is easy to explain the development of the enormous potentials which must evidently be present in order to cause discharges through the great distances through which lightning is known to "strike." It would seem that lightning flashes are oftentimes one half or three quarters of a mile in length, or even more. The potentials represented by these flashes are of course very great. The development of these high potentials is explained as follows:

Let it be assumed that the cloud is made up of minute particles of water vapor, each one bearing an infinitesimal charge of electricity. When condensation sets in and these small drops coalesce to form larger ones, the charges carried by the small drops are combined into larger charges upon the larger drops. Now it has been demonstrated (Section 267) that the capacity of a sphere is numerically equal to its radius. Any change in the size of a charged drop of water will therefore be accompanied by a change in its potential, since, as we have seen, the potential of a body varies inversely as the capacity, providing the charge remains the same. Now the volume of a sphere is proportional to the cube of its radius, therefore, eight small drops would combine to form one drop of twice the radius of the smaller drops. Furthermore, this large drop will have upon it eight times as much charge as the small drop, but since the capacity of the large drop is but twice as great as the capacity of one of the small drops, it will be evident that the potential to which the large drop is charged by these eight small charges combined upon its surface will be four times as great as the potential of the small drops when they exist singly. Thus, as condensation goes forward and the size of the drops increases, the potential of the cloud rapidly rises. Positive and negative charges are developed in nearly equal amounts among the clouds, and the large majority of lightning flashes are therefore from

cloud to cloud. The number of flashes which reach the earth is relatively small.

THE LIGHTNING ROD

275. The manner in which a lightning rod operates to prevent lightning discharge between a cloud and a building is as follows:

Let *A*, Figure 180, represent a charged cloud; *B*, a building provided with a lightning rod. The positive charge on the cloud charges the surface of the earth in the neighborhood of the cloud by induction as indicated in the figure, so that the surface of the ground immediately beneath the cloud, and the building *B* as well, are strongly electrified. This negative charge upon the building *B* is discharged by the sharp point of the rod, thus relieving the strain in the air in the neighborhood of the building.

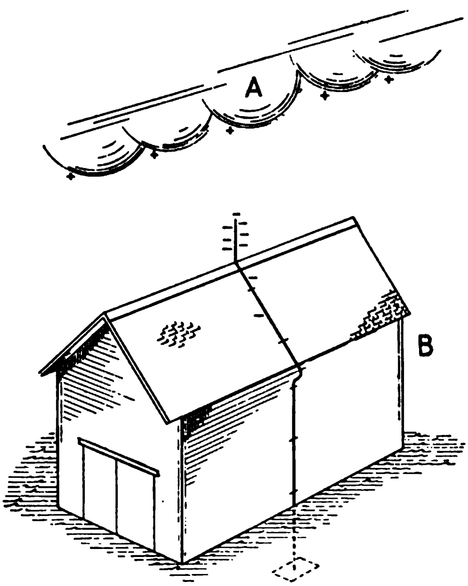

FIG. 180. — Action of the Lightning Rod.

Evidently, as this action goes forward the potential difference between *A* and *B* grows steadily less and the danger of a disruptive discharge is diminished. In the absence of the sharp point with its discharging action the dielectric strain between *A* and *B* might become greater and greater until it was relieved by a disruptive discharge between *A* and *B*, that is to say, until the building was "struck by lightning."

PROTECTION AFFORDED BY A LIGHTNING ROD

276. The lightning rod unquestionably protects the building upon which it is placed providing the dielectric strain which is

developed in the neighborhood of the building is developed slowly. When the dielectric strain is suddenly developed, a lightning rod seems to afford little or no protection against a disruptive discharge. It is customary to distinguish two kinds of dielectric strain as developed in the thunderstorm. The first is known as the condition of **steady strain.** The second condition is known as the condition of **sudden strain.**

The case of the **steady strain** may be illustrated in the following manner: Let *A* and *B*, Figure 181, represent the terminals of an electric machine. *C* is a condenser with its plates connected between the terminals as indicated. *D* and *E* are two conductors providing a spark gap for the discharge of the condenser *C*. Upon the conductor *D* is placed two terminals, one having a sharp point and the other terminating in a round metal knob. If, with the connections as represented, the machine is put into operation, the potential difference between the plates of *C* will tend to increase to higher and higher values. It will be found, however, that no spark will pass between the terminals *D* and *E*. The reason is that the discharging action of the point comes into play and the dielectric strain is steadily and continuously relieved. It will be noted that the strain which comes upon the dielectric between *D* and *E* is gradually developed.

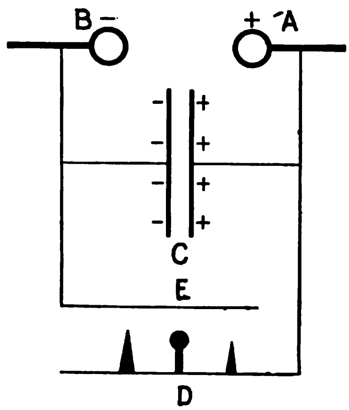

FIG. 181.—The Condition of "Steady Strain."

The case of **sudden strain** is illustrated in Figure 182. *A* and *B* are the terminals of the electric machine. C_1 and C_2 are condensers. The first plate of each condenser is connected to a terminal of the machine. The second plates of C_1 and C_2 communicate with one another by means of the conductors *D* and *E* and the spark gap between them. As the machine is put into operation, charges accumulate upon the condensers as indicated in the figure, the separation of electricities upon the

second plates of the condensers being made possible by the discharging action of the point at *D*. As the operation of the machine continues the charges upon the condensers increase, and the potential difference between the plates of the condensers and between *A* and *B* is steadily increased. When this potential difference becomes sufficiently great, a disruptive discharge takes place between *A* and *B*. The charges upon the first plates of C_1 and C_2 are neutralized by this discharge; **the charges upon the second plates of the condensers thus become suddenly free** and rush together, causing a disruptive discharge between the conductors *D* and *E*. This discharge will take place as often between *E* and the round knob as between *E* and the sharp point. In this case the dielectric strain which comes upon the air between *D* and *E* is developed suddenly.

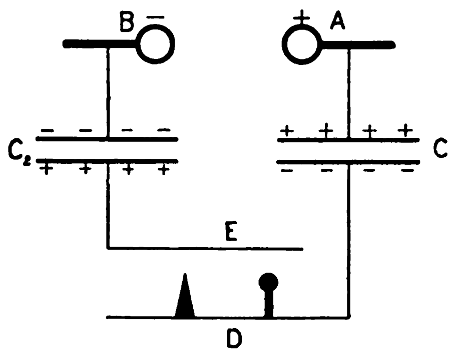

FIG. 182. — The Condition of "Sudden Strain."

Comparing the two cases represented by Figures 181 and 182, it will be seen that **in the first case the point operates to protect the rounded metal knob.** Whatever discharge takes place passes to the point and is prevented from going to the rounded knob. **In the second case the discharge will pass as often to the rounded knob as to the sharp point.** In this case, therefore, the sharp point affords no protection to the knob.

There are cases on record in which houses provided with good lightning rods have been struck by lightning, the discharge apparently not passing to the rod at all. It is probable that these cases are to be explained on the theory of the sudden strain. Fortunately the sudden strain condition as produced in a thunderstorm is, comparatively speaking, of rare occurrence.

THE ESSENTIALS OF A GOOD LIGHTNING ROD

277. The essentials of a good lightning rod are:

(*a*) Continuity.
(*b*) Sharp points.
(*c*) Good ground connections.

A lightning rod which is discontinuous or has bad joints in it is a source of danger, since whatever discharge passes the rod may be of the nature of a disruptive discharge at the poor connection. A heating effect is therefore produced which may result in setting the building on fire. Evidently in order that the discharging action referred to in Section 275 may take place readily, it is necessary that the rod be provided with a sharp point, or points, and that the connection between it and the moist earth upon which the negative charge is induced be a good one.

Problems

1. Two spheres of equal size are given charges of $+50$ and -30 respectively. They are brought into contact and then separated. How are the charges altered?

2. How are the potentials altered in problem 1?

3. A condenser of capacity 80 and charge 400 is connected by a poor conductor to earth. When the energy of the charged condenser is reduced to one sixteenth its initial value, what charge remains on the condenser?

4. How is the potential altered in problem 3?

5. A condenser with air between its plates has a capacity of 500. When glass is substituted for the air, the capacity of the condenser is 3200. What is the specific inductive capacity of the glass?

6. The force action between two charged plates separated by a distance d in air is F. What would be the force if the space between the plates were filled with a liquid having a specific inductive capacity of 2.4?

7. One plate of a condenser is connected to earth. A charge of 500 c.g.s. units on the other plate will raise its potential to 100. What is the capacity of the condenser?

8. What energy would the charged condenser of problem 7 possess?

9. Two condensers in series are charged to a potential difference of 1000 c.g.s. units. The capacities of the condensers are 5 and 15 c.g.s. units. What energy is stored in the charged condensers?

10. To what potential difference are the condensers of problem 9 individually subjected?

Omit

ELECTROKINETICS

CHAPTER XXIII

THE ELECTRIC CURRENT

278. When two conductors between which there is a difference of potential are connected by means of a wire, a transfer of electricity takes place. Under these circumstances there is said to be an **electric current** in the wire. According to the electron theory **this current consists of a procession of electrons moving toward the conductor which is most strongly charged positively.** This may be termed the electron current. Under the theories universally adopted before the advent of the electron theory, the current is assumed to flow from the body having the higher (positive potential) to that of lower potential. This is called the positive current. For example, referring to Figure 183, let A represent a body charged positively and let B represent a negatively charged body or one having a lower positive potential. If a wire w is made to form a connection between A and B, there will be a positive current in the wire in the direction indicated by the arrow.

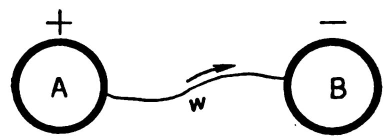

FIG. 183. — The Direction of the Positive Current.

If there is nothing to maintain the potential difference between A and B, the current in the wire will exist for a brief interval only after the wire is attached. If the potential difference between A and B is maintained, for example, by the continuous operation of an electrostatic machine, there will be a steady current in the wire.

THE STRENGTH OF THE CURRENT

279. The strength of the current in a wire is the rate at which electrostatic charge is transferred. Let it be assumed,

for example, that in the case described in the last section, Q units of charge pass from A to B in the time t. Then the ratio $Q \div t$ gives the average strength of the current in the wire during that interval, that is,

$$I = \frac{Q}{t} \tag{82}$$

in which I stands for the electric current.

If Q in Equation 82 is expressed in c. g. s. electrostatic units of charge and t in seconds, then I is expressed in terms of the c. g. s. electrostatic unit of current. This unit is much too small to be used conveniently in practical measurements. For this reason a unit equal to $3{,}000{,}000{,}000 = 3(10)^9$ c. g. s. electrostatic units of current has been agreed upon as the practical unit of current. This unit of current is called the **ampere.**

ELECTROMOTIVE FORCE

280. The term **electromotive force** (e. m. f.) is sometimes used instead of **potential difference** to specify that which causes a current to flow in a wire or other conductor. Evidently the unit of electromotive force is the same as that of potential difference. The c. g. s. electrostatic unit of electromotive force or potential difference (Section 256) is too large for convenience in practical measurements and by universal agreement a unit equal to $\frac{1}{300} (= \frac{1}{3} 10^{-2})$ c. g. s. electrostatic units has been adopted. This unit of electromotive force or potential difference is called the **volt.**

OHM'S LAW

281. It was discovered by Ohm that **the ratio of the potential difference between the terminals of a conductor and the current which flows in the conductor in response to that potential difference is constant,** so long as the physical state of the conductor remains the same. That is, calling the potential difference between the terminals of the conductor E and the current in the conductor C, then,

$$\frac{E}{C} = R = \text{a constant} \tag{83}$$

This relation is known as Ohm's Law. The constant R, which is found to depend upon the dimensions of the conductor and the material of which it is made, is called the **resistance** of the conductor.

Equation (83) is a definition for resistance. The unit of resistance defined by this equation is evidently that resistance through which unit potential difference will cause unit current to flow. That is to say, if, when unit potential difference is maintained between the ends of a conductor, unit current flows in the conductor, the conductor is said to have unit resistance.

If the potential difference is measured in volts and the current is measured in amperes, the resistance is measured in **ohms. A conductor has a resistance of one ohm when a potential difference between its ends of one volt will cause a current of one ampere to flow in it.**

FALL OF POTENTIAL

282. In order that water may flow in a pipe there must be **a difference of pressure** from point to point along the pipe, the pressure decreasing in the direction of flow. In order that heat may flow (be conducted) along a bar of metal there must exist **a difference of temperature** from point to point, the temperature decreasing in the direction in which the heat is flowing. In the same way there is **a difference of potential** or "fall of potential" from point to point along a conductor through which an electric current is flowing.

The current flows from a region of higher to a region of lower potential, and the potential difference between the ends of a conductor carrying current may be regarded as the cause of the flow of current in the conductor.

A portion of the e. m. f. of a dynamo or electric battery is therefore required for each part of the circuit through which it causes current to flow. According to Ohm's law a portion of the circuit which has high (large) resistance requires a large part of the total e. m. f., and a portion of the circuit which has small resistance requires a relatively small part of the e. m. f. In other words the e. m. f. is distributed throughout the cir-

cuit as **fall of potential** according to the relative resistances of the various parts.

Consider a system like that represented in Figure 184. A dynamo supplies current to a group of incandescent lamps, the lamps being connected to the dynamo by the lines (conductors) *ab* and *cd*. If the total current is I and the resistance of the group of lamps is R_3, the potential difference between the

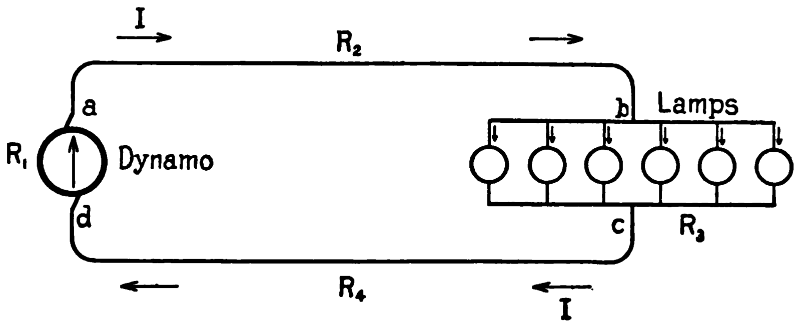

FIG. 184.

lamp terminals is R_3I, according to Ohm's law. Let R_2 be the resistance of the line *ab* and R_4 the resistance of the line *cd*. Then the **fall of potential** in *ab* (*i.e.* the difference of potential between *a* and *b*) is R_2I, and the fall of potential in *cd* is R_4I. Now the conductors through which the current flows in the dynamo also have resistance. Call this resistance R_1. The current flows through this resistance out over the line *ab*, through the lamps, back over *cd*, and again through R_1, and so on. Therefore a part of the e. m. f. generated by the dynamo must be apportioned to the resistance R_1. This part of the e. m. f., is of course, R_1I.

If E is the e. m. f. generated by the dynamo, evidently,

$$E = R_1I + R_2I + R_3I + R_4I$$

or,

$$E = \Sigma RI \tag{84}$$

that is, the total e. m. f. generated by the dynamo (applied to the circuit) is equal to the sum of the RI values around the circuit.

Evidently, in order that a large part of the e. m. f. may be effective at the lamps, the fall of potential in the lines must be small. The fall of potential in the lines may be made small by making R_2 and R_4, Figure 184, the resistance of the line, small. It is for this reason that large copper wires are often used for such lines. Large copper wires have relatively small resistance.

COMPARISON OF RESISTANCES BY FALL OF POTENTIAL

283. If two conductors having resistances K and X respectively are connected in series (*i.e.* so connected that the same current flows through each) and a current I is caused to flow through them, see Figure 185, the ratio of the resistances may be found by measuring and comparing the corresponding potential differences. Call the fall of potential in K, V_1, and that in X, V_2. We have, then,

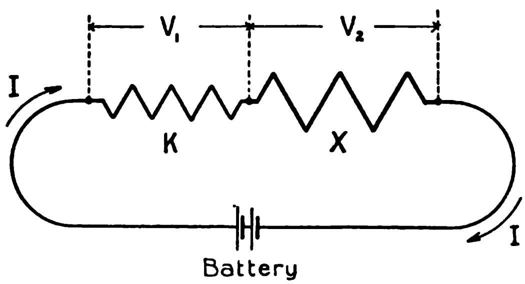

FIG. 185.

$$\frac{V_1}{V_2} = \frac{KI}{XI} = \frac{K}{X}$$

whence

$$X = K \cdot \frac{V_2}{V_1} \quad (85)$$

The fall of potential in a conductor may be measured by means of a voltmeter, which is an instrument for measuring e. m. f. or potential difference. If, therefore, with such an instrument V_1 and V_2 are measured, the ratio $K : X$ may be obtained from the above relation. If the value of K is known the equation may be solved for the numerical value of X.

SPECIFIC RESISTANCE

284. As stated above, the resistance of a conductor is found to depend upon the dimensions of the conductor and upon the material of which the conductor is made. Assuming that the physical state (temperature, etc.) remains constant,

these are the only things upon which the resistance of the conductor depends. It is found by experiment that the longer the conductor, other things being equal, the greater is its resistance. That is to say, the resistance varies directly with the length. It is also found by experiment that, other things being equal, the smaller the cross section of the conductor the greater the resistance; that is to say, the resistance is inversely proportional to the cross section of the conductor. This may be expressed algebraically as follows:

$$R \propto \frac{L}{q}$$

or,

$$R = k \cdot \frac{L}{q} \tag{86}$$

in which L is the length of the conductor and q its cross section. The proportionality constant k which appears in Equation (86) is known as the **specific resistance** of the material of which the conductor in question is made. Equation (86) is a definition for specific resistance. Evidently from the equation, **k is the resistance of a conductor of the given material having a length of one centimeter and a cross section of one square centimeter.**

The specific resistances of a few of the common metals are given in the following table of specific resistances:

Substance	Specific Resistance in Ohms at 20° C.
Silver, annealed	1.488×10^{-6}
Copper, annealed	1.580×10^{-6}
Platinum, annealed	8.957×10^{-6}
Iron, annealed	9.611×10^{-6}
Nickel, annealed	12.320×10^{-6}
German silver, pressed	20.76×10^{-6}

VARIATION OF RESISTANCE WITH TEMPERATURE

285. In discussing Ohm's Law and specific resistance the assumption has been made that the temperature of the material under investigation remained constant. It is found that the resistance of a conductor depends upon its temperature. The resistance of metals increases with a rise of temperature. Such

substances are said to have positive temperature coefficients. The resistance of some substances, for example, carbon, decreases with a rise of temperature. Such substances are said to have negative temperature coefficients. It is found that the law of increase in resistance for metals with rise of temperature may be expressed very approximately as follows:

$$R_t = R_0(1 + \alpha t) \tag{87}$$

in which R_t = resistance at $t°$ C. and R_0 = resistance at 0° C. α is known as the **temperature coefficient of resistance** for the material under investigation. For most pure metals it is about 0.4 of 1 per cent per Centigrade degree.

The temperature coefficient of resistance of alloys is, in general, less than that of the metals composing them. Thus, manganin, an alloy of manganese, copper, and nickel, has a temperature coefficient which is very nearly zero and is even negative for certain temperatures.

THE RESISTANCE OF CONDUCTORS IN PARALLEL

286. Let two conductors be connected in parallel between two points A and B, Figure 186. Under these circumstances the effective resistance between the points A and B is less than that of either of the conductors. The actual value of this effective resistance may be found as follows: Let the resistance of the upper branch be R_1, that of the lower branch R_2. Call the total current which divides at A between the two branches I.

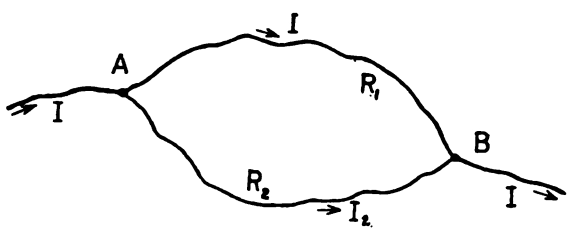

FIG. 186. — The Resistance of a Branched Circuit is Less than that of Either Branch.

Let I_1 be the current in the upper branch and I_2 the current in the lower branch. Then, evidently,

$$I = I_1 + I_2 \tag{a}$$

Calling the potential difference between the points A and B, E, it follows at once from Ohm's Law that

$$I_1 = \frac{E}{R_1} \tag{b}$$

and

$$I_2 = \frac{E}{R_2} \tag{c}$$

Now, let it be assumed that there is a third conductor of resistance R, such that, if placed between the points A and B, it would in every respect take the place of the branched circuit. Then, since the same total current I would flow through this resistance, if it were substituted for R_1 and R_2, therefore

$$I = \frac{E}{R} \tag{d}$$

Substituting the values of I_1, I_2 and I from (*b*), (*c*), and (*d*) in (*a*), we have

$$\frac{E}{R} = \frac{E}{R_1} + \frac{E}{R_2}$$

or dividing through by E,

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

or

$$R = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} \tag{88}$$

In other words, **the resistance of a branched circuit is equal to the reciprocal of the sum of the reciprocals of the resistances of the branches.**

THE SHUNT

287. A conductor connected to an instrument in such manner as to form a branched circuit with the instrument is called a shunt. The effect of a shunt is to switch or "shunt" a part of the total current past the shunted instrument. For example, let A, Figure 187, be an instrument, let us say, an ammeter (ampere meter), between the terminals of which has been placed a conductor having a resistance R_2. Evidently a current I coming to the instrument will divide, a part going

through the ammeter and a part going through the shunt. The current in the ammeter will be but a fractional part of the total current I. That part of the total current which flows through the ammeter may be predetermined by a proper adjustment of the resistance of the shunt.

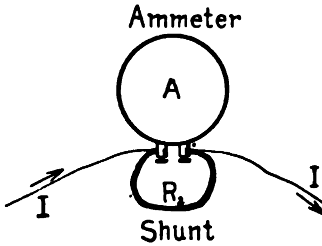

FIG 187.

Evidently the ammeter A takes the place of the resistance R_1 in the branched circuit of Figure 186. That portion of the total current which flows through the ammeter when connected in this manner is determined as follows: Let E be the potential difference between the terminals of the branched circuit as in the discussion of the last section. Then I_1, the current in the ammeter, equals $E \div R_1$, where R_1 is the resistance of the ammeter. But I, the total current, is equal to $E \div R$, where R is the resistance of the branched circuit. We have, therefore,

$$I = \frac{E}{R} = E\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \quad \text{(See Equation 88)}$$

$$= E\frac{R_1 + R_2}{R_1 R_2}$$

Whence,

$$E = I \cdot \frac{R_1 R_2}{R_1 + R_2}$$

Putting this value of E in the expression for I_1, $\left(I_1 = \frac{E}{R_1}\right)$ we have

$$I_1 = \frac{I}{R_1} \cdot \frac{R_1 R_2}{R_1 + R_2}$$

or,

$$I_1 = I \cdot \frac{R_2}{R_1 + R_2} \tag{89}$$

If, for example, $R_1 = 9$ ohms and $R_2 = 1$ ohm, then the fraction, $R_2 \div (R_1 + R_2) = 0.1$, that is, to say, 0.1 of the total current would under these circumstances pass through the ammeter.

WHEATSTONE'S BRIDGE

288. One of the most convenient and at the same time most accurate methods for measuring resistance is by the use of Wheatstone's bridge. Wheatstone's bridge consists essentially of a branched circuit containing four resistances, two in each branch, and a cross connection between the branches containing a sensitive current detector or galvanometer. The arrangement of parts is shown in Figure 188. R_1, R_2, R_3, and R_4 are the four resistances forming the branched portion of the circuit AC. G is the sensitive galvanometer in the cross connection BD as indicated. E is supposed to represent a battery which supplies the current used in the instrument. Let it be assumed that the resistances R_1, R_2, R_3, and R_4 have such values that **no current flows in the cross connection BGD. This will be the case only when the points B and D are at the same potential.** That this condition of affairs is possible will be readily understood by a moment's study of the hydraulic analogue represented in Figure 189. Let abc and adc be the branches of a stream of water flowing about the island I. Let it be further imagined that beginning at the point d, a ditch is dug across the island. Evidently if this ditch is joined to the upper branch of the stream at the proper point, there will be no tendency for water to flow in it in either direction. If the b end of the ditch is connected too far upstream, water will flow in the ditch g in the direction bgd. If it is connected too far downstream, there will be a flow of water in the ditch in the direction dgb. There is one point, b, therefore at which the ditch may terminate such that there will be no tendency for the water to

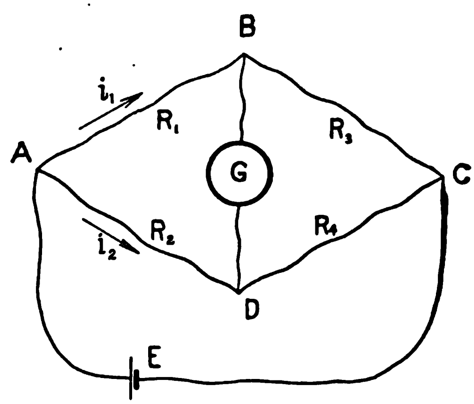

FIG. 188. – Network of the Wheatstone Bridge.

flow in either direction in the ditch g. **Evidently that point b is the one which is at the same level as the point d.** Returning to the discussion of Wheatstone's bridge, if the points B and D are at the same potential, there will be no flow of current through the galvanometer G.

Assuming that this condition has been reached, the following relation must hold among the four resistances. From Ohm's

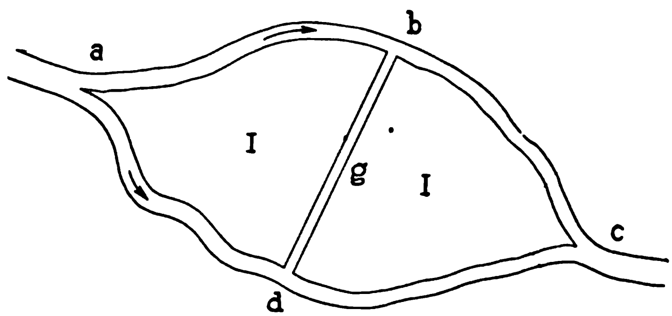

FIG. 189. — Wheatstone Bridge Analogue.

Law we have the potential difference A to B is I_1R_1, and the potential difference A to D is I_2R_2. But since B and D are at the same potential, these two potential differences must be equal. We may therefore write

$$I_1R_1 = I_2R_2$$

In the same manner, we may write

$$I_1R_3 = I_2R_4$$

Dividing the first equation by the second, we have,

$$\frac{R_1}{R_3} = \frac{R_2}{R_4}$$

or,

$$R_1R_4 = R_2R_3$$

The Wheatstone bridge is used for measuring an unknown resistance in the following manner: the resistance to be measured, call it R_1, is connected in series with a known resistance R_3, **the value of which can be varied**. The other side of the bridge is formed of two known resistances R_2 and R_4.

When the connections are complete **the resistance of R_3 is varied** until no current flows in G. Then,

$$R_1 = R_3 \cdot \frac{R_2}{R_4} \tag{90}$$

RESISTANCE THERMOMETER

289. Advantage may be taken of the fact that the resistance of a conductor varies with its temperature in estimating the variations of temperature to which the conductor is subjected. That is, it is possible to measure the temperature of a given region by comparing the resistance of a conductor when placed in that region with its resistance when kept at some standard temperature, say 0° C. The form which the resistance thermometer usually takes is that of a Wheatstone bridge with extended arms as represented in Figure 190. The extended arm *AB* is made up of large copper wires joined at their extremi-

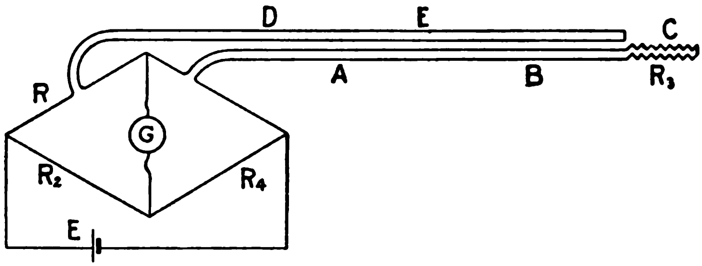

FIG. 190. — Connections of the Resistance Thermometer.

ties by a coil of fine platinum wire *C*. It is the variation in the resistance of *C* that is made use of in the estimation of temperature by the instrument. The variation in resistance of the connecting wires *A* and *B*, due to change of temperature, is usually compensated for by placing a similar pair of conductors *DE* minus the platinum wire *C* in an adjoining arm of the bridge. These conductors are placed alongside of the conductors *A* and *B* so that they are subjected to the same temperature variations. The change in resistance of *A* and *B* is thus automatically compensated. The change in the temperature of the wire *C* corresponding to a given variation in its resistance may be determined by means of Equation (87).

Problems

1. Upon what four things does the resistance of a wire depend?

2. When an electromotive force of 110 volts is applied to the terminals of an incandescent lamp, a current of 0.5 ampere flows through the lamp. What is the resistance of the lamp?

3. What potential difference must exist between the ends of a conductor having a resistance of 20 ohms in order that a current of 5 amperes may flow in the conductor?

4. A wire has a length of 10,000 cm. and a diameter of 0.2 cm. Its resistance is 0.5 ohm. What is the specific resistance of the metal of which the wire is made?

5. The resistance of a copper wire at 0° C. is 10 ohms. What is its resistance at 100° C.?

6. Two points A and B are connected by two wires in parallel. The resistances of these wires are 5 and 7 ohms respectively. What is the resistance A to B?

7. What is the resistance between two points when they are joined by three wires in parallel having resistances of 3, 5, and 7 ohms, respectively?

8. The resistance between two points in a circuit is 10 ohms. What resistance must be placed in parallel with this to reduce the resistance to 4 ohms?

9. An ammeter has a resistance of 0.27 ohm. What must be the resistance of a shunt for this instrument such that 0.1 of the total current will pass through the ammeter?

10. The terminals of a wire of 25 ohms resistance are at potentials + 50 and − 50. What current is flowing in the wire? When the potentials are + 80 and − 20? When the potentials are + 100 and 0?

11. Five hundred coulombs are carried along a wire in 25 sec. What is the average current in the wire during this interval?

12. A dynamo is connected to a group of 100 incandescent lamps in parallel. The resistance of the dynamo is 0.2 ohm. The total line resistance is 0.3 ohm. The resistance of each lamp is 200 ohms. If the dynamo e. m. f. is 100 volts, what current will flow in this circuit?

13. What part of the dynamo e. m. f. is effective at the lamps in problem 12?

14. What e. m. f. would the dynamo in problem 12 have to generate in order to supply the lamps with 100 volts at their terminals?

15. What would be the effect on the remaining lamps, problem 12, if 50 lamps were suddenly turned off, the e. m. f. of the dynamo remaining the same?

MAGNETISM

CHAPTER XXIV

MAGNETS

290. It was discovered by the ancients that a certain iron ore (now called magnetite) possessed the property of attracting and holding small bits of iron. At the present time this iron ore is found in Sweden and Spain, in Arkansas and elsewhere. Pieces of this ore are called **natural magnets** and the property which enables these magnets to attract and hold bits of iron is called magnetism. The name is derived from Magnesia, a province in Asia Minor in which natural magnets were first discovered.

In the tenth or twelfth century the discovery was made that if a magnet is suspended so as to be free to turn in a horizontal direction, it always sets itself in a definite position with respect to the points of the compass, a certain part of the magnet tending always to point toward the north, a certain other part to point toward the south. These parts of a magnet are called **magnet poles** and are distinguished as the **north-pointing pole and the south-pointing pole.** The magnetic property of a magnet is limited to its poles. If a magnet is dipped into iron filings, they will cling to the magnet only at its poles.

Magnetism is very readily imparted to a piece of iron or steel. A piece of steel possessing the properties of a natural magnet is sometimes called an **artificial magnet.** Such magnets are usually made in the form of straight bars or "horseshoes." A compass needle is a light bar magnet delicately poised on a pivot so as to be free to turn in a horizontal plane.

THE FORCE ACTION BETWEEN MAGNET POLES

291. A magnet is found to exert a force action, not only upon bits of iron and steel as pointed out above, but also upon

other magnets. It is found by experiment that **similar poles repel one another, while between unlike poles there is a force of attraction.** There is a force of repulsion between two north-pointing poles or between two south-pointing poles, while a north-pointing pole attracts a south-pointing pole.

The force action between two magnet poles depends upon the "strength" of the poles and upon the distance between the poles. A magnet has great pole strength if the force with which it acts upon other poles brought near to it is great. A rough notion of the pole strength of a bar magnet may be obtained by dipping one of the poles into iron filings. If the pole strength is great, a large mass of filings will adhere to the pole; if the pole strength is small, a smaller mass of filings will adhere. Experiment shows that **the force between two magnet poles is proportional to the strength of each pole and inversely proportional to the square of the distance between the poles.** This law may be expressed as follows:

$$F = \frac{mm'}{d^2} \tag{91}$$

in which F is written for the force between the poles, d the distance which separates them, and m and m' are the strengths of the magnetic poles.

Evidently Equation (91) is a definition for magnetic pole strength, and unit pole strength in the c. g. s. system would be defined as follows: **A magnet pole has unit strength if when placed at a distance of one centimeter from a pole of equal strength it is acted upon by a force of one dyne.**

MAGNETIC FIELD

292. The magnetic field of a magnet is that region of space into which the influence of the magnet extends. Theoretically, the field of a magnet is infinite in extent. Practically, for ordinary forms of magnets the field is quite limited.

The **lines of force in a magnetic field are** imaginary lines supposed to be drawn through the field in such manner that at each point they extend in the direction in which a small magnet pole would tend to move if placed at that point. Arrow-

heads are placed on the lines of force to show the direction in which a north-pointing pole would tend to move in the field. Evidently a south-pointing pole would tend to move in the opposite direction. **A compass needle, therefore, always tends to set itself tangent to the lines of force of the magnetic field in which it is placed.** From the above discussion it can be seen that magnetic lines of force emerge from a north-pointing pole and enter a south-pointing pole.

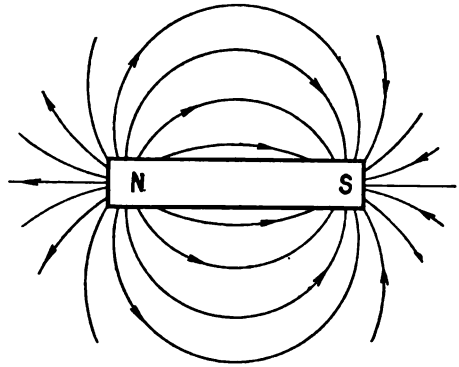

FIG. 191. — Magnetic Field of a Bar Magnet.

The general character of the magnetic field surrounding a bar magnet is represented in Figure 191.

The field about two bar magnets placed with the north-pointing pole of one opposite the south-pointing pole of the other is shown in Figure 192.

A convenient way of mapping the magnetic field about a magnet or system of magnets is to scatter iron filings in the

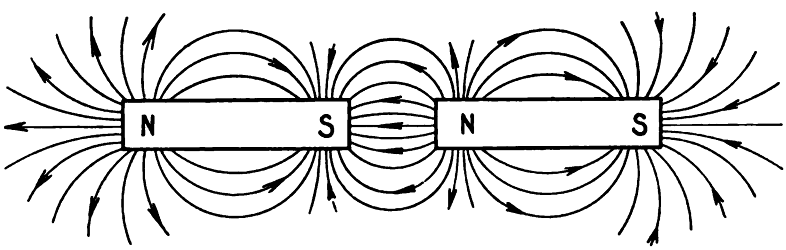

FIG. 192. — Magnetic Field surrounding two Bar Magnets.

field. The filings as they fall become magnetized and act like small compass needles, setting themselves tangent to the lines of force.

MAGNETIC SUBSTANCES

293. There are but few substances to which this property of magnetism may be imparted in any appreciable degree. These

substances are iron (steel), nickel, cobalt, manganese, chromium, and cerium. From a practical standpoint only the first three or four of the above mentioned substances are of importance. These substances are known as **magnetic substances.** Other substances, *e.g.*, copper, zinc, aluminum, are called **non-magnetic substances.**

MAGNETIZATION

294. A magnetic substance may be magnetized:

1. By contact with a magnet.
2. By means of the electric current.
3. By induction.

(1) **Magnetization by contact.** If a piece of steel, for example, a knitting needle, or a piece of a watch spring, is stroked with a magnet, it will acquire the property of the magnet or become magnetized. When a piece of steel has been magnetized in this manner, it may be used for magnetizing other pieces of steel by the same process, that is, by being rubbed upon them.

(2) **Magnetization by electric current.** It is found that if a wire is wrapped about a bar of magnetic substance and a current sent through the wire, the bar will acquire the properties of a magnet. Let *NS*, Figure 193, be a bar of iron, and *CD* a wire wrapped spirally about the bar as shown, and let *E* represent an electric battery connected to the wire *CD* in such manner as to send a current through it. Under these circumstances the bar *NS* becomes a magnet.

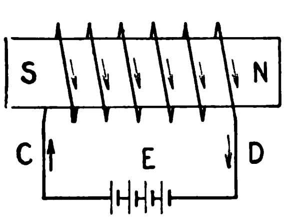

Fig. 193. — Magnetization by means of the Electric Current.

(3) **Magnetization by induction.** If a bar of magnetic substance is brought into a magnetic field, it tends to become magnetized. The magnetic substance acts as if it afforded a better or easier path for the lines of force than the air which it displaces. Because of this fact there is a tendency for the lines of force from right and left to bend into and pass through the bar of magnetic substance. Hence, **the system of lines surrounding it is very much like that surrounding an ordinary bar magnet and the bar of magnetic substance is found to possess the**

properties of a magnet. It is said to be magnetized by induction. The soft iron bar *ns*, Figure 194, brought into the presence of a magnet *NS*, becomes a magnet. The magnetic field surrounding the magnet and the soft iron bar is indicated in the

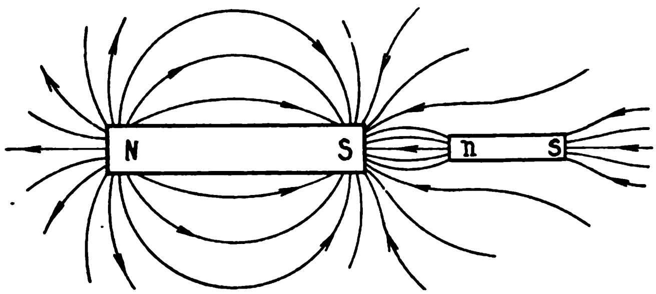

FIG. 194. — Magnetization by Induction.

figure. Under these conditions *ns* exhibits all the properties of a magnet, and is said to be magnetized by induction. It should be noticed that the end of the bar *ns* which is nearest to the inducing magnet *NS*, is of opposite polarity to that of the inducing pole *S*.

THE RETENTION OF MAGNETISM

295. If a bar of hard steel is magnetized by any of the methods above mentioned, it will be found to be more or less difficult to magnetize it strongly, but once it is magnetized, it will retain its magnetism for a great length of time. On the other hand, if a bar of soft iron is magnetized by the same method, it will be found to yield more readily to the magnetizing influence brought to bear upon it, but also to lose whatever magnetism is imparted to it, almost, if not quite completely, as soon as it is removed from the magnetizing influences. **Permanent magnets** therefore can be made of steel only. Iron is used in magnetic devices in which it is desired to quickly change the magnetic condition of the iron from time to time.

If a magnet is roughly handled, for example, if it is dropped on the floor or hammered, it is found to lose its magnetism much more rapidly than would be the case if it were not subjected to such treatment. The effect of heat upon a magnet with respect to its retention of magnetism is very marked. It

is found that if a magnet is strongly heated it loses a large part of its magnetism, and if it is heated to what is known as the critical temperature its magnetism disappears.

That property of a substance which enables it to retain its magnetism is usually referred to as the **retentivity** of the substance. The retentivity of steel is great. The retentivity of soft iron is almost zero. Hard cast iron oftentimes has considerable retentivity.

THE CRITICAL TEMPERATURE

296. Reference was made above to what is known as the critical temperature. This temperature may be defined as the temperature at which the magnetic properties of a magnetic substance disappear. It is found that if any magnetic substance is sufficiently heated, it loses its magnetic properties completely. When this temperature is reached, the substance is said to be at its critical temperature. The manner in which the magnetic properties of a magnetic substance change with the rise of temperature is indicated in Figure 195. The curve *ABC* is plotted in the following manner: Distances measured horizontally, that is, abscissæ, are temperatures on the Centigrade scale.

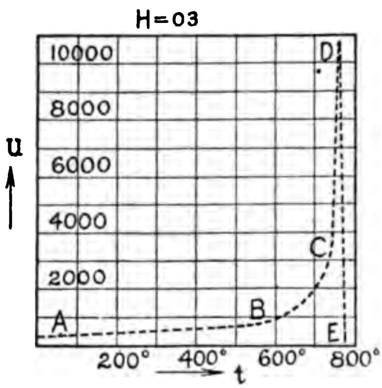

FIG. 195.—Curve showing the Effect of Temperature upon the Magnetic Property of Iron.

Distances measured vertically (ordinates) represent the magnetic properties of the magnetic substance at the different temperatures. The curve indicates that as the temperature rises, the magnetic properties of the material become more and more pronounced until a certain temperature known as the critical temperature is reached. At this point the material becomes almost, if not quite, completely non-magnetic. This is indicated by the sudden drop in the curve. The critical temperature for iron is about 786° C. The critical

temperature for nickel is about 350° C. The significance of critical temperature is not well understood, but it is known that the sudden disappearance of the magnetic properties of a magnetic substance at the critical temperature is accompanied by other marked molecular changes. Certain other physical properties change abruptly at this temperature, and it would seem as if an entire rearrangement of the molecular parts takes place.

THE THEORY OF MAGNETISM

297. The theory of magnetism which is most commonly accepted at the present time is that **each molecule of a magnetic substance is in reality a permanent magnet.** In the unmagnetized body it is supposed that these small molecular magnets are arranged in heterogeneous fashion so that they neutralize one another in their effects upon outside bodies. A magnet is considered to be a body in which these molecular magnets are turned so that they all face in the same direction. Evidently, if this were the case, then at one end of the magnet there would be a number of molecular south poles which, combined, would constitute the south pole of the magnet, while at the other end of the body there will be a group of molecular north poles, constituting the north pole of the magnet.

Evidently, under this theory, the process of magnetization is simply the process of turning these molecular magnets so as to cause them all to face in the same direction. A body exhibits a small amount of magnetism if but a few of these molecular magnets are so turned. Its magnetic properties become more marked as larger numbers of these molecular magnets are turned in the same direction.

The electron theory assumes that the magnetism of the molecule is due to the revolution of electrons about the positive part of the atom in the same way that the earth revolves about the sun. These revolving electrons constitute electric currents flowing around the molecule. The molecule is, therefore, magnetized in much the same way as the iron bar in Figure 193.

MAGNETIC FIELD INTENSITY

298. The force which a given magnet pole experiences at a given point in a magnetic field depends upon the strength of the pole and the **field intensity** at the point. A definite notion of this quantity, called magnetic field intensity, is best obtained in the following manner: Let P, Figure 196, represent a point in the neighborhood of a magnet NS, that is to say, a point in a magnetic field. Let it be imagined that different magnet poles are brought up to this point P and that the force which each pole experiences when brought to that point is carefully measured. It will be found in this experiment that in each case the force F is proportional to the strength of the magnet pole m placed at P.

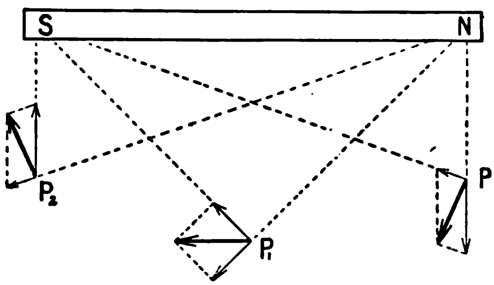

FIG. 196

That is,

$$F \propto m$$

or,

$$F = H \cdot m \tag{92}$$

The constant H in this equation, which evidently pertains to the point P, is called the **magnetic field intensity** at that point. The field intensity in a given region is sometimes called the magnetizing force in that region. Evidently,

$$H = \frac{m'}{d^2} \tag{93}$$

(Compare Equations 91 and 92)

If the magnetic pole considered in the last paragraph is thought of as unit magnet pole, then evidently $H = F$. That is to say, **the field intensity at a point is numerically equal to the force which unit magnet pole will experience if placed at that point.**

SPECIFICATION OF FIELD INTENSITY BY NUMBER OF LINES OF FORCE

299. In the discussions of the magnetic lines of force which have thus far been given, the lines have been supposed to represent the direction of the field only. It is possible to represent also the field intensity at each point by the lines of force, by making the number of lines of force per square centimeter equal to the field intensity at that point. The field intensity may then be completely specified by stating the number of lines of force per square centimeter.

INDUCTION

300. When a magnetic substance is placed in a magnetic field of given intensity, it becomes magnetized to an extent which depends upon the magnetic substance itself, and upon the field intensity or magnetizing force to which it is subjected. **The number of lines of force per square centimeter which thread through the magnetic substance is called the *induction*.**

PERMEABILITY

301. If different magnetic substances are subjected to the same magnetizing force or field intensity, the induction will in each case be different. That is to say, if a piece of iron is subjected to a given magnetizing force, the induction in the iron will have a certain value. If a piece of nickel is subjected to the same magnetizing force, the induction will be quite different. This is usually expressed by saying that the magnetic permeability is different for these different substances. The induction in iron, for example, would be in the ordinary case very high as compared with the induction in nickel or cobalt. We say, therefore, that iron is more permeable to the lines of force or has a higher permeability. **Permeability is defined as the ratio of the induction in the substance to the field intensity or magnetizing force to which the magnetic substance is subjected.** That is,

$$\mu = \frac{B}{H} \tag{94}$$

in which the symbol μ is written for the permeability, B for the induction, and H for the "magnetizing force."

The permeability of any magnetic substance depends upon the intensity of the field which is acting upon it. The permeability of iron increases for a time as the field intensity is made greater up to a certain point. After this point, known as the point of saturation, is reached, the permeability grows less with a further increase of field intensity. This fact is clearly brought out by the following table, in which are shown the corresponding values of B, H, and μ for a certain sample of wrought iron.

B	H	μ
41	0.3	128
1460	2.2	670
11540	4.9	2350
14840	10.2	1450
16900	78.0	215

The table shows that for small values of H, B is also small (relatively) and μ is small. For larger values of H both B and μ increase until H reaches a value of about 5. **For larger values of H, although B continues to increase, the ratio $\frac{B}{H}$, that is, the permeability, becomes smaller.**

The permeability of air is taken arbitrarily as unity. The permeability of non-magnetic substances, for example, wood, rubber, copper, aluminum, etc., for practical purposes are also equal to unity.

THE CURVE OF MAGNETIZATION

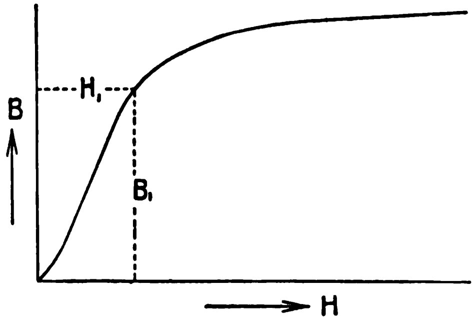

FIG. 197.—Magnetization Curve for Wrought Iron.

302. It will be evident from the statements made in the last section that the induction in a given magnetic substance is not proportional to the magnetizing force. The manner in which the induction changes in wrought iron as the

magnetizing force increases is shown by the curve, Figure 197. This curve is plotted in the following manner: The magnetizing force H to which the magnetic substance is subjected is laid off horizontally. The induction in the substance when subjected to this magnetizing force is laid off vertically. Thus the point C of the curve indicates that when the magnetic substance is placed in the field of field intensity H_1 the induction in it is B_1.

THE TORQUE ACTION ON A BAR MAGNET IN A UNIFORM FIELD

303. A uniform magnetic field is one in which the lines of force are straight, parallel, and equidistant. The torque action upon a bar magnet when placed in such a field is determined from the following considerations: Let *mm*, Figure 198, represent a bar magnet in a uniform magnetic field. Let it be

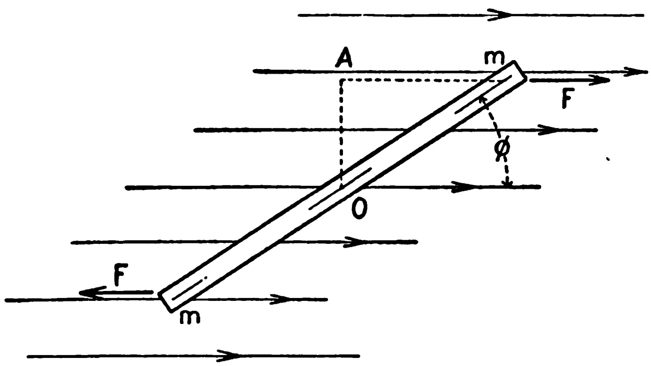

FIG. 198.

assumed that the strength of this uniform field is H and that m represents the strength of the magnetic pole. Let it be further assumed that the distance between the poles of the magnet is L. The north-pointing pole of the magnet will be acted upon by a force $F = mH$ (Equation 92) urging it toward the right, that is, in the positive direction of the lines of force. This will give rise to a torque action about the center of the magnet O equal to F times OA, in which OA is the perpendicular distance between the center of the magnet O and the line of action of the force F. Calling the angle which the magnet

makes with the lines of force ϕ, then $OA = \frac{L}{2} \cdot \sin \phi$. We have, therefore,

$$T_1 = F \cdot OA$$
$$= mH \cdot \frac{L}{2} \sin \phi$$

This is a negative (clockwise) torque.

The torque action due to the force acting upon the south-pointing pole is determined in the same manner to be

$$T_2 = mH \cdot \frac{L}{2} \sin \phi$$

This is also a negative torque. Therefore, the total torque acting upon the magnet is

$$T = mHL \sin \phi$$

The product mL, that is, the pole strength of a magnet by its length, is called the **magnetic moment**, M, of the magnet. Substituting M for mL in the above equation, we have

$$T = M \cdot H \cdot \sin \phi \qquad (95)$$

THE TIME OF VIBRATION OF A BAR MAGNET IN A UNIFORM MAGNETIC FIELD

304. A bar magnet in a uniform field in the position represented in Figure 198 is acted upon by a torque tending to turn the bar into a position parallel to the lines of force. If the bar magnet is assumed to be free to move about the point O, it will turn as indicated; but because of its inertia it will not stop when it reaches a position parallel to the lines of force. Its inertia will carry it beyond this position. It will then be acted upon by a positive torque which will turn it back. Its inertia will again carry it beyond the position of equilibrium, and so on, that is to say, the bar magnet will oscillate to and fro through the position of equilibrium. The rapidity with which these oscillations succeed one another is best determined in the following manner. Referring to Equation (95), it will be seen that for small values of ϕ this expression may be written as follows :

$$T = M \cdot H \cdot \phi$$

since for small angles the sine of the angle is numerically equal to the angle itself when measured in radians.

Since M and H are constants, the magnet satisfies the condition for simple harmonic motion (Sections 49 to 52). But any body so conditioned that it will execute simple harmonic motion of rotation will vibrate in such manner that,

$$T = -4\pi^2 n^2 K\phi \qquad \text{(Equation 17}a\text{)}$$

in which n is the number of vibrations per second, K is the moment of inertia of the vibrating body, and ϕ is the small angular displacement. Comparing this equation with the one given above, it will be evident that

$$4\pi^2 n^2 K = MH$$

Solving for n, the number of vibrations which a bar magnet will execute in one second under the conditions assumed is

$$n = \frac{1}{2\pi}\sqrt{\frac{MH}{K}} \qquad (96)$$

THE MAGNETISM OF THE EARTH

305. The earth is surrounded by a magnetic field which varies both **in direction and intensity** from point to point on the earth's surface. The earth's magnetic field is **such as would exist if a central portion of the earth were magnetized so as to have a south-pointing pole in the northern hemisphere and a north-pointing pole in the southern hemisphere**, both poles being considerably displaced from the axis of the earth and far below the surface. In Figure 199 is shown the magnetized central portion the existence of which would, in a general way, account for the earth's magnetic field. The circle $ABCD$ represents the earth's surface. S, N are the poles of the magnetized central portion. The dotted lines show the field in a plane passing through the magnet poles N, S and the axis of earth.

MAGNETIC DIP

306. A bar magnet suspended in such manner as to be free to turn in all directions tends to set itself parallel to the mag-

netic field in which it is placed. An inspection of Figure 199 will show that at the points *B* and *D* (and all other points on the great circle passing through *B* and *D* and perpendicular to *NS*) the lines of force are **horizontal,** that is, parallel to the surface of the earth. At *A* and *C* the lines of force are **vertical** or perpendicular to the surface of the earth.

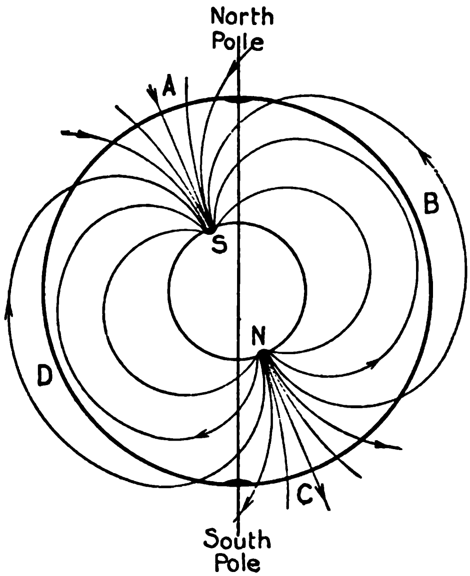

Fig. 199. — The Magnetic Field of the Earth.

The magnetic dip at any point is the angle between the magnetic field at that point and the horizontal. The magnetic dip at *B* and *D*, Figure 199, is zero, at *A* and *C*, 90°, and for intermediate points its value ranges between these limits.

The point *A* is on the west coast of Hudson's Bay at about 70° north latitude. At this point a bar magnet tends to stand on end (parallel to the plumb line).

DECLINATION

307. The magnetic declination at a point is the deviation of the compass needle from the true (geographic) north and south. The magnetic declination for points near *B* and *D*, Figure 199, on the great circle *ABCD* is zero. For other points the declination has a value and direction depending upon the latitude and longitude of the point. In the United States the declination ranges from about 17° west in Maine to 23° east in the state of Washington.

HORIZONTAL INTENSITY

308. Compass needles are mounted on pivots so as to be free to turn in a horizontal plane only. When a needle is so

mounted, it responds to the horizontal component of the earth's field and is unaffected by the vertical component.

In Figure 200 let F represent in magnitude and direction the earth's magnetic field at a given point. Through the upper end of the line F draw the horizontal line H and through the lower end of F the vertical line V. Then H and V are respectively the horizontal and vertical components of the field F.

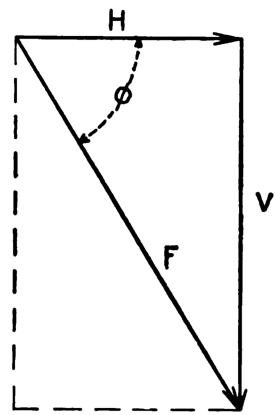

FIG. 200.

The horizontal intensity of the earth's magnetic field is the intensity of its horizontal component. At B and D, Figure 199, the vertical component of the magnetic field of the earth is zero, and the total field intensity is effective in directing the compass. At A and C the **horizontal component is zero**, and the compass needle will stand indifferently in any position. It is found that over a large area in the neighborhood of A the compass needle is very sluggish in its action. That is, in this region the horizontal intensity is so small that the needle is scarcely affected by it.

The statements made above with respect to the magnetic elements (dip, declination, and horizontal intensity) of the points B, D, A, and C, must be regarded as correct only in the general sense. As a matter of fact, the magnetic elements of any point on the earth's surface depend not only upon the geographical location of the point, but oftentimes upon the presence near at hand of deposits of iron ore, etc. Furthermore, the magnetic elements of a point are subject to slight recurrent variations, both daily and annual, and to slow progressive changes extending over long periods of time.

Problems

1. Two equal magnet poles are placed 10 cm. apart. The force action between them is 16 dynes. What is the pole strength of each?

2. Two north-pointing poles of pole strength 50 and 60 are placed 5 cm. apart. What is the force of repulsion between them?

3. The pole strength of the north-pointing pole of a bar magnet is 80 c. g. s. units. What is the field intensity at a distance of 3 cm. from this pole? Neglect effect of south-pointing pole.

4. A bar magnet has a pole strength of 50 c. g. s. units. The distance between its poles is 10 cm. What is the field intensity at a point 5 cm. from the north-pointing pole and 10 cm. from the south-pointing pole of this magnet? At a point for which the distances are 6 and 6 cm. respectively? At a point for which the distances are 5 and 14 cm.?

5. Sketch roughly the lines of force in the field surrounding three equal magnet poles placed at the corners of an equilateral triangle. Two of the poles are north-pointing, the other, south-pointing.

6. What is the field intensity at the center of the triangle of problem 5? Assume pole strength = 20 and side of triangle = 10 cm.

7. A sample of iron when subjected to a magnetizing force of 5 c. g. s. units shows an induction of 12,000 lines per square centimeter. What is the permeability of the iron?

8. A piece of iron has a permeability of 1000. What magnetizing force will give an induction of 3600? Would twice this magnetizing force give twice the induction? Explain.

9. A compass needle makes 200 vibrations per minute when placed in a magnetic field having an intensity of .2 c. g. s. units. What would be its period in a field of which the intensity is .1?

10. The horizontal intensity of the earth's magnetic field at a point is .18 c. g. s. units. The dip is 70°. What is the total intensity of the field at this point?

2nd semester 1914

ELECTROMAGNETISM

CHAPTER XXV

OERSTED'S EXPERIMENT

309. A wire carrying an electric current is surrounded by a magnetic field. This fact was first discovered by Oersted in 1819. The experiment which led to Oersted's discovery is as follows: Let *AB*, Figure 201, represent a wire carrying current in the direction from *A* to *B* as indicated by the arrow, and *NS* represent a compass needle mounted just below the wire. It is supposed that we are looking at the apparatus from above. Let it be assumed further that the wire *AB* lies in the **magnetic** north and south direction, and therefore that the compass needle stands parallel to the wire before the current is turned on. When the current flows, the compass needle will swing into some such position as that represented in the diagram, the north-pointing pole having moved toward the west and the south-pointing pole toward the east.

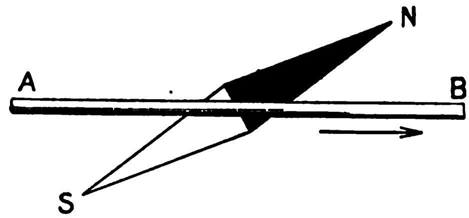

FIG. 201.

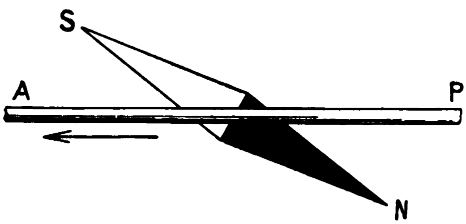

FIG. 202.

When the current flows in the opposite direction, that is, from *B* to *A*, the wire still remaining above the compass needle, the deflection of the needle is like that indicated in Figure 202. That is to say, the north-pointing pole moves toward the east, the south-pointing pole toward the west.

The angle through which the needle is deflected in this experiment depends upon the strength of the current in the wire and the intensity of the magnetic field in which the compass is placed before the current is turned on.

The fact that the compass needle is deflected when the current flows in the wire is evidence that **the current in the wire produces a magnetic field.** Evidently the poles of the compass in this, as in all other cases, tend to move in the direction of the field in which they are placed. Therefore if the compass needle stands northwest and southeast instead of north and south we must conclude that the magnetic field in which it is placed has an east to west component. But the magnetic field of the earth extends north and south. Therefore a new field extending east and west must have been introduced by the passage of the current through the wire, which, combined with the earth's field in the north and south direction, gives the resultant field.

THE MAGNETIC FIELD WHICH SURROUNDS A WIRE CARRYING CURRENT

310. The magnetic field surrounding a wire carrying current is of such nature that the lines of force are concentric circles having their centers at the axis of the wire and their planes perpendicular to the wire. The field is most intense close to the wire and falls off rapidly as the distance from the wire increases. The general character of the field may be determined by the following experiment: Let *AB*, Figure 203, represent a wire carrying current in the vertical direction *AB* as indicated by the arrow. Let it be assumed that a number of small compasses *e*, *f*, *g*, etc., are arranged about the wire supported by the cardboard *CD*. The compass needles will arrange themselves as indicated in the diagram. If iron filings are scattered upon the cardboard *CD* and the cardboard is gently tapped, they will arrange themselves in concentric circles as indicated by the dotted lines.

The relation between the direction of the current in the wire and the direction of the field may be stated as follows: If one imagines himself in such a position that he can **look along the**

wire in the direction in which the current is flowing, then the positive direction of the field is that in which the hands of a clock move.

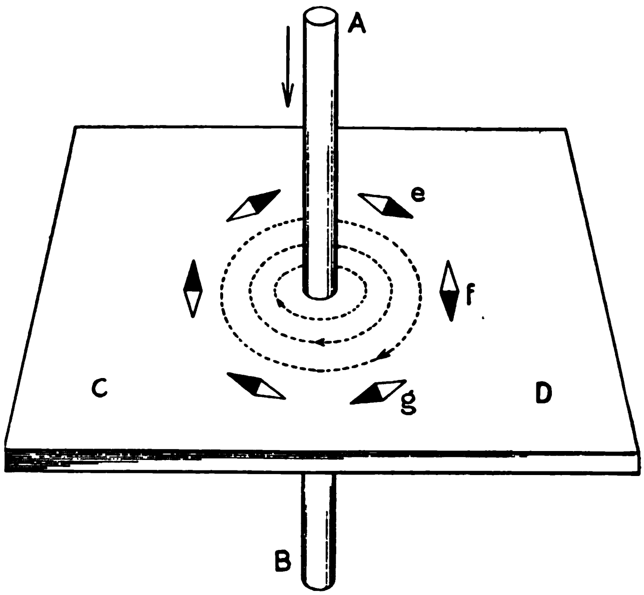

FIG. 203. — The Magnetic Field about a Wire carrying Current.

THE FORCE ACTING UPON A WIRE CARRYING CURRENT AND LYING IN A MAGNETIC FIELD

311. Ampère demonstrated that **a wire carrying a current and lying at right angles to a magnet field is acted upon by a force which tends to move the wire in a direction perpendicular to both the field and the wire.** A simple device for exhibiting this effect is shown in Figure 204. *AB* is a wire suspended

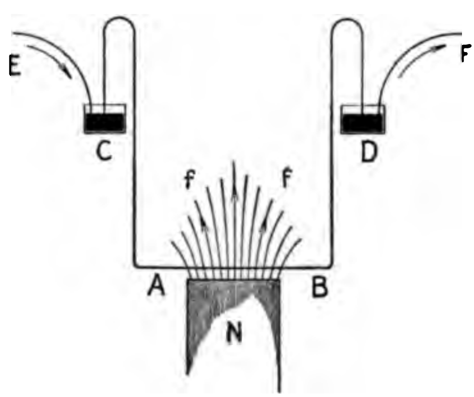

FIG. 204.

from two small cups *CD*, filled with mercury, which serves to make good contact between the wire *AB* and the wires *E* and *F* which convey the current to and from the apparatus. *N* represents the north-pointing pole of a bar magnet. The lines of force spread from *N* in the manner indicated by the dotted lines. With the arrangement shown in the figure the wire *AB* is perpendicular to the lines of force of the field *f* of the magnet. When a current flows in the wire, it is pushed either toward or from the observer according to the direction of the current.

AMPÈRE'S LAW

312. When a wire carrying current lies at right angles to the lines of force in a magnetic field, it experiences a force action which is proportional to the field intensity, to the length of the wire, and to the current which is flowing in the wire. In other words,

$$F \propto ILf$$

in which I is the current in the wire, L is the length of the wire lying in the magnetic field, f is the field intensity which is supposed to be uniform over the entire length of the wire. This may be written in the form of an equation by introducing a proportionality constant, or by what amounts to the same thing, the choosing of a new unit of current. Thus,

$$F = ILf \qquad (97)$$

If this relation is written thus in the form of an equation, it becomes a definition for what is known as c. g. s. electromagnetic unit of current. Evidently **the c. g. s. electromagnetic unit of current is that current which flowing through a wire one centimeter long lying at right angles to a magnetic field of unit strength experiences a force action of one dyne.**

1 c. g. s. electromagnetic unit of current = 10 amperes.

The direction of the force contemplated in Equation (97) depends upon the direction of the field and the direction of current in the wire. It is found, by experiment, that the direction of the force action is always related to the directions of these two quantities in a simple manner. A good rule for

determining the direction of the force action when the direction of the field and the direction of the current are known, is the following **left-hand rule. If the thumb and first and second fingers of the left hand are held in such position as to be at right angles to one another and the forefinger is made to point in the direction of the field while the second finger points in the direction of the current in the wire, then the thumb will indicate the direction of the force.**

THE FORCE ACTION BETWEEN TWO WIRES CARRYING CURRENT

313. Since there is a magnetic field about a wire when it is carrying current, it is evident that there may be a force action between two wires which lie near one another when electric currents are flowing through them. Experiment shows that **parallel wires carrying current in the same direction attract one another and parallel wires carrying current in opposite directions repel one another.** A simple experiment illustrating the attraction between parallel wires carrying currents in the same direction is that represented in Figure 205. *AB* represents a spiral of copper wire supported at its upper end. The lower end of the wire dips into a cup of mercury, *C*. When a current flows through the spiral, the adjacent turns attract one another. The result is that the spiral as a whole contracts, thereby lifting the lower end of the wire from the mercury; but since the circuit is completed through the mercury cup, the lifting of the wire from the mercury breaks the circuit. When the circuit is broken, there is no longer any attraction between the successive turns of wire, and the spiral falls back. The circuit is thus once more completed and the operation is repeated.

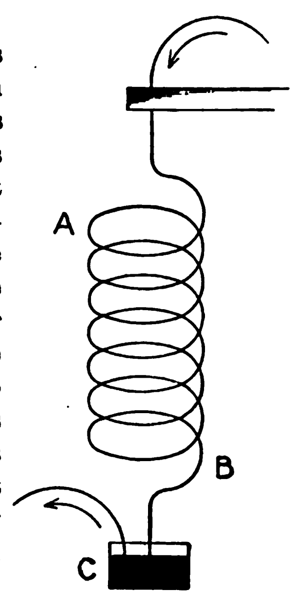

FIG. 205.

THE MAGNETIC FIELD OF A SOLENOID

314. A solenoid is a spiral of wire, the successive turns of which are of the same diameter. Such a coil is obtained by winding the wire spirally upon a cylinder.

The magnetic field of a solenoid is represented in a general way in Figure 206. The lines of force which would extend in concentric circles about the individual coils of the solenoids, if these coils were isolated, unite, forming continuous lines of force from end to end within the solenoid and extending back from end to end without the solenoid in curves similar to those which surround a bar magnet. In fact, the **external field of a solenoid is like that of an ordinary bar magnet.**

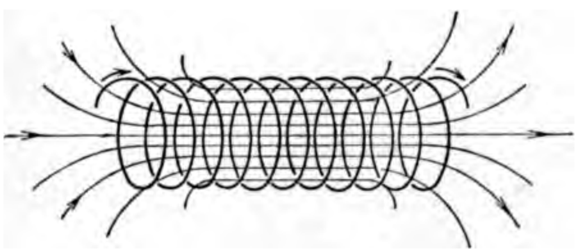

FIG. 206. — Magnetic Field of a Solenoid.

The direction of the lines of force surrounding a solenoid is easily determined by the following rule: **If one imagines himself placed at that end of the solenoid from which the electric current appears to run clockwise about the coils, then he will be looking in the direction of the lines of force through the solenoid.**

Another rule which is often made use of in this connection is known as the **right-handed screw rule.** If one imagines a right-handed screw placed in the axis of a solenoid and turned in the direction in which the current is flowing about the coils of a solenoid, then the screw will advance in the positive direction of the lines of force.

THE ELECTROMAGNET

315. The electromagnet in its simplest form consists of a solenoid having a core of soft iron. The effect of placing a bar of soft iron within a solenoid is to increase the number of lines of force set up by the current in the coils. This will, of course, increase the intensity of magnetic field at each and every point

in the neighborhood of the solenoid, since all of the lines of force which thread through the core of the solenoid must return through the space surrounding it.

The increase in the number of lines of force within the solenoid due to the iron is explained by saying that the permeability of the iron is many times as great as that of air, so that the same magnetizing force is enabled to establish or set up a larger number of lines of force.

The electromagnet in its most efficient form is so designed that the **magnetic circuit**, that is to say, the path through which the lines of force of the magnet extend, is as nearly as possible all iron. Thus, in the electric bell, the electromagnet is given the form shown in Figure 207. One of the essential parts of a dynamo or electric motor is a strong electromagnet. Great care is taken in the design of such machinery to limit what is known as the **air gap** as far as possible, for the reason that the number of lines of force developed for a given current in the coils is greatest when the magnetic circuit is as nearly as possible all of iron.

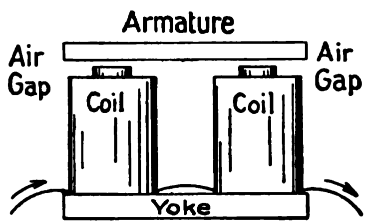

FIG. 207.

THE MAGNETIC FIELD OF A CIRCULAR LOOP OF WIRE

316. Since the lines of force about a wire carrying current are concentric circles whose planes are at right angles to the axis of the wire, it will be evident that the lines of force about a circular loop of wire carrying current will lie in planes which pass through the axis of the coil, and that the current in each part of the circular coil will contribute to the magnetic field at the center. These statements will be more readily understood by reference to Figure 208, in which A and B represent the ends of a circular loop of wire which has been cut by a plane passing through its axis CD. Let it be imagined that the current is flowing out at the top of the coil and in at the bottom. Then the lines of force about the wires in the plane of the

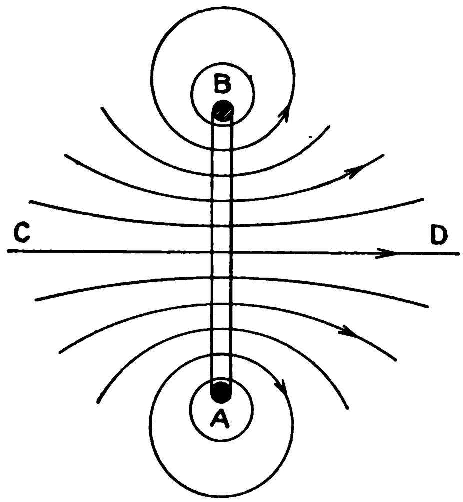

FIG. 208.

paper will be as indicated in the diagram. From the figure it will be evident that the current in both the A and B portions of the wire contributes to the field at the center of the coil O. In the same manner it will be evident that each and every part of the loop adds its portion to the field at the center of the coil. At the very center of the coil O the lines of force are parallel and equidistant, that is to say, the field at the center of the coil is uniform.

FIELD INTENSITY AT THE CENTER OF A CIRCULAR LOOP OF WIRE

317. As defined above (Section 298), the field intensity at any point is that constant which multiplied by the strength of a magnetic pole brought to that point will give the force which acts upon the pole. Let Figure 209 represent a circular

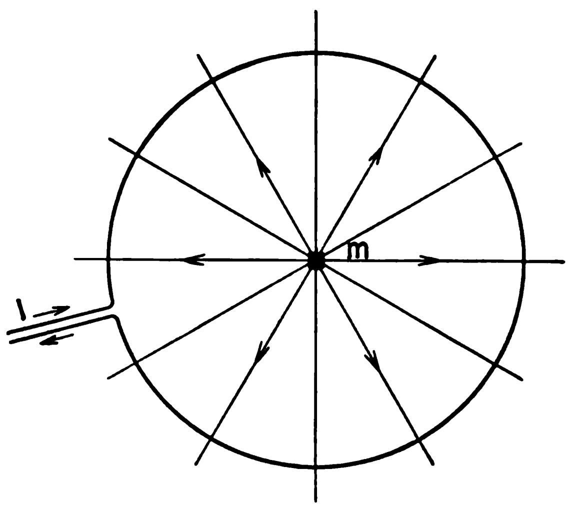

FIG. 209.

loop of wire of radius r, carrying a current I. Let it be imagined that there is placed at the center of this coil an isolated north-pointing magnet pole of strength m. The lines of force radiating from this pole will cut the circular loop of wire at right angles. Furthermore, the magnetic field intensity at the wire due to the magnet pole m is equal to $m \div r^2$ (Equation 93). According to Ampère's Law the circular coil of wire under these circumstances will experience a force action tending to move it at right angles to the radial lines of force, that is, to lift it perpendicular to the plane of the paper in the figure. The magnitude of this force action which the coil experiences is obtained from Ampère's Law. That is,

$$F = ILf$$
$$= I \cdot 2\pi r \cdot \frac{m}{r^2}$$

Since $2\pi r$ is the length of the wire lying in this field and the field intensity as shown above is $\frac{m}{r^2}$, therefore,

$$F = \frac{2\pi I}{r} \cdot m$$

If the coil has two turns of wire in it, evidently the force action will be twice that given by the above equation, since each coil is acted upon by the force F given by the above expression. If there are n turns of wire in the coil, then,

$$F = \frac{2\pi n I}{r} \cdot m \tag{98}$$

This is the expression for the force acting upon the coil which tends to lift it vertically, assuming that the coil of Figure 209 lies in a horizontal position. Since action is equal to reaction, the magnetic pole m must be acted upon by a force of equal value but oppositely directed. That is to say, m is acted upon by a force the magnitude of which is given by the above equation, the direction of which is downward.

Referring again to the definition for field intensity, it will be evident that the quantity $2\pi nI \div r$ is an expression for the field intensity at the center of the coil due to the current in the coil. This must be evident from the fact that it is this group

of constants which, multiplied by the strength of the pole m, gives the force which acts upon it.

Problems

1. A wire 20 cm. long lies at right angles to a magnetic field of 50 c. g. s. units intensity. What is the force acting upon the wire when a current of 30 amperes flows in it?

2. Assume that in the last problem the wire extends in a vertical direction, the current flowing from top to bottom, and the direction of the magnetic field is from north to south. What is the direction of the force?

3. A wire 1 km. in length is stretched horizontally on poles and carries a current of 100 amperes. The vertical component of the earth's field is .3 c. g. s. units. What is the total force urging this wire in a horizontal direction?

4. A circular coil of 1 turn carries a current of 10 amperes. What is its radius if the field intensity at its center is unity?

5. Two circular coils of wire lie in the same plane. One coil consists of 4 turns; the other of smaller radius has but one turn. What must be the ratio of their radii in order that the field intensity at their common center may be zero when they carry the same current in opposite directions?

6. A circular coil of wire has a radius of 20 cm. There are 50 turns of wire in the coil and the current flowing is 6 amperes. What is the field intensity at the center of the coil?

7. Why does a solenoid tend to shorten when a current is passed through it?

8. A circular coil of wire carrying current is suspended in the earth's magnetic field. Explain the torque action on the coil when its plane is vertical and extends north and south. When its plane extends vertically east and west. When its plane is horizontal.

THE HEATING EFFECT OF THE ELECTRIC CURRENT

CHAPTER XXVI

JOULE'S LAW

318. The electric current produces a heating effect in any conductor through which it flows. For example, the filament of an incandescent lamp is so strongly heated by the current which flows through it as to become incandescent.

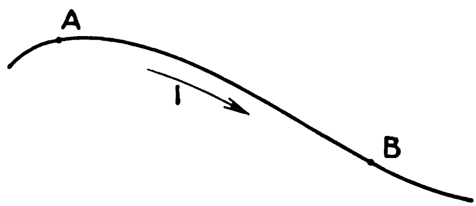

FIG. 210.

An understanding of this heating effect may be obtained from the following considerations. Let AB, Figure 210, represent a portion of a wire of resistance R carrying a current I. Let it be assumed that the potential difference between the points A and B is E. The quantity of electricity which flows down the wire from A to B in t seconds is evidently

$$Q = It \qquad \text{(Equation 82)}$$

When this quantity of electricity passes from A to B, the potential energy of the system is decreased by an amount which we may call W. Then $W = EQ$, since, from the definition of potential difference, it is evident that this amount of work must be done upon the charge Q to carry it back from the point B to the point A and thus restore the original conditions. Since Q is equal to It, therefore,

$$W = EIt \qquad (a)$$

But from Ohm's Law, $E = IR$

Therefore, $W = IR \cdot It$

or, $$W = I^2Rt \qquad (99)$$

This apparent loss of potential energy in the system appears in the conductor AB in the form of heat. Therefore, Equation (99) is an expression for the amount of heat developed by a current I in a conductor of resistance R in the time t. This is known as Joule's Law.

If E is expressed in c.g.s. units of potential difference, that is, in ergs per c.g.s. unit charge, I is given in c.g.s. units of current and t in seconds, so that It expresses the charge in c.g.s. units of charge, then evidently W (Equation a) is given in ergs. **If E is expressed in volts and I in amperes, W is given in *joules*.** This is proven as follows: Assume that all quantities are given in c.g.s. units. In order that E (Equation a), may be reduced from c.g.s. units to volts it must be multiplied by 300 (Section 280). To reduce I from c.g.s. units to amperes it must be divided by 3×10^9 (Section 279). Thus the right-hand member of the equation is in effect divided by 10^7. In order that equality may be preserved the left-hand member must also be divided by 10^7. But W in ergs divided by 10^7 gives W in joules, since 1 joule $= 10^7$ ergs (Section 60). Hence also if, in Equation (99), I is given in amperes and R in ohms, W is given in joules.

THE HEAT DEVELOPED BY A CURRENT

319. It is oftentimes desirable to express W (Equation 99) in calories, since the energy appears in the form of heat. To do this, W as given in Equation (99) must be divided by the number of joules in one calorie. Now,

$$\text{one joule} = 10^7 \text{ ergs} \qquad \text{(Section 60)}$$

$$\text{one calorie} = 4.187 \times 10^7 \text{ ergs} \qquad \text{(Section 218)}$$

Therefore

$$\text{one calorie} = 4.187 \text{ joules}$$

We have, therefore,

$$W(\text{in calories}) = \frac{I^2Rt}{4.187}$$

or

$$W(\text{in calories}) = 0.24\, I^2Rt \text{ (very approximately).}$$

THE POWER EXPENDED IN HEATING AN ELECTRIC CONDUCTOR

320. Power is defined as the rate of doing work (Section 86). That is to say, the power expended by any agency is equal to the total work done by that agency divided by the time in which the work is accomplished. Dividing Equation (99) by t, we have

$$P = \frac{W}{t} = I^2R$$

That is to say, **the power expended in heating a conductor is equal to the product of the resistance of the conductor and the square of the current flowing in that resistance.** From Ohm's Law IR is equal to E where E is the potential difference between the terminals of the resistance R in which the current I is flowing. Therefore the equation above may be written,

$$P = EI \tag{100}$$

Evidently from the above discussion of units, when E is expressed in volts and I is expressed in amperes, P will be expressed in watts (see Section 86).

This discussion leads to the conclusion that **in any circuit which absorbs electric power the total power absorbed is obtained by multiplying together the current in the absorbing circuit and the potential difference between the terminals of that circuit.** In the same way the total electric power delivered to a circuit by an electric generator giving a steady current is obtained by multiplying together the electromotive force of the generator and the current which it is supplying.

EXAMPLES. If an incandescent lamp when subjected to a potential difference of 110 volts has a current of $\frac{1}{2}$ ampere flowing through it, the power absorbed by the lamp is

$$P_1 = 110 \times \tfrac{1}{2} = 55 \text{ watts.}$$

Again, if a dynamo having an electromotive force of 500 volts supplies 50 amperes to a circuit to which it is connected, then the power supplied by the dynamo is,

$$\begin{aligned} P_2 &= 500 \times 50 = 25{,}000 \text{ watts} \\ &= 25 \text{ k.w.} \qquad \text{(See Section 86.)} \end{aligned}$$

ELECTRIC HEATING

320. Heating by means of the electric current is accomplished by causing current to flow through a suitable resistance, the value of the resistance being so chosen that when connected to the given circuit, the proper value of current will flow through it for developing the amount of heat required. Joule's law is used in determining the value of the resistance necessary for such purpose.

ELECTRIC COOKING

322. If it is desired to heat a liquid, a coil of insulated wire protected by a copper tube and bent in the form of a spiral is employed, see Figure 211. This spiral copper tube containing the resistance coils is lowered into the vessel containing the liquid to be heated. When it is desired to heat a vessel from below, an electric stove is employed. This consists of suitable resistance coils arranged immediately beneath an iron plate, which becomes strongly heated when the electric current flows in the resistance coils. An electric oven is made of sheet iron. On the inner walls of the oven are mounted resistance coils through which the electric current is caused to flow, thus heating the oven.

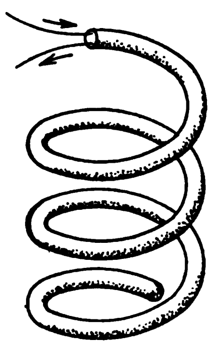

FIG. 211. — Immersion Coil for heating Liquids by Means of the Electric Current.

ELECTRIC LIGHTING

323. When a body is strongly heated, it becomes incandescent and gives out light. The **incandescent lamp** is a device in which a conductor of high resistance is heated to incandescence by the electric current, thus becoming a source of light. In the ordinary form of incandescent lamp the conductor is of **carbon.** Since carbon oxidizes readily at high temperature, it is necessary to inclose a conductor used in this way within a glass bulb from which the air has been exhausted. Under these circumstances, there being no oxygen present, the carbon filament may

be heated to incandescence without danger of oxidation. The **Nernst** lamp glower is of magnesium oxide and similar substances which, being already in an oxidized condition, are stable chemically in air even when the filament is raised to a very high temperature. Recently incandescent lamps are being made of the metals, **tantalum** and **tungsten.** The principal advantage claimed for the incandescent lamp in which tantalum or tungsten has been substituted for carbon is that its efficiency is greater, that is to say, much larger returns in the way of light are secured for a given input of electric power. The efficiency of an incandescent lamp is usually specified in terms of the watts absorbed per candle power of light delivered. Thus in the carbon lamp the efficiency is roughly 4 watts per candle power, for the Nernst and tantalum lamps about 2 watts per candle power, and for the tungsten lamp about one and one fourth watts per candle power.

The **arc lamp** is another device made use of in lighting by electricity. In this device the incandescent body is the tip of a **carbon rod** which has been brought into momentary contact with a second carbon rod and then slightly separated therefrom. **The potential difference employed between the carbon rods tends to maintain the flow of current between the rods even after they are separated.** The resistance at this point of the circuit being very high, an intense heating effect is produced. In the presence of this heat the carbon is vaporized and forms a sort of bridge of vapor between the ends of the carbon. This vapor bridge is sufficient to maintain the current, and therefore the heating action of the current, which keeps the tip of the carbon white hot. In place of carbon a rod of **magnetite** is sometimes employed for the negative terminal of the arc. The magnetite arc has a higher efficiency than the carbon arc for the reason that the magnetite, when strongly heated in the vapor state, is brilliantly luminous, while the carbon vapor under the same circumstances gives but little light. Another form of high efficiency arc lamp known as the **flaming arc** employs carbon rods impregnated with the salt of some metal of such nature that the vapor produced is strongly luminous. Calcium fluoride or calcium borate is usually employed for this purpose.

The efficiency of the arc lamp varies from about 2 watts per candle power in certain forms of carbon arc to about $\frac{1}{4}$ of one watt per candle in some forms of the flaming arc.

THE ELECTRIC FURNACE

324. The highest temperature known in the laboratory is that produced in the "crater" of the electric arc, carbon rods being used as electrodes. Certain chemical changes take place at this high temperature which do not take place at lower temperatures. Thus, it is possible to effect certain chemical combinations in the electric furnace which cannot be brought about by any other means. It is by means of the electric furnace that such compounds as calcium carbide are made. Calcium carbide is a compound of calcium and carbon. It is impossible to secure a combination of these two elements except at the very high temperature of the electric furnace.

ELECTRIC WELDING

325. If two pieces of iron are brought end to end and a strong current is sent through them, they will become strongly heated at the point at which they are in contact, because at this point there is large resistance to the flow of the current. If the current is sufficiently large, this heating effect is very pronounced, and after the lapse of a short time the ends of the iron rods may be raised to a welding temperature. When sufficiently heated in this manner, they may be compressed or pounded together, and in this manner welded.

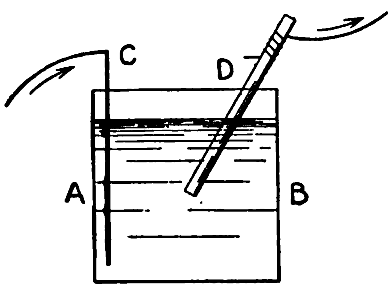

FIG. 212. — The Electric Forge.

THE ELECTRIC FORGE

326. The electric forge affords another way of heating a piece of metal to a high temperature by means of the electric current. This apparatus is illustrated in Figure 212. *AB* is a vessel containing a solution of sodium carbonate. *C* is a plate of lead and *D* the bar of iron to be heated. When a

current is sent through the apparatus from the lead to the iron, a layer of gas forms on the surface of the iron, which, because of its high resistance, gives rise to a strong heating effect. With such a forge a piece of iron may be brought to a welding heat in a few seconds.

FUSES

327. The heating effect of the electric current is taken advantage of in the use of fuses for protecting circuits against excessive currents. The fuse in its simplest form is a piece of wire formed of an alloy of lead and tin. This alloy is chosen because of its low fusing point. The size of the fuse is so chosen that it will carry the maximum allowable current without becoming excessively heated. If a larger current flows, the fuse is "burned" and the circuit opened before the excessive current injures the other parts of the circuit. The manner in which fuses are placed in a circuit is illustrated in Figure 213. *M* represents an electric motor. *FF* are the fuses. If, from any cause, an excessive current begins to flow from the line to the motor *M*, the fuses will burn out and damage to the motor will be prevented.

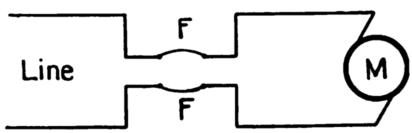

Fig. 213.

Problems

1. A current of 10 amperes flows through a resistance of 10 ohms. How much heat (in joules) is developed per minute?

2. A certain electric oven has a resistance of 9.6 ohms. At what rate is heat developed in the oven when a current of 12.5 amperes is flowing, — (*a*) in watts? (*b*) in cal./sec.?

3. What power is absorbed by an arc lamp which is supplied with 9 amperes at 50 volts?

4. A wire having a resistance of 10 ohms is connected to a dynamo having an e.m.f. of 50 volts. What power is absorbed by the wire? What would be the effect as to the power absorbed if the length of the wire doubled? Halved?

5. A certain 16 candle power incandescent lamp having a carbon filament takes 0.56 ampere at 100 volts. A tantalum lamp of 22 candle power takes 0.44 ampere at the same voltage. How do the efficiencies of the lamps compare?

6. An electric motor requires 15 amperes at 110 volts. The two wires leading from the dynamo to the motor have a resistance each of 0.3 ohm. What e. m. f. must the dynamo supply? How much power is lost in the line?

7. A current flows through 3 wires of copper, platinum, and silver of the same length and diameter, connected in series. What are the relative amounts of heat developed in the 3 wires?

8. If the 3 wires of problem 7 are connected in parallel, what are the relative amounts of heat developed in them?

THE CHEMICAL EFFECT OF THE ELECTRIC CURRENT

CHAPTER XXVII

ELECTROLYSIS

328. When an electric current is caused to flow through a liquid having a complex molecule, it tends to break up the complex molecular structure, reducing it to some simpler form. If, for example, an electric current is passed through acidulated water, the water molecule is broken up into oxygen and hydrogen. This effect of the electric current in bringing about chemical change is called **electrolysis.** The liquid acted upon is called the **electrolyte.**

IONS

329. Modern theory assumes that in a solution many of the molecules are normally separated into positively and negatively charged parts. Under this theory the action of the electric current when passing through such a solution is largely to assemble these positively and negatively charged molecular parts at the points at which the current enters and leaves the electrolyte, although it probably serves at the same time to break up other molecules which before the passage of the current were more or less stable.

Under the electron theory, the negatively charged part of the molecule has an excess of electrons, the positive part a deficit. These charged parts of the molecule tend to move in the electrolyte in response to the electric forces which are present. The positively charged parts tend to move with the (positive) current (Section 278), the negatively charged parts tend to move against the (positive) current. These charged parts of molecules which are supposed to wander about in this manner in the electrolyte are called **ions.**

It is not believed that the ions move in straight and unbroken paths through the electrolyte; but that they are continually forming combinations and again breaking away from such combinations to wander for a brief interval, perhaps entirely free, only to unite a moment later with some other free ion of opposite sign, to form a complete molecule, and so on. During all of these changes they are steadily progressing in response to the electric forces which are urging them forward.

ELECTROLYTIC TRANSFORMATIONS

330. A general idea of the transformations due to electrolysis may be obtained from a discussion of the following simple cases. Let *AB*, Figure 214, represent a glass vessel containing an electrolyte, into which dip terminals (**electrodes**) of an electric circuit. Such an arrangement is called an **electrolytic cell.** The electrode by which the current enters the cell is called the **anode**, that by which it leaves the cell the **cathode.**

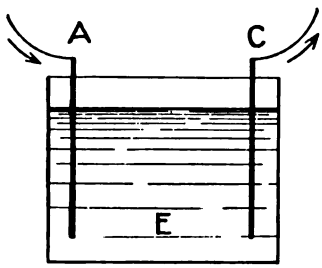

FIG. 214. — Electrolytic Cell.

Let it be assumed that the electrolyte in the cell shown in the figure is copper sulphate ($CuSO_4$) and the electrodes metallic copper. When electrolysis takes place, the copper sulphate breaks up into Cu ions (+) and SO_4 ions (−). The Cu ions move toward the cathode and unite with it, thereby increasing its weight. The SO_4 ions coming into the presence of the anode combine with it, forming $CuSO_4$, thereby decreasing the weight of the anode. Evidently the average concentration of the electrolyte remains the same.

If the electrolyte, instead of being copper sulphate, is dilute sulphuric acid and the electrodes are of platinum, the products of the electrolytic action of the current are gaseous, and layers of gas gradually accumulate on both electrodes of the electrolytic cell, finally rising in bubbles to the surface of the liquid. Under these circumstances the hydrogen of the water molecule, or the H_2SO_4 molecule, acts like the copper in the $CuSO_4$ solu-

tion in the cell described above. That is to say, the hydrogen accumulates at the cathode. Probably in this case it is the H_2SO_4 molecule which is broken up; the SO_4 ion coming into the neighborhood of the anode combines with a molecule of water, forming H_2SO_4 and free oxygen.

FARADAY'S LAWS

331. The mass of any ion deposited from an electrolyte by an electric current is proportional to the quantity of electricity passed through the electrolyte. That is, $M \propto Q$, or, since $Q = It$, we may write, $M \propto It$. Hence,

$$M = \epsilon \cdot I \cdot t \tag{101}$$

in which ϵ, the proportionality constant, is called the **electrochemical equivalent** of the substance deposited.

If $I = 1$ and $t = 1$ in Equation (101), $\epsilon = M$. Hence, **the electrochemical equivalent of any substance is the mass of that substance which is deposited by unit current in unit time.** The law expressed by Equation (101) is known as Faraday's first law of electrolysis.

If the same quantity of electricity is passed through a number of electrolytic cells, each containing a different electrolyte, the mass of each substance deposited is proportional to its chemical equivalent. Thus the same quantity of electricity will deposit 1 gram of H, 35.46 grams of Cl, 107.9 of Ag, etc. This is called Faraday's second law of electrolysis.

From Faraday's second law it appears that the electrochemical equivalents of different substances are widely different. Below is a table which gives the electrochemical equivalents of a few substances:

ELECTROCHEMICAL EQUIVALENTS

SUBSTANCE	ELECTROCHEMICAL EQUIVALENT
Silver	0.001118
Copper	0.0003271
Nickel	0.000304
Hydrogen	0.000010
Oxygen	0.000082
Water	0.000093

The electrochemical equivalents given in this table are in grams per ampere-second. It is convenient, when the products of electrolysis are gaseous, to have the electrochemical equivalent expressed in terms of the volume of gas liberated in the cell, that is to say, cubic centimeters per ampere-second. Since the volume of a gas depends upon its temperature and the pressure to which it is subjected, the electrochemical equivalent must be given in terms of the standard temperature, 0° C., and the standard pressure, 760 millimeters of mercury. The electrochemical equivalents of oxygen, hydrogen, and water specified in this manner are given in the table below:

Substance	Electrochemical Equivalent
Oxygen	0.0578
Hydrogen	0.1156
Water	0.1734

THE COULOMB METER

332. The electrolytic cell represented in Figure 214 may be used for the measurement of an electric current. The operation is as follows: Using, say, copper electrodes in an electrolyte of $CuSO_4$, the cathode is carefully weighed before placing it in the electrolyte and again after the unknown current has been passing through the cell for an observed time t. The difference between these two weights is the gain in weight of the cathode, *i.e.* the amount of copper deposited by the current in the time t. This is the mass contemplated in Equation (101). Since the electrochemical equivalent of copper is known, we have M, ϵ, and t of Equation (101), which may therefore be solved for the value of I. Evidently the mass of copper deposited is a measure of the quantity of electricity which has passed through the cell rather than the current. This is evident from the fact that ten amperes in one second will deposit as much metal as one ampere in ten seconds. Hence the apparatus, properly speaking, is a **quantity meter**, or coulomb meter, the coulomb being the practical unit of quantity (1 coulomb = 1 ampere-second).

APPLICATIONS OF ELECTROLYSIS

333. Electrolysis is used extensively in practical operations. Among the more important applications are the following:

Electrometallurgy. — Very pure copper is obtained by electrolytic refining. For this purpose an electrolyte of $CuSO_4$ is used. The impure copper is connected as anode, a thin plate of pure copper serving as cathode. When the current flows, pure copper is deposited upon the cathode, the "sludge" (impurities) falling to the bottom of the cell. Metallic aluminum is reduced from the oxide of aluminum by subjecting the oxide in a fused condition to electrolysis.

Electroplating. — Objects made of the baser metals may be gold or silver plated by causing them to serve as cathodes in an electrolytic cell containing a suitable electrolyte. The electrolytes used are the double cyanides of potassium and gold or potassium and silver as the case may be. Surfaces of brass or steel are often nickel plated to prevent tarnishing or rusting. For nickel plating a double sulphate of nickel and ammonium is used as the electrolyte. In order that the density of the electrolyte may remain constant, the anode used must in each case be a plate of the same metal as that deposited upon the cathode.

Electrotyping. — Most books which are printed in large editions are printed from copperplate copies of the pages of type as set up in the ordinary way. A wax impression or mould is made of the type, and this mould is copper plated by electrolysis. This electrolytic copy of the original type is used in the press.

The electrolytic process is also used in the manufacture of various chemicals, such as caustic soda and potassium chlorate.

Problems

1. How many coulombs would be required to deposit 10 g. of silver?

2. How long would it take a current of 10 amperes to deposit 1 lb. of copper?

3. How many grams of copper will be deposited by 5 amperes in 1 hr.?

19000a
.501)10000

4. The same current is made to pass through a silver voltmeter and a copper voltmeter in series. What are the relative amounts of silver and copper deposited?

5. In an experiment with the coulombmeter, the weight of the cathode before it was placed in the electrolyte of $CuSO_4$ was 10 g. After the unknown current had passed through the cell for 20 min. the cathode was found to weigh 11 g. What was the average value of the current?

6. How much water would be decomposed by a current of 1 ampere in 1 hr.?

THE VOLTAIC CELL

CHAPTER XXVIII

GALVANI'S EXPERIMENT

334. In 1786 it was discovered by an Italian by the name of **Galvani** that some freshly prepared frog legs, which were suspended from a copper hook attached to an iron railing, twitched when they came into contact with the iron. This effect being like that produced by an electric discharge, Galvani recognized it as an electric effect and attempted to explain it by assuming that an electric discharge was generated in the muscle of the leg. This explanation was rejected by **Volta**, a professor in the University of Padua, who maintained that the true source of the effect was the contact of dissimilar metals. Volta proceeded to verify his theory by devising experiments in which dissimilar metals were brought into contact directly and in other cases through the medium of a suitable liquid. The most marked effects, that is, the greatest potential differences, were secured by an arrangement like that represented in Figure 215, which is commonly known as a Voltaic cell. It consists essentially of a containing vessel *A*, an electrolyte *B*, and two dissimilar metals *C* and *D* dipping into the electrolyte, in other words, **Volta's cell is an electrolytic cell having dissimilar electrodes.**

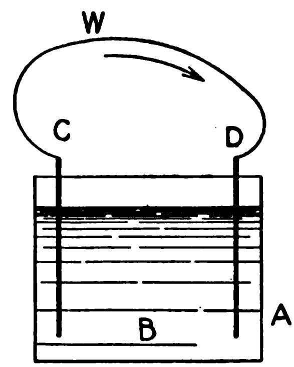

FIG. 215. — The Voltaic Cell.

A simple and effective cell is made by using strips of copper and zinc as electrodes and a dilute solution of sulphuric acid as an electrolyte.

Upon examining such a cell it will be found that **there is a difference of potential between the copper and the zinc so long as they are allowed to stand in the electrolyte**, and furthermore, if the upper end of the copper and zinc electrodes are joined by a wire *w* as indicated in the figure, an electric current will flow in the direction indicated by the arrow. This current will continue to flow so long as the conditions represented in the diagram are maintained. **The source of the energy represented by this current is the chemical transformation which is going forward within the electrolyte.** It is found that the zinc gradually disappears as metallic zinc, being taken up by the acid and converted into zinc sulphate ($ZnSO_4$).

As intimated above any dissimilar metals may be employed in place of the copper and zinc but **the potential difference between the terminals depends upon the nature of the materials employed.** In the same way it is found that there are many liquids which will take the place of dilute sulphuric acid as an electrolyte, but **the potential difference between the electrodes depends upon the nature of the electrolyte.**

It is also found upon experiment that with two given metals the one may be positive and the other negative when dipped in one electrolyte, while the reverse relation will exist if they are placed in a second electrolyte. For example, if the metals, lead and copper, are placed in dilute HCl, the copper is at a higher potential than the lead. If a solution of potassium sulphate is used as an electrolyte, the lead is positive with respect to the copper.

Evidently there is a large choice of materials which might be employed in the cell both with respect to the electrode and the electrolyte. The advantages of several of the more important combinations will be taken up in detail in the discussion given below.

THE CHEMICAL ACTION IN THE SIMPLE VOLTAIC CELL

335. When a simple voltaic cell like that represented in Figure 215 is supplying current, that is to say, when its electrodes are connected by a wire, electrolysis takes place in the cell. Thus H_2SO_4 when used as an electrolyte is broken up into hydrogen

on the one hand and SO_4 on the other. It is found that the hydrogen set free in the operation of the cell tends to accumulate upon the copper, that is upon that electrode which is at the the higher potential (*i.e.* the cathode), and which is commonly referred to as the positive terminal of the cell. SO_4 is found to accumulate in the neighborhood of the negative terminal of the cell, and being very active chemically it immediately combines with the metallic zinc of this terminal, forming zinc sulphate. The hydrogen does not enter into combination with the copper terminal in the neighborhood of which it appears, but collects in the form of hydrogen gas on this terminal.

POLARIZATION

336. Evidently, as the chemical action described in the last section goes forward, the copper terminal or positive terminal of the Voltaic cell becomes coated with a layer of minute hydrogen bubbles. The effect of this layer of bubbles is to prevent contact between the electrolyte and the electrode, and may result in a very decided diminution of the potential difference between the electrodes of the cell. This effect is known as polarization.

Various means are employed for preventing polarization. One method used is to give the positive electrode a large size and a rough surface. This method only serves to postpone the effeet. It requires, of course, a longer time for the bubbles to cover a large surface than would be required for a smaller one. The best method is to take up the hydrogen as it is formed, preventing in this way its accumulation as a layer of minute bubbles against the electrode. If, for example, the electrode is surrounded by an oxidizing agent with which the hydrogen combines readily, the formation of free hydrogen in the neighborhood of the electrode will be prevented. Chemical agents employed for this purpose are known as **depolarizers.**

LOCAL ACTION

337. Another difficulty which is encountered in the operation of the Voltaic cell in a practical way is that which is

known as local action. Local action arises from the fact that commercial zinc which is so extensively employed in Voltaic cells contains a certain amount of impurities in the way of iron, carbon, and the like. **These particles of dissimilar substances embedded in the surface of the zinc constitute with the zinc, when dipped into an electrolyte, small Voltaic cells.** This will be understood by reference to Figure 216, in which *AB* represents a piece of zinc, *C* a bit of carbon embedded in the surface of the zinc. When the zinc is dipped into dilute sulphuric acid a difference of potential exists between *C* and *AB*. This will give rise to small electric currents circulating through the zinc and the carbon and the electrolyte in the immediate neighborhood as indicated by the small arrows. **The effect of these local currents is to cause the zinc to be gradually converted into zinc sulphate even when the battery proper is not in operation.** The effects of local action may be largely done away with by what is known as amalgamation. The zinc rod is amalgamated by dipping it into dilute sulphuric acid and then into mercury. The mercury forms an amalgam with the zinc. This amalgam spreads over the surface of the rod, covering up all of the particles of foreign matter which may be embedded in its surface and thus preventing contact between them and the electrolyte.

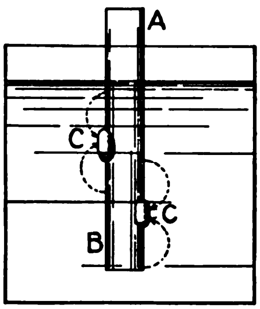

FIG. 216. — Illustrating Local Action.

THE GRAVITY BATTERY

338. One of the more common forms of modern Voltaic cell is the **gravity battery.** This has been evolved from the original form of what was known as the **Daniell cell.** The Daniell cell in its original form consisted of copper and zinc electrodes dipping into zinc and copper sulphate solutions, the copper electrode dipping into the copper sulphate, the zinc electrode being surrounded by the zinc sulphate, and the two solutions being kept separate by placing the zinc sulphate in a porous cup, that is, an unglazed earthenware cup. This cup, while it prevented

the mixing of the solutions, did not separate them electrically. The copper sulphate in this cell acts as the depolarizer, — the copper of the copper sulphate, being displaced by the free hydrogen, is thrown down against the copper terminal. The SO_4 ion combines with metallic zinc at the zinc electrode, thus increasing the amount of zinc sulphate present in the cell.

In the gravity battery the copper electrode and solution of copper sulphate are placed in the bottom of a vessel, the zinc electrode and zinc sulphate being placed in the top of the vessel. The two solutions are kept separate by gravity, the copper sulphate being more dense than the zinc sulphate. This battery is also called the **crowfoot** battery because of the form which is commonly given to the electrodes.

THE GROVE CELL

339. In the Grove cell the electrodes are of platinum and zinc. The platinum in the form of a thin strip is placed in a small porous cup and surrounded by strong nitric acid. This porous cup is then placed in a solution of sulphuric acid into which the zinc electrode is also placed. Evidently in this cell the strong nitric acid is the depolarizer. The free hydrogen coming into the presence of the nitric acid combines with it in such manner as to form nitric oxide (N_2O_2) and water (H_2O). The nitric oxide escaping from the cell takes up additional molecules of oxygen when it comes into contact with the air, forming nitrogen tetroxide. One of the serious drawbacks to the use of this cell is the formation of these noxious fumes. Another objectionable feature from a commercial standpoint is the high cost of the platinum electrode.

THE BUNSEN CELL

340. To overcome the objectionable features of the Grove cell, Bunsen substituted carbon for the platinum and potassium bichromate ($K_2Cr_2O_7$) for the nitric acid. It is found that when potassium chromate combines with a small amount of sulphuric acid a very strong oxidizing agent, chromium trioxide (CrO_3), is formed.

Bunsen also discovered that when potassium bichromate is

used as a depolarizing agent, the porous cup may be dispensed with, the two electrodes dipping into the same solution. A cell made up in this manner is usually called a **bichromate cell.**

THE LECLANCHÉ CELL

341. In this cell the positive electrode is of carbon and is surrounded by or mixed with manganese dioxide (MnO_2), which acts as a depolarizer. The electrolyte is ammonium chloride (NH_4Cl). The negative electrode of the cell is zinc.

THE DRY BATTERY

342. The batteries described in the foregoing sections all possess the disadvantage of containing liquids which evaporate when used for any appreciable length of time and which are apt to spill as the cells are carried about. The dry battery is largely free from these defects. As usually constructed it consists of a zinc vessel which serves as the negative electrode and at the same time as the containing vessel for the electrolyte. Against the walls of this vessel are placed several layers of damp blotting paper. Within this is a moist mixture of manganese dioxide, graphite, plaster of paris, and calcium chloride, in the midst of which is placed the carbon or positive electrode. The manganese dioxide serves as a depolarizer. The calcium chloride is used because of its property of taking up moisture from the air. Evidently the contents of the cell must be kept moist in order that it may continue in operation.

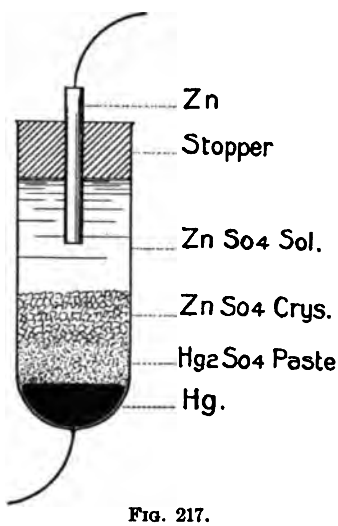

FIG. 217.

THE STANDARD CLARK CELL

343. The Clark cell is used as a standard of potential difference and is usually made up in small sizes, oftentimes conveniently in a test tube. The positive electrode is mercury. The

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depolarizer is mercurous sulphate (Hg_2SO_4). The electrolyte is a saturated solution of zinc sulphate, and the negative electrode is pure metallic zinc. One of the more simple forms of the Standard Clark cell is shown in Figure 217, in which the arrangement of the various parts is shown. The e.m.f. of the Standard Clark cell is very constant, changing only slightly with the temperature. At 15° C. its e.m.f. is 1.434 volts.

E.M.F.'S OF COMMON BATTERIES

344. The electromotive forces of the cells described above are given in the following table: Ex

NAME OF CELL	E.M.F.
Gravity Battery	1.08 volts
Grove Battery	1.9 volts
Bichromate Cell	2.1 volts
Leclanché Cell	1.5 volts
Dry Battery	1.35 volts
Clark Cell	1.434 volts

THE REVERSIBILITY OF THE VOLTAIC CELL

345. In certain forms of Voltaic cell it is possible to completely reverse the chemical transformations which take place in the cell when it is supplying current by forcing a current through it in the reversed direction. Let it be imagined, for example, that three gravity batteries are connected in series as shown in Figure 218. *A* and *B* are joined in the same sense, that is to say, in such manner that the potential difference between the zinc and the copper in the cell *A* is

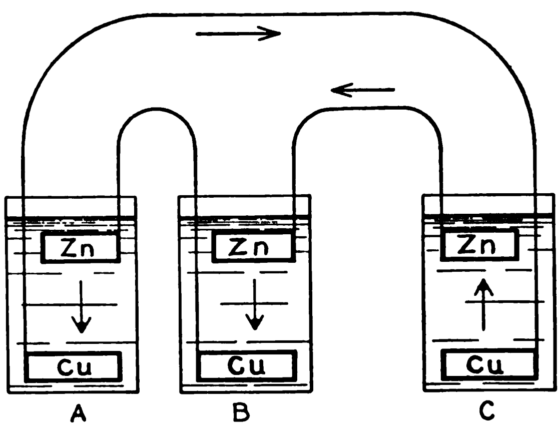

FIG. 218.

added to the potential difference between the zinc and the copper in the cell *B*. The cell *C* is connected in the opposite sense. Cells *A* and *B* work together to send a current around the circuit in the direction indicated by the arrows. The cell *C* is so connected that it tends to oppose this flow of current, that is, it tends to cause current to flow in the opposite direction. Since *A* and *B* are working together, they will overcome the effect of the cell *C* and will force current through the cell *C* in the reverse direction. Under these circumstances the chemical transformations which take place in the cell *C* are the reverse of those which take place in the cells *A* and *B*. In *A* and *B* the hydrogen of the H_2SO_4 molecule appearing in the neighborhood of the copper electrode displaces the copper of the copper sulphate, thus freeing the metallic copper, which is deposited on the copper plate. The SO_4 ion going into the presence of the zinc electrode combines with metallic zinc, forming zinc sulphate. In the cell *C* the hydrogen of the H_2SO_4 molecule, which is broken up by the current, goes into the presence of the zinc plate, displaces the zinc in the zinc sulphate, thus freeing the zinc, which is deposited on the zinc electrode. The SO_4 ion coming into the presence of the copper plate combines with the metallic copper, forming copper sulphate. That is to say, in the cells *A* and *B* the weight of the zinc decreases and zinc sulphate is formed. In the cell *C* the weight of the zinc increases and zinc sulphate grows less in amount. In the cells *A* and *B* the copper plate increases in weight and the copper sulphate grows less in amount, while in the cell *C* the copper plate decreases in weight and more copper sulphate is formed.

The thought here suggests itself that it might be possible to restore a gravity battery which is pretty well worn out by connecting it to other batteries in the manner in which *C* is connected in Figure 218, and sending through it a current in the reverse direction. This is possible. Evidently when the cell has again exhausted itself, that is to say, when it becomes discharged, it may again be charged in the manner indicated. **A cell used in this manner is called a storage battery.**

THE STORAGE BATTERY

346. Evidently the essential features of a storage battery are that the chemical reactions which take place when the cell is used as a source of energy shall be completely reversible. In addition to this the nature of the electrodes and electrolyte must be such that there is no local action of any sort when the cell is standing idle.

THE LEAD STORAGE CELL

347. The simplest form of storage battery consists of lead electrodes dipping into dilute sulphuric acid. When a current is sent through a cell of this character, the anode becomes strongly oxidized, its surface being coated with a dark brown layer of peroxide of lead. The other plate is gradually formed into spongy lead. If such a cell, after being charged in this manner, is connected to a receiving circuit, it will be found capable of furnishing electric current. The potential difference between its electrodes when charged is about 2.1 volts. As the cell discharges the oxidized condition of the positive plate gradually disappears, the oxide being reduced to a lower oxide and this finally to metallic lead. When the oxide entirely disappears, the action of the cell may be restored by again charging it. In the practical form of the storage battery it is customary to make the plates in the form of grids upon which is placed a paste of lead oxide. This construction simplifies the process of charging the cell and makes the cell more efficient in that larger surface is exposed to the action of the current than would be the case if the plate were made up in the solid form.

The chemical reactions in the cell are as follows: Assume the cell to be charged, then one plate is spongy lead Pb, and the other lead peroxide PbO_2. During discharge the H_2 of the electrolyte (H_2SO_4) goes to the cathode, the SO_4 going to the anode; we have, therefore,

$$\left\{\begin{array}{l|l}\text{Plate } P & PbO_2 + H_2 + H_2SO_4 = 2\,H_2O + PbSO_4 \\ & \qquad\;\uparrow \\ \text{Plate } N & Pb \quad + SO_4 \qquad\qquad = PbSO_4\end{array}\right.$$

The arrow shows the direction of the current **in the cell.** The products of the reaction are lead sulphate (on the plates) and water. Evidently **during discharge the concentration of the electrolyte decreases.**

For the reactions during charge we have,

$$\left\{\begin{array}{l|l} \text{Plate } P & \text{Pb SO}_4 + \underset{\downarrow}{\text{SO}_4} + 2\,\text{H}_2\text{O} = \text{PbO}_2 + 2\,\text{H}_2\text{SO}_4 \\ \text{Plate } N & \text{Pb SO}_4 + \text{H}_2 \qquad\qquad\qquad = \text{Pb} \quad + \quad \text{H}_2\text{SO}_4 \end{array}\right.$$

During the charging operation the plates P and N return to their original chemical form, and the **concentration of the electrolyte increases.**

ELECTRICAL MEASURING INSTRUMENTS

CHAPTER XXIX

GALVANOMETERS

348. A galvanometer is an instrument for measuring electric current. There are several kinds of galvanometers, the more important of which are described in the following paragraphs.

THE TANGENT GALVANOMETER

349. The tangent galvanometer consists essentially of a circular loop of wire of one or more turns standing in a vertical position and having suspended at its center a small bar magnet or compass needle. In the use of the instrument the coil is carefully adjusted to stand in a **magnetic north and south** direction. Evidently the compass needle at its center will, under these circumstances, be parallel to the plane of the coil. When current is sent through the coil, an east and west magnetic field is set up at the center of the coil, as pointed out in the discussion of the magnetic field at the center of a circular loop of wire (Section 316). This east and west field due to the current in the coil combines with the earth's magnetic field, forming a resultant field in the direction of which the compass needle tends to set itself. Let H, Figure 219, represent the horizontal component of the earth's field. Let f represent the field due to the current in the coil. The resultant, R, of these two fields is indicated in magnitude and direction by the diagonal of the

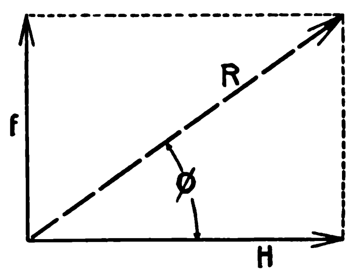

Fig. 219.

rectangle in the figure. Let θ be the angle between the resultant field R and H. Then evidently,

$$\tan\theta = \frac{f}{H}$$

but f, the field due to the current in the coil, is $2\pi nI \div r$ (Section 317). Substituting this value of f in the expression for tan θ, we have,

$$\tan\theta = \frac{2\pi nI}{rH}$$

Solving this equation for I,

$$I = \frac{rH}{2\pi n}\cdot\tan\theta$$

r and n are constants depending only on the dimensions of the instrument; and H, the horizontal component of the earth's magnetic field, may also be taken as a constant for any one point. It will be seen, therefore, that the coefficient of tangent θ in the above equation is a constant. We may therefore write,

$$I = K\cdot\tan\theta \tag{102}$$

in which K has been written for $rH \div 2\pi n$. That is to say, the tangent of the angle θ through which the compass needle is deflected is proportional to, and therefore a measure of, the current which flows in the coil of the instrument.

K is known as the constant of the instrument. It is best determined by sending through the instrument a known current and observing the corresponding deflection. Knowing I and tan θ, the constant K may be calculated from the equation.

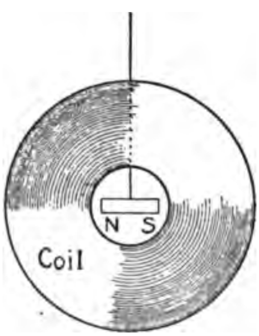

FIG. 220.

THE THOMSON GALVANOMETER

350. The equation of the tangent galvanometer indicates that a given current will produce the largest effect in the instrument when the number of turns of wire is large and the radius of the coil is small. It will be evident, therefore, that a "sensitive" galvanometer, that is, one which will respond to very small currents, should be made up in this way.

An instrument having a small coil of many turns of fine wire, with a small magnet suspended at its center, is called a Thomson galvanometer. With an instrument of this type currents of less than one ten-billionth of an ampere may be measured. Figure 220 represents a Thomson galvanometer in its simplest form. The magnet is supported by a slender fiber of silk or quartz, and its deflections are observed by means of a small attached mirror. **The law of the tangent galvanometer cannot be applied to the Thomson instrument for the reason that the field in which the needle turns is not uniform.**

THE D'ARSONVAL GALVANOMETER

351. The D'Arsonval galvanometer is an instrument in which the arrangement of parts is the reverse of that found in the Thomson galvanometer, that is, in this instrument it is the coil carrying the current which moves, the magnet producing the field in which the coil lies remaining stationary. A common form of the D'Arsonval galvanometer is represented in Figure 221.

N and *S* are the poles of a strong horseshoe magnet mounted in an upright position. Between these poles is suspended a vertical coil of fine wire, one end of the wire serving as a support for the coil and at the same time as one of its terminals, the other end of the wire extending downward and serving as the other terminal of the suspended coil as represented in the diagram. Let it be imagined that the current passes clockwise around the coil as indicated. Applying the left-hand rule, it will be evident that the left-hand side of the suspended coil will tend to move toward the observer

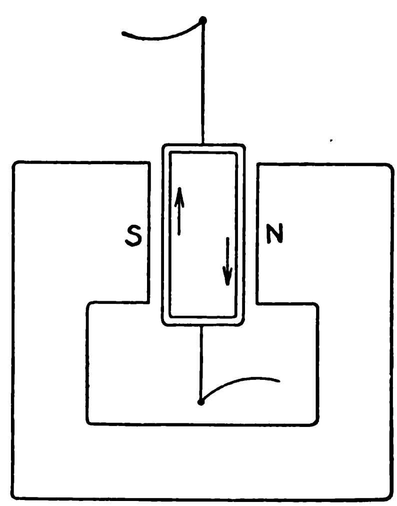

FIG. 221. — Essential Parts of a D'Arsonval Galvanometer.

and the right-hand side from the observer. This tendency of the suspended coil to turn is opposed by the twist in the supporting wires. Evidently the larger the current which is flowing, the larger the force action on each side of the coil, and the larger the angle through which the coil will turn. This instrument is capable of being made very sensitive. It possesses the distinct advantage over the Thomson galvanometer that **it is but little affected by external magnetic influences.**

THE PLUNGER INSTRUMENT

352. Another type of electrical measuring instrument which depends for its indications upon the magnetic action of the current is known as the plunger type. The essential parts of this instrument are shown in Figure 222. It is seen to consist essentially of a solenoid *AB*, through which the current to be measured is passed, and a soft iron plunger *C*, which becomes magnetized in such way that the plunger is drawn down into the solenoid. For example, if the current in the solenoid flows clockwise in the coils as seen from above, the positive direction of the lines of force is that indicated in the diagram. The lower end of the soft iron plunger therefore becomes a north-pointing pole and the upper end a south-pointing pole. The north-pointing pole tends to move in the positive direction of the lines of force, that is, toward the center of the solenoid, while the south-pointing pole tends to move in the opposite direction.

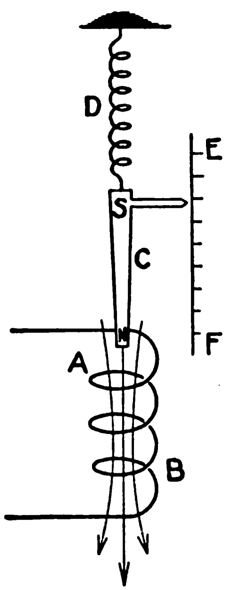

Fig. 222.

The field intensity, however, is greater in the neighborhood of the north-pointing pole or lower end of the plunger than it is in the neighborhood of the upper end of the plunger. Hence the tendency of the north-pointing pole to move in a downward direction is greater than that of the south-pointing pole to move in the opposite direction. The plunger as a whole moves downward. This tendency to move

downward is opposed by the spiral spring. The extent to which the plunger moves downward is indicated by the pointer attached to the plunger which plays over the scale *EF*. Evidently the stronger the current in the solenoid the greater the intensity of the magnetic field within it, and hence the greater the distance to which the plunger is drawn into the coil.

THE ELECTRODYNAMOMETER

353. In the electrodynamometer a movable coil mounted like the coil of a D'Arsonval galvanometer is placed at the center of a coil (large) like that used in the tangent galvanometer. The two coils are arranged to stand perpendicular to each other. When a current is passed through the two coils, the small coil tends to turn about its suspending wires. This tendency to turn will be understood from the following considerations. Let *A*, Figure 223, represent the larger coil and *D* the smaller coil. When a current flows in the coil *A*, a magnetic field is established in the neighborhood of *D*, the direction of which is perpendicular to the plane of coil *A*. If a current is caused to flow through the coil *D*, it will tend to turn in this magnetic field, being acted upon by forces proportional to the current in the coil *D* and the field in the neighborhood of *D* due to the current in the larger coil. But the field at *D* due to the current in *A* is proportional to the current in *A* (Section 317). Therefore the torque on *D* is proportional to the product of the currents in *D* and *A*, that is, $T \propto I \cdot i$ or $T = KIi$. If the same current I flows through both coils, then

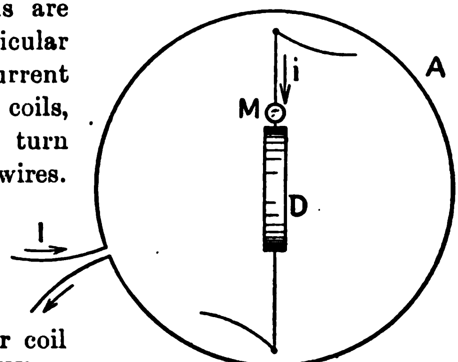

FIG. 223.

$$T = K \cdot I^2$$

If there were nothing to oppose this tendency to turn, the coil *D* would set itself parallel to the coil *A*. In turning, however,

it twists the suspending wires. The opposing torque introduced in this manner balances the torque due to the action of the current in the coil. The coil *D* will therefore turn through a definite angle for each value of the current which flows through it. The deflections of the coil *D* may be observed by means of a mirror *M* attached to the suspending wire near the coil.

THE HOT WIRE INSTRUMENT

354. In this instrument the heating effect of the electric current is taken advantage of. Consider a wire *AB*, Figure 224, supported at its ends and attached at its center to a spring *c*. If a current is passed through *AB*, it will heat the wire, causing it to increase in length (Section 158); the "slack" will be taken up by *c*. The pointer *SP* pivoted at *O* is attached at *P* to the end of the spring and moves with it. Hence as *AB* lengthens, the pointer moves over the scale *S* according to the current effect in *AB*.

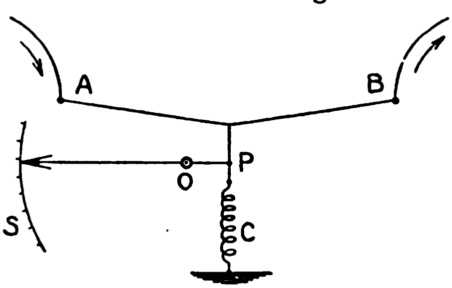

FIG. 224.

AMMETERS, VOLTMETERS, AND WATTMETERS

355. An *ammeter* is a low resistance galvanometer provided with a scale so marked as to indicate directly the current which passes through the instrument.

A *voltmeter* is a high resistance galvanometer provided with a scale so marked as to indicate directly the electromotive force applied to its terminals.

The method of connecting ammeters and voltmeters is shown in Figure 225. *A* represents an ammeter and *V* a voltmeter; *L* is a group of lamps being supplied with current from the dynamo *D*. With this arrangement **the total current going to the lamps passes through and is measured by the ammeter *A*. The e. m. f. which is applied to the lamps acts also upon the volt-**

meter V, sending through it a current $i = \frac{E}{R}$. Hence the indications of V are proportional to E, the e. m. f. acting upon the lamps. Obviously, in order that A and V may **absorb** but little power,

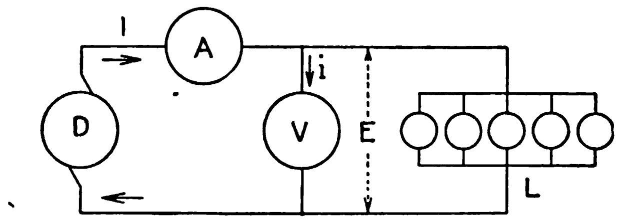

FIG. 225. — Diagram of Circuit showing Connections of Ammeter and Voltmeter.

A, which carries the **whole current,** must have **small resistance,** and V must be of **large resistance** in order that the current which flows through it may be small.

***A Wattmeter* is an electrodynamometer having one coil (the current coil) of low resistance and one coil (the pressure coil) of high resistance, and provided with a scale so marked as to indicate directly the power absorbed by the circuit to which it is connected.**

The manner in which a wattmeter is connected to a circuit is shown in Figure 226. W is the wattmeter which is to measure the power supplied to the lamps L by the dynamo D. The circle represents the current coil and the ellipse the pressure coil of the wattmeter. With the connections indicated in the figure the **total current I which goes to the lamps passes through the current coil of the wattmeter. The pressure coil of the wattmeter is connected as a shunt to the lamps.** Let E be the e. m. f. acting on the lamps. Then the current which will flow through the pressure coil is $i = \frac{E}{R}$, in

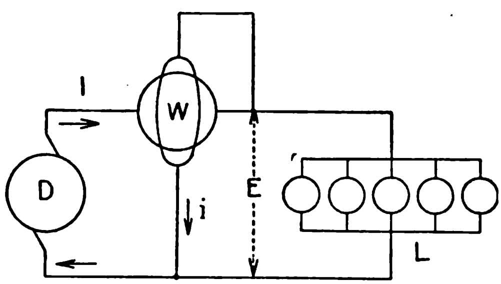

FIG. 226. — Diagram of Circuit showing Connections of Wattmeter.

which R is the resistance of the coil. It has already been pointed out (Section 353) that the torque acting on the suspended coil of an electrodynamometer is proportional to the product of the currents in the coils, therefore $T = K_1 \cdot I \cdot i$ or. since $i = \frac{E}{R}$, $\therefore$

$$T = \left(\frac{K_1}{R}\right) EI = \text{a const.} \times EI.$$

i.e.
$$T = K \cdot EI \tag{103}$$

But EI is the power absorbed by the lamps L (Section 320); therefore the torque acting on the movable coil of the wattmeter is porportional to the power (watts) absorbed by the circuit to which the instrument is connected.

Problems

1. When a certain tangent galvanometer is in adjustment, it is found that a current of 5.5 amperes in the coil will deflect the needle 45°. What is the constant of the galvanometer?

2. A current of I amperes deflects the needle of a tangent galvanometer 60°. A current of i amperes gives a deflection of 30°. What is the ratio $\frac{I}{i}$?

3. What current will deflect the needle of the galvanometer of problem 1 50°?

4. The coil of a tangent galvanometer has a mean radius of 20 cm. and consists of 40 turns of wire. If used where the horizontal intensity of the earth's field is .2 c. g. s. units, what is the constant of the instrument?

ELECTROMAGNETIC INDUCTION

CHAPTER XXX

INDUCED ELECTROMOTIVE FORCE

356. It was discovered by Faraday in 1831 that **whenever a current is started or stopped in an electric circuit, there is a momentary current in any other closed circuit in its immediate neighborhood.** The circuit in which the current is started or stopped is called the **primary** circuit. The circuit in which the momentary current circulates at the moment of starting or stopping the current in the primary is called the **secondary** circuit. The temporary currents which circulate in the secondary at the moment of starting or stopping the current in the primary are called **induced currents.** The e. m. f.'s in the secondary in response to which the induced currents flow are called **induced electromotive forces.**

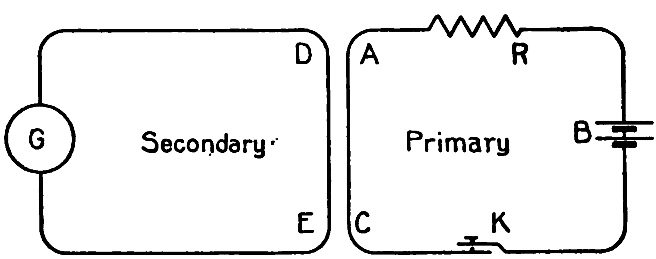

FIG. 227. — Illustrating Electromagnetic Induction.

The statement made in the foregoing paragraph will be more readily understood by reference to Figure 227, in which ABC represents an electric circuit containing a battery B joined by a wire AC to a resistance R and a key K. DEG represents a second circuit consisting of a galvanometer, the terminals of which are connected by a wire DE. It is supposed that DE

and AC are parallel portions of the two circuits which lie close to one another. With the arrangement shown in the diagram it will be observed that, at the moment of closing the key K, and thus starting the current in the circuit ABC, the galvanometer G will indicate the presence of a current in the circuit DEC. Again, upon stopping the current in the primary circuit ABC, the galvanometer G will indicate a momentary current in the secondary circuit DEG, in the opposite direction to that which was produced when the current was started in the primary.

A careful study of the conditions represented in the diagram will discover that a momentary current is produced in the secondary, not only upon starting or stopping the current in the primary, but **whenever any change is made in the primary, either with respect to the magnitude of the current flowing in the primary or with respect to its position relative to the secondary.** In other words, an induced current is present in the secondary: (*a*) when the current is started in the primary; (*b*) when the current is stopped in the primary; (*c*) when the value of the current in the primary is changed (either increased or diminished); (*d*) when the primary is moved nearer to or farther from the secondary, the current in the primary remaining the same.

A more convenient arrangement of apparatus for studying these effects is that shown in Figure 228. A is a coil of wire connected to a battery E, a resistance R and a key K. This is the primary circuit. B is a coil of wire connected to a sensitive galvanometer G. This constitutes the secondary circuit. Induced currents are present in the coil B whenever the current in A is started, stopped, or changed in magnitude, or whenever the coil A, with a steady current flowing in it, is caused to approach or recede from the coil B.

Since the coil A, Figure 228, is surrounded by a magnetic field which changes with respect to the coil B when the current in the coil A is changed or when the position of the coil A is changed, we very naturally conclude that **the induced electromotive forces in the coil B are in some way associated with the changing magnetic field due to the coil A.**

If it is the changing magnetic field in the neighborhood of

the coil *B* which gives rise to the induced electromotive force in *B*, it ought to be possible to demonstrate the presence of such electromotive forces in *B* whenever the magnetic field in its neighborhood changes from other causes. It ought, for example, to be possible to demonstrate the presence of induced electromotive forces in the coil *B* when a bar magnet is brought up into its presence or removed from its neighborhood, since under these circumstances there would be a changing field in the neighborhood of the coil *B*. That there are induced electromotive forces in the coil *B* under these circumstances may be shown by means of the apparatus represented in Figure 229.

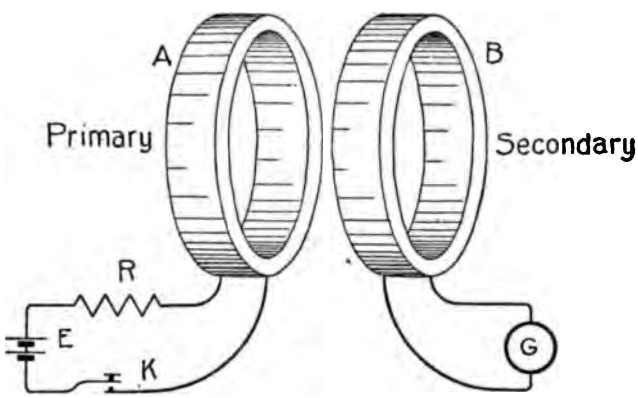

Fig. 228.

NS is a permanent bar magnet which is supposed to have been thrust into the coil *B* from the left to its present position. Upon making the experiment, it is found that there is an induced electromotive force in the coil *B* while the magnet is moving up to its present position. If the magnet is now withdrawn, there will again be an induced electromotive force in the coil *B*, but in the opposite sense.

Again, if it is the changing field in the neighborhood of *B* which is responsible for the induced electromotive force, it ought to be possible to induce electromotive forces in such a coil by turning it over in a magnetic field. For example, in Figure 230, let *B* represent a coil lying in a horizontal position.

Under these circumstances a certain number of lines of force due to the earth's magnetism are threading through the coil in the direction indicated. If the coil is reversed, evidently, when it comes into an edgewise position with respect to the direction of the lines of force, there will be no lines of force threading through it. And, finally, when it is turned into the reverse position, this same number of lines of force will be passing through the coil in the opposite direction. Under these circumstances it is found that when the coil is reversed there is an induced electromotive force in the coil.

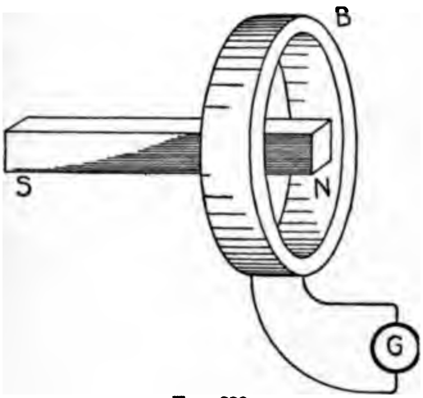

Fig. 229.

THE INDUCED ELECTROMOTIVE FORCE DEPENDS UPON THE RATE AT WHICH THE MAGNETIC FIELD CHANGES

357. In all of the experiments outlined in the preceding paragraphs it can be very readily determined that the induced electromotive force depends upon the rate at which the magnetic field in the neighborhood of the coil B is changing. Thus if, in the arrangement of apparatus represented in Figure 229, the north pole of the bar magnet is very suddenly thrust into the coil B, the induced electromotive force in the coil is correspondingly great. If the pole is inserted slowly, the induced electromotive force is correspondingly small. Again, if in the experiment represented in

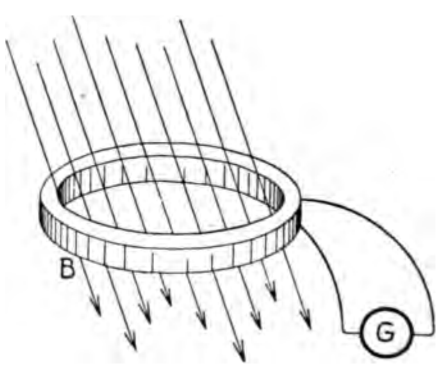

Fig. 230.

Figure 230, the coil is very quickly reversed, the induced electromotive force is greater than when the coil is slowly revolved in the magnetic field.

THE LAW OF INDUCED ELECTROMOTIVE FORCES

358. Thus it may be demonstrated by experiment that there is an induced e. m. f. in any circuit through which the magnetic flux (*i.e.* total number of lines of force) is changing, and that the magnitude of the induced e. m. f. depends upon the rate at which the flux is caused to change. There are two statements of the law of induced e. m. f.'s, both of which are useful in the discussion of the various applications of the principle of electromagnetic induction.

(*a*) In terms of changing flux:

There is an induced electromotive force in any coil when the number of lines of force threading through it is changing, and the value of the electromotive force is equal to the rate at which the number of lines of force through the coil is changing.

(*b*) In terms of cutting lines of force:

There is an induced electromotive force in any conductor which is cutting (*i.e.*, moving across) lines of force, and the value of the induced electromotive force is equal to the rate at which lines of force are cut by the conductor.

MAGNITUDE OF THE INDUCED ELECTROMOTIVE FORCE

359. Let it be imagined that N lines of force are withdrawn from a coil of wire in t seconds. The induced electromotive force in the coil under these circumstances is given by

See exp 91 manual

$$e = \frac{N}{t} \tag{104}$$

This then is the algebraic expression of statement (*a*). (See above.)

If a conductor moves through a magnetic field in such manner that it cuts N lines of force in t seconds, the induced electromotive force is again given by the above equation. Hence, this is also the algebraic expression of statement (*b*). In other

words, Equation (104) is the general law of induced electromotive force.

It should be observed that the above expression is really a defining equation for electromotive force. That is, a new unit of e. m. f. is here contemplated such that if one line of force is cut per second the induced e. m. f. is unity. It can be shown that one volt is equal to 100,000,000 such units. Therefore, if e is to be expressed in volts, we have, —

$$e \text{ (in volts)} = \frac{N}{t} \cdot \frac{1}{10^8}$$

DIRECTION OF THE INDUCED ELECTROMOTIVE FORCE

360. Experiment shows that **the induced current in any circuit is in such direction as to oppose that change of conditions which gives rise to the induced current.** This is known as **Lenz's Law.** As examples of the application of Lenz's Law consider the following. In the experiment illustrated in Figure 229 it is the approach of the magnetic pole which gives rise to the induced electromotive force in the coil B. According to Lenz's Law the induced current in the coil B will be in such direction as to oppose the approach of this north-pointing pole. In other words, the induced current in the coil B under the assumed conditions will be in such direction as to establish a magnetic field within the coil of such nature that the lines of force pass through the coil from right to left. **As the north-pointing pole is pushed into the coil it is therefore being carried forward in opposition to this magnetic field.**

Again, considering the experiment illustrated in Figure 228, we have seen that when a current is flowing in the coil A, an induced current is present in the coil B if it is caused to approach nearer to the coil A. Applying Lenz's Law to this case, it will be understood that the induced electromotive force in the coil B, under these circumstances, is in such direction as to oppose by its magnetic reaction on the coil A this approach or coming together of the two coils; that is, **the direction of the induced current in the coil B will be opposite to that in the coil A,** since, as has been demonstrated, parallel currents flowing in

opposite directions repel one another. If we consider the induced electromotive force in the coil *B* when it is drawn back from the coil *A*, we can see that, according to Lenz's Law, the induced current in *B* would be in the same direction as the current in *A*, since parallel currents flowing in the same direction attract one another; and this force of attraction would therefore constitute a resistance to the separation of the coils, or in other words, would tend to oppose that motion which separates them.

In determining the direction of the induced electromotive force in a **conductor**, the **right-hand rule** may be used. This rule is as follows: Holding the thumb and first and second fingers of the right hand in such manner that they are at right angles to one another, if the first finger points in the direction of the field or lines of force, and the thumb points in the direction in which the conductor is moving, the second finger gives the direction of the induced electromotive force in the conductor. Figure 231 is designed to illustrate the right-hand rule and to show the relative directions of these three quantities. For determining the direction of an induced electromotive force in a coil the following rule is applicable: Let it be imagined that one is looking through the coil in the direction in which the lines of force extend. Then a decrease in the number of lines of force in the coil will give rise to an induced current which will flow clockwise in the coil. An increase in the number of lines of force will give rise to an induced electromotive force which will flow counterclockwise in the coil. See Figure 232. The curved arrows represent the direction of the induced electromotive force in the coil

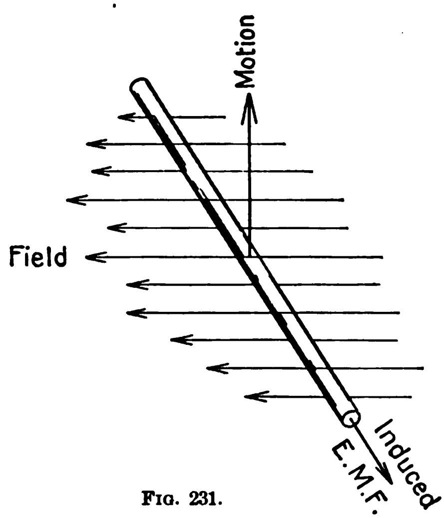

Fig. 231.

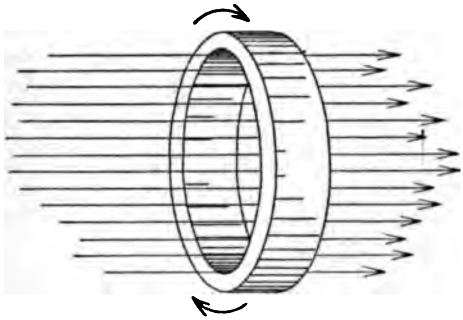

FIG. 232.

when the number of lines of force threading through the coil from left to right is decreasing.

The effect of decreasing the number of lines of force threading through a coil from left to right is the same as that produced by increasing the number of lines of force threading through the coil from right to left.

EDDY CURRENTS

361. "Eddy currents" are currents which eddy or circulate locally in masses of metal in the neighborhood of which the magnetic field is changing. Consider, for example, a disk of copper *A*, Figure 233, which is rotating between the poles of a horseshoe magnet.

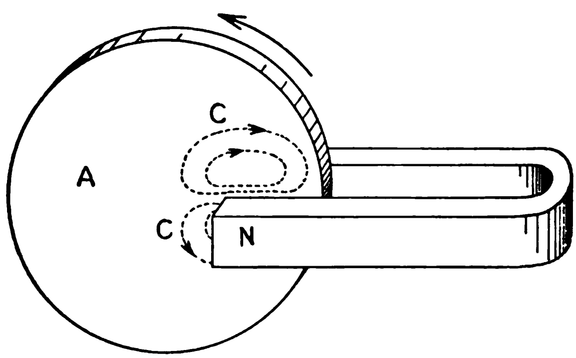

FIG. 233. — Eddy Currents.

Consider any radial element of the disk, for example, that one lying horizontally to the right and between the poles of the magnet. According to statement (*b*) of the law of induced electromotive force, an electromotive force will be induced in this element as it cuts the lines of force due to the magnet, the direction of which is from the right toward the

left as may be determined by the right-hand rule. In response to this induced electromotive force induced currents will flow along this element of the disk from the circumference toward the center and back through the adjacent parts of the disk as represented by the dotted lines *C*. These currents circulating in small closed paths within the copper are known as eddy currents. They are true electric currents, being characterized by the effects of the electric current. Thus, they produce heating effects and magnetic effects. Since their circulation in the copper disk means the expenditure of energy, it will be evident that work must be done upon the disk to supply this energy. **That is to say, it will require greater expenditure of energy to rotate the disk when these eddy currents are present than would be required in their absence.** Furthermore, the disk will become heated if it is caused to rotate between the poles of a strong horseshoe magnet as contemplated in the discussion.

ARAGO'S EXPERIMENT

362. An interesting example of the generation of eddy currents is that afforded by Arago's experiment. Figure 234 represents a thick copper disk rotating about its center *O* in the direction of the arrows *CD*. Let *NS* represent a small bar magnet pivoted at *O* and standing just in front of the copper disk. Under these conditions it will be found that the magnet tends to follow the copper disk in its rotations, although every precaution is taken to shield it from any other than the magnetic influence of the eddy currents in the disk *AB*. The explanation of this rotation of the magnet *NS* is as follows: Consider that radius of

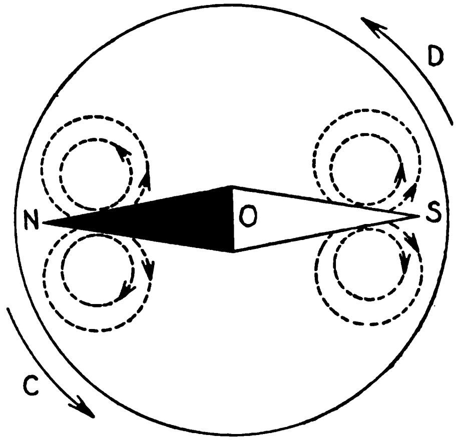

FIG. 234. — Arago's Rotations

the disk which is just passing beneath the north-pointing pole of the bar magnet. Some of the lines of force which radiate from N pass directly through the copper disk. These lines of force are cut by each radius of the revolving disk. Applying the right-hand rule, it is easily determined that the induced electromotive force on that radial element of the disk which is just passing beneath the north pole of the bar magnet is from circumference to center along the radial element as indicated. The eddy currents which are set up in the copper will circulate as indicated by the curved dotted lines. If now we consider the reaction of these currents upon the magnetic field of the bar magnet, it will become at once apparent that the north pole of the bar magnet is urged in the direction in which the disk is moving. To determine the magnetic reaction between the eddy current and the north-pointing pole of the bar magnet, apply the left-hand rule (see Section 312). The application of this rule shows that **the eddy current in this portion of the disk is urged in a direction opposite to that of the rotation**; but since reaction is equal to action and oppositely directed, therefore **the force acting on the north-pointing pole of the magnet is in the direction in which the disk is rotating**. If we consider the radial element of the disk which is passing under the south-pointing pole of the bar magnet, it is evident that the radial eddy current in this part of the disc is from center to circumference, the direction of the field and the direction of motion being both reversed. Hence the force acting upon the south-pointing pole of the bar magnet under these circumstances urges it in the direction in which the disc is rotating.

THE PREVENTION OF EDDY CURRENTS

363. Since power is required to maintain eddy currents and through their agency energy is transformed into heat, it is desirable to eliminate the eddy current effect as far as possible from commercial electrical apparatus. All massive metal parts which are subject to fluctuating magnetic fields and all iron parts which are repeatedly magnetized and demagnetized will have eddy currents generated in them except in those cases in which the circulation of such currents is prevented.

The method of preventing the flow of eddy currents most commonly employed is to **laminate** (cut in thin sheets) the metal parallel to the magnetic field and perpendicular to the plane in which the eddy currents tend to circulate. For example, let *A*, Figure 235, represent a mass of iron which is being magnetized in the direction indicated by the arrows *B*. Since the outer portions of this mass of

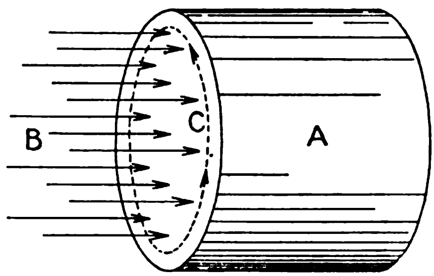

FIG. 235.

iron *A* constitute a closed conductor about these lines of force, it will be evident that induced currents will tend to flow in these outer portions as indicated by the dotted lines. The arrowheads indicate the direction in which the induced current will flow in the mass of iron when the number of lines of force threading through the mass *A* from the side *B* is increasing. Now the mass of iron *A* may be laminated in any plane parallel to the lines of force *B* without breaking its magnetic continuity in the direction *BA* ; but such lamination will interrupt the continuous path *C* in which the eddy currents tend to circulate. The metallic path for the eddy currents being broken up, the eddy currents are largely prevented, especially so if the laminations are insulated from one another. Evidently in the case represented in Figure 235, the same effect might be secured by using a bundle of small iron wires.

SELF-INDUCTION

364. A careful consideration of the law of induced electromotive force leads to the conclusion that **induced electromotive forces are present in any coil in which current is being started or stopped or changed in magnitude**. Consider, for example, coil *A*, Figure 228. When the key *K* is closed and the current begins to flow in the coil *A*, a magnetic field is set up about the coil.

In other words, the starting of the current in the coil amounts to a threading of a number of lines of force through the coil.

But according to statement (*a*) of the law of induced electromotive force, this threading of a number of lines of force through the coil will result in an induced electromotive force. In the same way, if after the current is established in the coil *A*, and the magnetic field surrounding it becomes constant at each and every point, the circuit is opened and the current stopped, there will again be an induced electromotive force in the coil *A*, since to stop the current is in effect to withdraw those lines of force which are threading through the coil. Thus we see that induced electromotive forces are present in a coil in which a current is started or stopped, and in the same way in a coil in which the current is varied in magnitude. These electromotive forces are termed self-induced electromotive forces or electromotive forces of **self-induction** since they are developed in the coil in which the current which produces the changing magnetic field about the coil is flowing.

To determine the direction of this e. m. f. of self-induction we have only to apply Lenz's Law of induced currents. For example, imagine the current to be increasing in the coil. This means an increasing number of lines of force threading through the coil in response to the increasing current. But Lenz's Law states that the induced current will oppose that which gives rise to the induced current, namely, the increasing number of lines of force threading through the coil. In other words, the induced current under these circumstances will be opposed in direction to the current supplied by the battery, since an opposing current would tend to establish a magnetic field in the opposite direction, or what amounts to the same thing, to oppose the introduction of this increasing number of lines of force. Again, considering the moment of opening the circuit and stopping the current from the battery, the decreasing current, under these circumstances, means a decreasing number of lines of force threading through the coil. Therefore, according to Lenz's statement, the induced current will be in the direction of the current from the battery, since a current in this direction will tend to prevent that which gives rise to the induced current, namely, the decreasing number of lines of force threading through the coil.

Stated briefly, **the self-induced electromotive force is in such direction as to oppose the current from the battery when the current is increasing in magnitude or when the current is being started, and in such direction as to tend to maintain the current from the battery when the current is decreasing or is being stopped.**

The presence of the self-induced e. m. f. in a circuit may be demonstrated by means of the apparatus represented in Figure 236. *MM* is an electromagnet having a laminated core of soft iron. This electromagnet is connected to a battery *B* and a key *K*. *L* is an incandescent lamp connected as a shunt across the terminals of *MM*. Let it be assumed that the key is closed and a current is flowing in the direction indicated by the arrow *C*. There will also be a current *i* in the lamp as indicated. **At the moment of opening the circuit at *K* there will be a self-induced e. m. f. in *MM* in such direction that it tends to prevent the decrease in the current in *MM*.** This self-induced e. m. f. will send a **reverse** current through *L*. If *MM* and *B* are properly chosen, the lamp *L* will glow brightly for an instant under the induced current in *MM* even when the battery e. m. f. is too small to "light" the lamp directly.

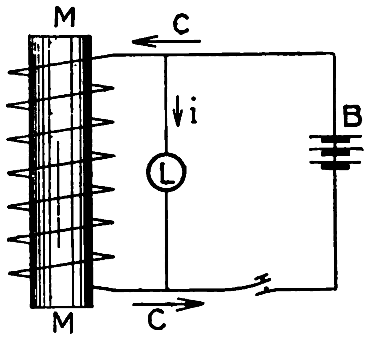

FIG. 236. — Arrangement of Apparatus for showing Self-induced E. M. F.

THE COEFFICIENT OF SELF-INDUCTION

365. Experiment shows that the magnetic flux (*i.e.* the total number of lines of force) which is established by a current in a coil is proportional to the current. That is, $N \propto I$, in which N represents the magnetic flux and I the current. Hence, we may write

differentiate to+ for definition of L

$$N = LI \tag{105}$$

in which L is a constant for the coil under consideration and is called the **coefficient of self-induction of the coil.**

Equation (105) holds rigidly only for coils surrounded by air or some other medium of constant permeability (Section 301). If a coil has an iron core, the magnetic flux through the coil is not proportional to the current (Section 302), *i.e.* the relation $N \propto I$ does not hold. The coefficient of self-induction of such a coil depends upon the value of the current flowing in it.

A **non-inductive** circuit is one in which the conductors are so disposed with respect to one another that their individual magnetic effects are neutralized. For example, two wires lying closely side by side and carrying the same current in opposite directions form a practically non-inductive system. Similarly a coil of wire consisting of two layers wound in opposite directions is non-inductive.

THE INDUCTION COIL

366. The induction coil is a device which is used for developing high electromotive forces by taking advantage of the principle

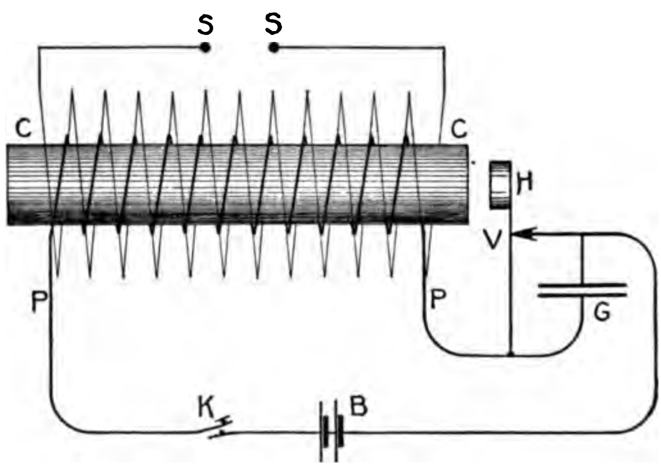

FIG. 237. — The Induction Coil.

of electromagnetic induction. Its essential features are a core or bundle of iron wires, a few turns of heavy insulated copper wire which is known as the primary coil and a secondary coil consisting of a very great number of turns of fine insulated

copper wire. The arrangement of parts and the connections for the apparatus are shown in Figure 237. *PP* is the primary and *SS* the secondary coil. The primary coil is connected to a battery *B* and a key *K* by means of which the primary current is started or stopped. When the key *K* is closed, the current flows from the battery through the primary circuit and magnetizes the core *CC*. This means that a large number of lines of force are threaded through each turn of wire wound upon the core. This again means that induced electromotive forces are present in each turn of wire while the number of lines of force is changing. According to statement (*a*) of the law of induced electromotive force, **the induced e. m. f. is numerically equal to the rate at which the number of lines of force is changing in the coil.** Considering the secondary circuit, it will be evident that the induced electromotive forces in the successive turns are in the same direction and that they act together to produce a large electromotive force between the terminals *SS*. Since the value of this electromotive force depends simply upon the rate at which the lines of force are being threaded through each turn, and the number of turns, evidently **by using a large number of turns and causing the magnetic flux through each turn to change rapidly, large values of electromotive force may be established between the terminals *SS*.**

When the circuit is opened and the current stops in the primary coil, this bundle of lines of force is in effect suddenly withdrawn from each and every turn of wire in the secondary circuit. Therefore at this moment there will again be an induced electromotive force in each of the turns and between the terminals of the secondary.

To secure a rapid interruption of the primary current a spring vibrator *V*, carrying a piece of soft iron *H*, is included in the circuit, as shown in the figure.

From the foregoing statements the effectiveness of the coil is greatest when the current in the primary is very quickly started or very quickly stopped. Now it is found in practice that, because of the effects of self-induction in the primary circuit, a large spark will be produced at the point at which the primary circuit is opened. This spark is in effect an arc which enables

the current to continue for a certain interval of time after the circuit is actually broken. The effect of this is to allow the current in the primary circuit to die away slowly instead of stopping suddenly as it should do in order to secure the maximum effect in the secondary. In order to obviate this difficulty of sparking at V in the primary circuit a condenser G is shunted around the spark gap at V. **With this arrangement the self-induced current in the primary**, instead of causing a spark at the key K, **tends to charge the condenser, which after a very brief interval discharges again in the reverse direction through the battery and the primary circuit.** This device not only does away in a large measure with the effect of the spark at the key K, but makes it possible to reduce the magnetic flux in the core from its maximum value to zero in a very brief interval of time. Without the condenser the core retains some of its magnetism after the primary circuit is broken. When the condenser is used, the reverse current from the discharging condenser just after the primary circuit is broken **demagnetizes** the core, thus doing away with any residual magnetism.

Induction coils may be constructed in this manner to give very high voltages between the terminals of the secondary, voltages of such magnitude as to cause a discharge through air of several inches (several feet even) and give effects analogous to those produced by the discharge from the electrostatic machine.

As was stated above, the core of an induction coil is made of a bundle of small iron wires instead of one large mass of iron such as is sometimes employed in an electromagnet. The object in using a bundle of iron wires instead of one large mass of iron is to prevent eddy currents in the iron (Section 363).

THE TESLA COIL

367. The effects secured by means of the Tesla coil illustrate in a very striking manner the principle of induced electromotive forces. This apparatus is essentially an induction coil without an iron core. In other words, it consists of two coils, one of a few turns of coarse wire which is called the primary,

and the other of many turns of fine wire called the secondary. The secondary is placed within the primary.

When a rapidly varying current is passed through the primary, e. m. f.'s are induced in the secondary. In order that the apparatus may be effective, a **very rapidly** changing current must be present in the **primary**. A convenient way of securing such a primary current is to make of the primary a discharging circuit for a Leyden jar. Under suitable conditions the discharge of a Leyden jar is oscillatory, that is, at each discharge of the jar the discharging current surges to and fro in the discharging circuit (Section 272). These surgings succeed each other very rapidly, often at the rate of a million per second.

The connections for a Tesla coil are given in Figure 238. *C* is a Leyden jar connected to the terminals *PP* of the primary of the Tesla coil. *G* is a spark gap across which the discharge of the jar takes place. *A* and *B* are connections extending to some suitable source for charging the jar, *e.g.* the secondary of an induction coil. When the apparatus is in operation and a rapid succession of discharges passes the spark gap *G*, very high e. m. f.'s are induced in the secondary *SS*. These e. m. f.'s are of very **high frequency**, *i.e.* they change rapidly in magnitude and direction. The discharge from such a coil has peculiar properties, among which is that of passing through or over the surface of the human body with but little sensible effect.

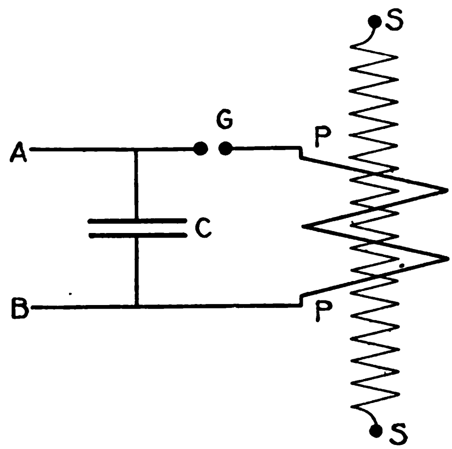

FIG. 238. — Diagram of Connections of a Tesla Coil.

THE DYNAMO

368. The dynamo is a device for transforming mechanical energy into electrical energy. It depends for its action upon

Get building up and dying down of current

the principle of electromagnetic induction. It consists essentially of a powerful electromagnet and a series of conductors which are made to move rapidly through the magnetic field, due to the electromagnet. One of the simplest forms of dynamo is represented in Figure 239, in which *NS* are the poles of an electromagnet *NABS*. When a current is flowing in the coils *AB*, there is established a strong magnetic field between *N* and *S* from left to right as indicated by the horizontal lines. Let it be imagined that in the space between *N* and *S* there are a number of conductors extending in a direction perpendicular to the plane of the paper in the diagram, and all revolving about the point *C*, which is the center of the magnetic field between *N* and *S*. It will be evident that each of these conductors cuts all of the lines of force extending across from *N* to *S* twice for each revolution which it makes about the point *O*. It is evident, therefore, that in each and all of these moving conductors, electromotive forces will be induced. By properly connecting these conductors the individual induced electromotive forces may be added together so as to get one large electromotive force acting through the entire series. In the practical form of the dynamo the revolving conductors are mounted upon a soft iron cylinder having its axis at *O*. This diminishes the air space through which the lines of force must flow, and very materially increases the magnetic flux of the electromagnet. Furthermore, it gives a solid support for the revolving conductors. This revolving part of the dynamo is called the armature. The iron core of the armature is laminated in order to prevent the eddy currents which would otherwise be developed in the iron core as it revolves in the magnetic field. The laminations extend at right angles to the axis of rotation. In other words, the core is made up of a series of thin disks.

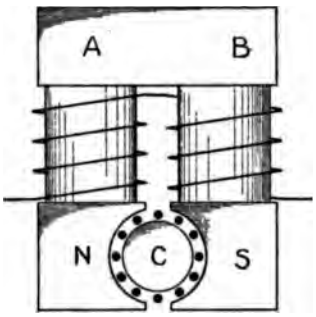

FIG. 239. — Simple Dynamo.

INDUCED ELECTROMOTIVE FORCE IN A COIL REVOLVING IN A MAGNETIC FIELD

369. The general character of the induced electromotive force in a coil which is caused to revolve in a magnetic field will be understood from a discussion of the following simple case: Let *AB*, Figure 240, represent a rectangular coil of wire of one turn arranged to rotate on the axis *CD*. Let it be imagined that this coil is lying in a uniform magnetic field extending from left to right as indicated in the figure, and that the coil is caused to rotate counterclockwise as seen from *C*. Evidently when the coil is in the position represented, there will be induced electromotive forces in the sides *A* and *B*, in the directions indicated by the dotted arrows. That is to say, the electromotive force in the side *A* is from the observer, while that in the side *B* is toward the observer. Therefore, if the coil forms a closed circuit, the induced current will circulate in a counterclockwise direction about the coil as seen from above.

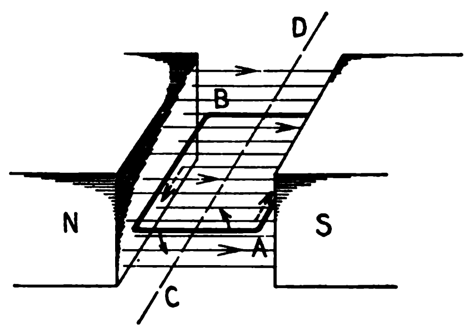

FIG. 240.

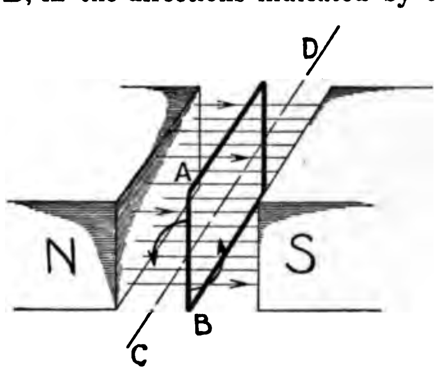

FIG. 241.

When the coil has rotated through 90 degrees and has come into the position represented in Figure 241, there are no induced electromotive forces present. In this position the greatest number of lines of force are threaded through the coil, but it will be remembered that the induced electromotive force depends, not upon the total magnetic flux present, but upon the rate at

which it is changing. It is evident that in this position of the coil the sides *A* and *B* are moving parallel to the lines of force, hence the flux through the coil is momentarily constant.

When the coil comes into the position represented in Figure 242, the sides *A* and *B* are again cutting lines of force and therefore have induced electromotive forces in them. It will be noticed, however, that the directions of the induced electromotive forces in the sides *A* and *B* are opposite to those in the first position as represented in Figure 240. That is, the induced electromotive force in the side *A* is now toward the observer, and that in the side *B* from the observer. The induced e. m. f.'s are, as before, in a counterclockwise direction as seen from above, but the coil has been reversed so that the induced current flows in each part of the coil in a direction opposite to that in which it was flowing in the first position.

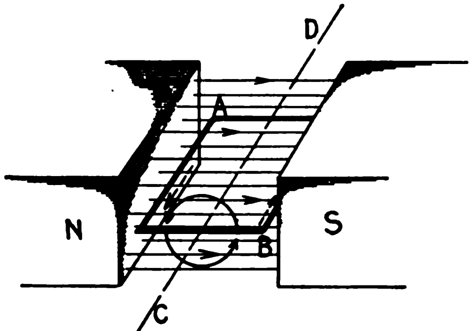

FIG. 242.

When the coil has made another quarter revolution and is again in a vertical position, the induced e. m. f.'s will again be zero. It will be evident that it is only when the coil is exactly vertical that the induced e. m. f.'s are zero, since when it has turned only slightly from a vertical position the sides *A* and *B* will begin to cut lines of force. The sides *A* and *B* have induced e. m. f.'s in them throughout the entire revolution of the coil except for the brief instant during which the coil is in a vertical position. The rate at which the sides *A* and *B* cut the lines of force steadily increases from zero up to a maximum when the coil lies in a horizontal position, and then steadily decreases as the coil turns once more into the vertical position.

ALTERNATING AND DIRECT CURRENTS

370. Evidently during one revolution of the coil described in the last section the induced e. m. f. (or current), rises to a

maximum value twice, and twice during the revolution is equal to zero. Furthermore, the two maximum values attained in each revolution are in opposite directions in the coil. Such a current is called an alternating current. That is, **an alternating current is one which begins to flow in one direction, rises to a maximum value, and then falls off to zero, then begins to flow in the opposite direction, rises to a maximum value, and then falls to zero, and so on repeatedly.**

A direct current is one which flows continuously in the same direction.

THE ALTERNATING CURRENT GENERATOR

371. The alternating current generator is a dynamo so constructed that the alternating currents which are developed in its rotating coils are transmitted to the outside circuit, with which it is connected, as alternating currents. The principle of the method employed for accomplishing this will be understood by reference to the simple case represented in Figure 243. *AB* represents a rotating coil like that described in the last section. *S* and *S'* are "slip rings," that is to say, insulated metallic rings to which the terminals of the coil *A* and *B* are connected. The side *A* of the coil is connected to the ring *S*, and the side *B* is connected to the ring *S'* as indicated. These rings are attached to the axis upon which the coil *AB* is mounted, and hence rotate with the coil. If two strips of metal *E* and *F*, connected to an outside circuit, for example an incandescent lamp circuit, *L*, are pressed against the slip rings *S* and *S'* while the coil *AB* is rotating, evidently the induced alternating current in the coil *AB* will circulate as an alternating current through the lamp circuit, *L*.

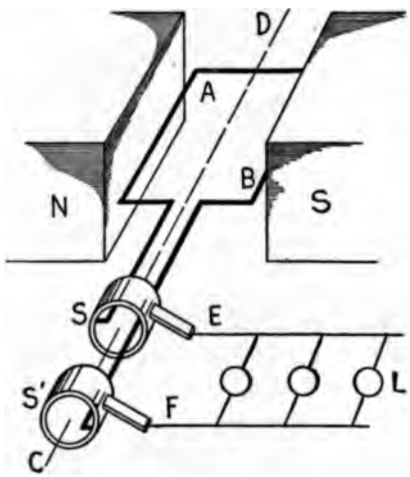

FIG. 243. — Simple Alternating Current Generator.

THE TRANSFORMER

372. Since induced electromotive forces depend for their existence on a varying condition of magnetism, it will be evident that induced electromotive forces must always accompany alternating currents. **It is possible by means of the alternating current to transfer electric energy from one circuit to another with which it has no metallic connection by utilizing the effect of electromagnetic induction.** This is done by a means of a device called a transformer.

The transformer depends for its action upon the principle of electromagnetic induction. Its action will be understood from the following discussion: Let *AB*, Figure 244, represent a frame of soft iron upon which are wound two coils of wire *P* and *S*, in the manner indicated in the diagram. Such an arrangement is called a transformer. Let it be imagined that an alternating current is flowing in the coil *P*. **This alternating current will give rise to an alternating magnetic condition of the iron frame *AB*.** The lines of force which extend through the iron when it is magnetized by current in the coil *P* will pass around the frame in the direction of the dotted lines. Since they pass through the coil *S*, evidently induced electromotive forces will be present in the coil *S* whenever these lines are being threaded through or withdrawn from that coil. If the terminals of the coil *S* are connected through any circuit, induced currents will flow in this coil in response to these induced electromotive forces. Thus energy is transmitted from the coil *P* to the coil *S* by means of the fluctuating magnetism in the iron frame.

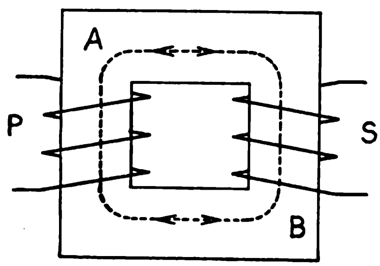

FIG. 244.—Illustrating the Principle of the Transformer.

Transformers are used for raising or lowering the electromotive force of an electric system. For example, electric current is distributed over the city at an electromotive force of 1000 volts. It would be dangerous in many ways to use such volt-

age in dwellings. It is, therefore, necessary to lower or "step-down" the voltage of such a system before the current is carried into the houses. This is done by means of transformers. If 100 volts are desired, a "ten to one" transformer is used, *i.e.* one having ten times as many turns in its primary as in its secondary coil. The secondary e. m. f. will then be 100 volts.

THE DIRECT CURRENT GENERATOR

373. The direct current generator is a dynamo so constructed that the alternating currents which are developed in its rotating coils are transmitted to the outside circuit, with which it is connected, as a direct current. This is accomplished as follows: Let *AB*, Figure 245, represent a rotating coil like that described in Section 369. The terminals of this coil are connected respectively to *C* and *C'*, the two parts of a metal cylinder which has been divided lengthwise as indicated in the figure. *C* and *C'* are insulated from one another and mounted upon the axis upon which the coil *AB* rotates. If two strips of metal (brushes) *E* and *F*, connected to an outside circuit, for example an incandescent lamp circuit, *L*, are caused to make contact with *C* and *C'* at the extremities of the horizontal diameter of the cylinder *CC'*, then as the coil *AB* rotates, a direct current will flow through the lamp. This will be evident from the following considerations: Consider the moment at which the coil *AB* reaches its vertical position. At this instant the induced e. m. f. in the coil is zero. As the coil passes the vertical position, the direction of the induced e. m. f.'s in it is changed in direction, but at this instant the brush *E* passes from the

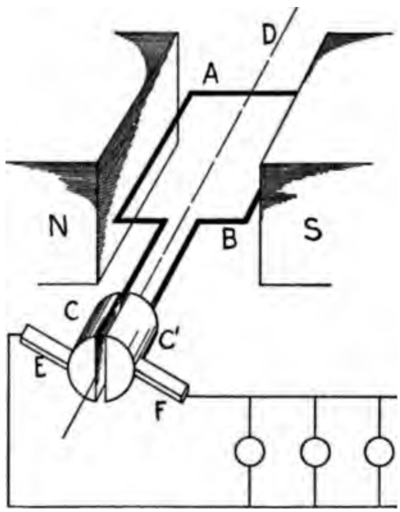

FIG. 245. — Simple Direct Current Generator.

C to the C' part of the cylinder and F passes from C' to C. That is to say, **at the moment in which the e. m. f. in the coil *AB* changes direction, the connections with the lamp circuit *L* are reversed.** It follows, therefore, that the direction in which the current flows in the circuit ELF remains unchanged as indicated by the arrow. The divided cylinder CC' is called **a commutator.**

THE ELECTRIC MOTOR

374. The electric motor is a device for transforming electrical into mechanical energy. It is essentially a dynamo, which, being supplied with electrical energy from some outside source, becomes a source of mechanical energy. Most dynamos are reversible, that is to say, they may be used either as generators of electricity, in which case they are supplied with mechanical energy, or they may be used as electric motors by supplying them with electric energy.

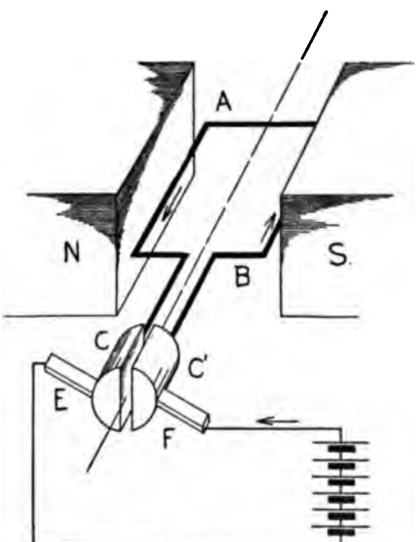

FIG. 246. — Simple Direct Current Motor.

The motor action of a dynamo will be understood from a consideration of the following simple case. AB (Figure 246) represents a coil like that discussed in the preceding section. It is provided with a commutator CC' and is connected by means of "brushes" EF to a battery as shown. Let it be assumed that to begin with the coil is stationary in the position shown. The battery current will flow through AB in the direction indicated by the arrows. Evidently the sides A and B are acted upon by forces tending to move them at right angles to the magnetic field in which they are lying. Applying the left-hand rule (Section 312), it is evident that A is acted upon by an upward force and B by a downward force so that **the coil**

as a whole tends to rotate clockwise as seen from the commutator. The forces acting upon A and B continue to be more or less effective in producing rotation of the coil until as the coil turns the vertical position is reached.

As soon as the coil has passed the vertical position, which the inertia of the moving coil will enable it to do, the current from the battery will be reversed in the coil; but the coil having been inverted with respect to the magnetic field, evidently the forces acting upon A and B will tend to continue the clockwise rotation. Hence a steady current flowing from the battery to the moving coil will maintain a continuous rotation.

The rotating coil described in the last paragraph may be kept in continuous rotation by supplying it with an alternating current instead of a continuous current such as is obtained from a battery, provided a definite relation between the speed at which the coil rotates and the alternations of the alternating current exists. The relation referred to is as follows: The coil must revolve through 180° while the current is making one alternation. It will be understood from the description of the direct current motor above, that the function of the commutator is to convert the steady current from the battery V into an alternating current in the coil A, since the commutator reverses the current in the coil A every half revolution. It follows, therefore, that if an alternating current from some outside source is conducted by means of slip rings to the coil AB, the condition for continuous rotation will be secured as before, provided, as stated above, that the coil revolves through 180° for each alternation of the current, and also that the coil is in its horizontal position when the alternating current is changing from positive to negative, or *vice versa*. This, of course, means that in order to cause such a coil to rotate by means of an alternating current it must be set in rotation first and made to revolve at a definite speed before the alternating current is turned on. Alternating current motors which operate on this principle are called **synchronous motors.**

There is another form of alternating current motor, called **an induction motor, which depends for its operation upon a rotating magnetic field.** One of the simplest ways of developing a rotating magnetic field by means of alternating currents is as follows:

Let $AA'BB'$, Figure 247, represent 4 coils of wire placed upon a soft iron ring RR'. If the coils AA' are supplied with current, the iron ring will be magnetized in such manner as to have its magnet poles at B and B'. When current is caused to flow in the coils BB', the current in AA' being zero, the ring will be magnetized in such a manner as to have its magnet poles at A and A'. When current is flowing in both sets of coils, the magnet poles will lie between the coils. For example, if the current in the coils AA' is in such direction that it tends to produce a north-pointing pole at B, and the current in the coils BB' is in such direction that it tends to produce a north-pointing pole at A', then the combined effect of the currents in both coils will produce a north pole at R', and the south pole at R. Evidently it is possible to cause the condition of magnetism in the ring RR' to shift or rotate by properly switching the currents on to the pairs of coils AA' and BB'. The eight successive stages of this shifting magnetic field, together with the corresponding directions of the currents in the coils A and B which produce them, are shown in Figure 248. In order that this rotating field may

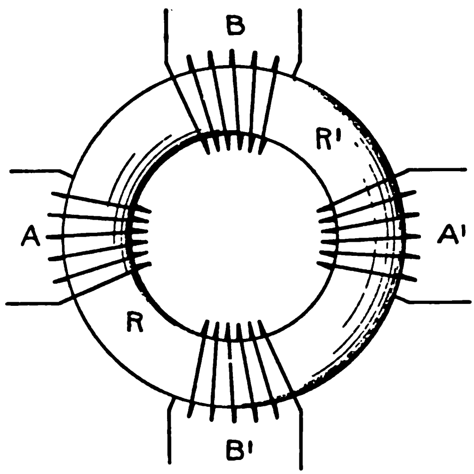

FIG. 247.

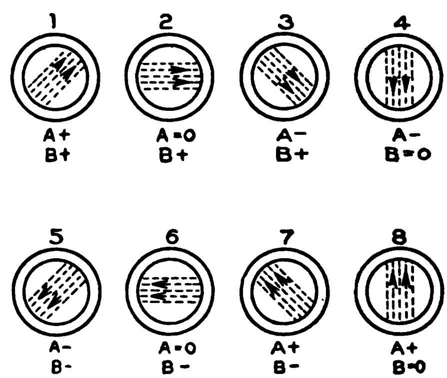

FIG. 248.

exist, it is only necessary to supply the coils AA' and BB' with alternating currents which are "out of step" in their alternations. Under these circumstances the magnetic effect produced by the A coils will reach its maximum value before the B coil effect reaches its maximum value. Hence, the condition for rotating magnetic field is secured. Alternating currents which are "out of step" in this way are usually said to differ in phase, and such an alternating current system is known as a two-phase system.

If now within the iron ring RR' there is placed a copper cylinder so mounted as to be free to turn, it will tend to rotate with the rotating magnetic field which passes through it, because of the eddy current effect which at once arises in the metal cylinder. This apparatus as described constitutes an induction motor. In the practical form of the apparatus the rotating part is filled with laminated iron.

Problems

1. A wire is moved across a uniform magnetic field cutting 10,000 lines of force in 1 sec. What is the induced e. m. f. in the wire?

2. A wire 1 m. long is moved through a uniform magnetic field having an intensity of 1000 lines per square centimeter. The wire moves perpendicular to its own length and at right angles to the field. If the velocity of the wire is 20 cm./sec., what is the induced e. m. f. in the wire?

3. If the direction of motion of the wire in problem 2 makes an angle of 60° with the field, what is the induced e. m. f.?

4. A circular coil of wire of 50 turns having a mean radius of 20 cm. lies on a table. $H = 0.18$, dip $= 70°$. The resistance of the coil is 0.01 ohm. The coil is picked up and turned over in one second. What is the average induced current in the coil?

5. What is the total quantity of electricity set in motion in turning the coil of problem 4? Would it make any difference in the quantity if the coil were turned slowly or quickly?

6. A coil of 50 turns having an area of 4 sq. cm. is "snapped" from between the poles of an electromagnet where the field intensity is 8000 lines per square centimeter to a point where the field intensity is negligibly small in 0.01 sec. What is the average induced e. m. f. in the coil?

7. A coil is rotated uniformly about a horizontal north and south axis. The average induced e. m. f. is 60,000 c. g. s. units. When the same coil is rotated about a vertical axis at the same speed, the average induced e. m. f. is 40,000. What is the dip of the earth's field?

8. A rectangular loop of wire 20 × 30 cm. is rotated in a uniform magnetic field of 5000 lines per square centimeter, at a speed of 1800 R. P. M. The axis of the coil is at right angles to the field. What is the average induced e. m. f.?

9. Would it make any difference in problem 8 whether the axis is parallel to the long side or short side of the coil? Whether it is at the center or the edge of the coil? If it extended along one of the diagonals?

10. If the rotating coil of problem 8 had 5 turns instead of 1 and formed 1 closed circuit (5 turns in series) of 0.5 ohm resistance, at what rate would heat be generated in the coil?

11. What average torque would be required to rotate the coil of problem 10? In what positions of the coil would the torque have its maximum and minimum values?

Review here Jan 10

TELEGRAPHY AND TELEPHONY

CHAPTER XXXI

THE ELECTRIC TELEGRAPH

375. The electric telegraph is a device for the transmission of intelligence in which advantage is taken of the magnetic action of the current. **An impulse of current sent over a line is made to magnetize a distant electromagnet.** This electromagnet attracts a small armature or piece of soft iron with sufficient force to produce an audible click. The electromagnet arranged to be used in this way is called a sounder. The sounder gives an audible click for every current impulse that goes over the line. Hence by a system of prearranged signals

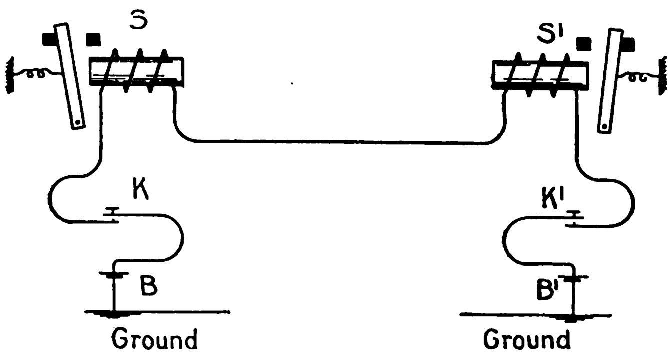

FIG. 249. — Simple Telegraph Circuit.

it is possible to transmit intelligence by means of this device. The simplest form of telegraph circuit is that shown in Figure 249. *K* is a key, *B* a battery, and *S* a sounder. A short-circuiting switch (*i.e.* a switch which closes the gap left by the open key), is provided for each key. If the short-circuit switch at either end of the line is opened and the key tapped, both

sounders will respond. In this manner signals may be sent to the farther end of the line. When not in use the key is short-circuited so as to complete the circuit for incoming signals. As indicated in the figure, a single wire extends from one station to the other. Both ends of the wire connections are "grounded," and the circuit is completed through the earth.

THE RELAY

376. For long distance telegraphy it is found that the line currents are too feeble to operate a sounder. Under such circumstances a "relay" is used. **The relay is a sensitive electromagnet which, responding to the feeble line currents, opens and closes a local circuit containing a battery and sounder.** The arrangement of apparatus will be understood by reference to Figure 250. *R* is the relay which is connected to the line *L*

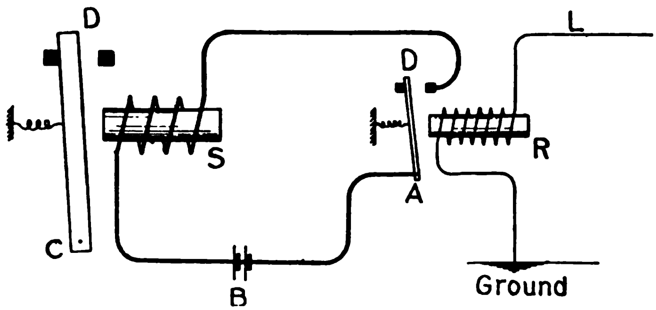

FIG. 250. — Relay Circuit.

and the ground *G* as indicated. *AD* is a very light, soft iron lever held to the left by a slender spiral spring. When *R* is energized, this lever *AD* is attracted and makes contact with the stop on the right, thus closing the local circuit *ABSD*. The armature or lever *CD* of the sounder *S* will evidently repeat the movements of the lever *A*. The current in the local circuit may, of course, be made sufficiently strong to render audible the click of the lever *CD*. Evidently this arrangement of relay, local circuit, and sounder may be used in place of the ordinary sounder in any circuit in which the line current is found to be too feeble to operate the ordinary sounder.

DUPLEX TELEGRAPHY

377. **In *duplex telegraphy* the apparatus is so arranged that signals may be sent in opposite directions over the same line at the same time.** This may be accomplished by the use of the **differential** magnet, upon which the coils are so placed that outgoing currents do not magnetize the core while incoming currents produce the usual effect, and hence record signals from the distant station.

The arrangement of apparatus is shown in Figure 251. S and S_1 are the differential magnets. When the key K is closed, a current flows over the line from the battery B. It does not,

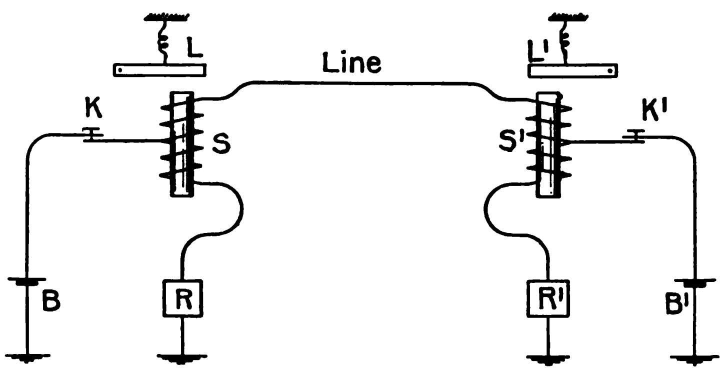

FIG. 251. — Circuit for Duplex Telegraphy.

however, magnetize the core of the electromagnet S, since the current is caused to divide as it enters the coil of this magnet, one half of the current flowing in a clockwise direction through one half of the coil, the other half of the current flowing in a counterclockwise direction through the other half of the coil. In the practical form of the apparatus one half of the coil is superimposed upon the other. Thus the magnetizing effect of one half of the coil is neutralized by that of the other. That half of the current which traverses the line to the distant station will energize the magnet S' and cause its armature to respond to the key K. In the same way, when the key K_1 is closed, the current from the battery B_1 divides in the magnet S_1, producing no effect upon L', but the part of the current which flows to S

energizes that magnet, causing its armature L to respond to the motions of the key K_1. In order that the differential magnet may be used successfully in this way, it will be evident that the current from the home battery must be divided equally between the two halves of the coil of the differential magnet, since if the currents in the two halves of the coil are unequal, their magnetic effects will be unequal, and they will not completely neutralize one another. This equal division of the current between the two halves of the coil is secured by adjusting the resistance R which is placed in series with that half of the coil which is grounded at the home station. When R is properly adjusted the current divides equally between the two halves of the coil. Because of the effects of electrostatic capacity in the line it is found necessary in practice to employ a condenser in the ground connection SR. It is connected as a shunt to the resistance R.

THE POLARIZED RELAY

378. The **polarized relay** is a relay which responds to a reversal of the current in the circuit in which it is placed, but does not respond to a change in the strength of the current. This mechanism will be understood from the sketch given in Figure 252. NS is a C-shaped bar of soft iron upon which is wound a magnetizing coil C. ns is a permanent steel magnet pivoted at s so as to be free to move back and forth between the poles NS of the electromagnet. Let it be assumed that a current is flowing in the electromagnet in a direction indicated by the arrows. Then the north and south poles of the electromagnet will be as indicated in the figure. Now the north-pointing pole of the steel magnet ns will be attracted by the south-pointing pole S of the electromagnet, and repelled by the north-pointing

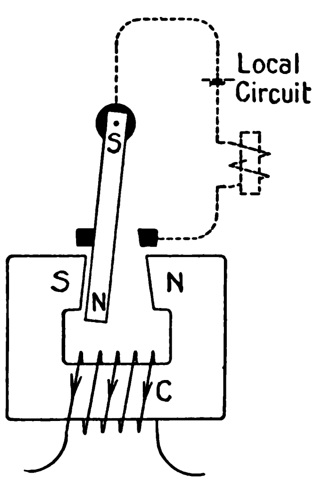

FIG. 252. — Polarized Relay.

pole N. **So long as the direction in which the current is flowing in the coil C remains unchanged, the steel magnet ns will remain in the position shown, even though the strength of the current in C is caused to vary between wide limits.** A change in the direction of the current in C will be followed at once by a change in the position of ns, since a change in the direction of the current means a reversal of the magnetism in the iron. When the current is reversed, the right-hand pole of the electromagnet becomes south-pointing, and the left-hand pole, north-pointing, and the small steel magnet then moves to the right. Thus **the polarized relay is a device which is unaffected by changes in the strength of the current but which responds at once to a reversal of its direction.**

DIPLEX TELEGRAPHY

379. In diplex telegraphy arrangements are made for sending two messages simultaneously in the same direction over the same wire. One of the simple arrangements by means of which this is accomplished is the following: In Figure 253, P and I

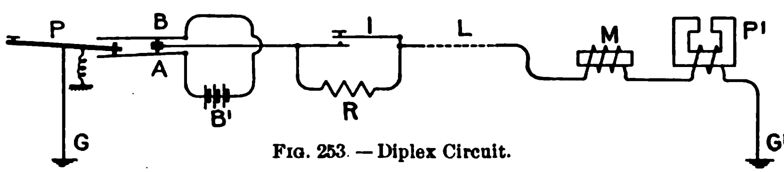

FIG. 253. — Diplex Circuit.

represent the two keys at the home station, which are used simultaneously, for sending messages over the line L. I is an ordinary key connected in shunt with the resistance R. Evidently the effect of closing the key I is to short-circuit the resistance R, thereby diminishing the resistance of the line and hence increasing the current which flows from the battery B to the distant station. The key P is called a "pole changer" for the reason that when it is manipulated it reverses the connections of the battery B^1. When the key P is at rest, it is held down by the spring, so that it makes contact with the lever A and the positive pole of the battery. The line at the same time is in communication with the negative pole of the battery through the lever B. Now if the right-hand end of the key P

is caused to rise, it makes contact with the lever *B* and the negative pole of the battery, while the line is placed in communication with the positive pole of the battery through *A*. When, therefore, *P* is operated, the current in the line is reversed in direction.

At the distant station are two relays represented by *M* and *P′* in the same figure. *M* is an ordinary relay which is so adjusted as to respond to relatively strong currents without regard to the direction in which the current is flowing in the line. *P′* is a polarized relay which responds only when the current in the line is reversed. It will be evident that with this arrangement of apparatus *M* will respond to every motion of *I*, while *P′* will respond to the motions of *P*. Therefore, *M* will record all messages transmitted by *I*, *P′* will record all messages transmitted by *P*, and each system of transmitter and recorder will be independent of the other.

QUADRUPLEX TELEGRAPHY

380. By combining the principles of duplex and diplex telegraphy it is possible to send simultaneously, two messages in each direction over one and the same wire. This is called quadruplex telegraphy.

THE TELEPHONE

381. By taking advantage of the principle of electromagnetic induction it is possible to transmit speech electromagnetically.

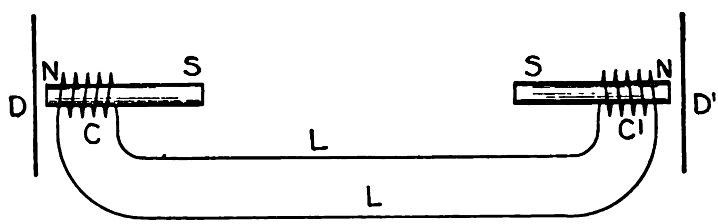

FIG. 254.—Simple Telephone Circuit.

The telephone is the apparatus by means of which this is accomplished. Figure 254 represents the simplest form of electromagnetic telephone. The apparatus at each end of the line

consists of a permanent magnet *NS*, a coil of fine wire *C* placed over one end of this magnet, and a soft iron diaphragm *D* placed near the end of the bar magnet which is surrounded by the coil. The two coils are connected in series by the lines *LL* as indicated in the figure. The operation of this telephone will be understood from the following considerations. Let it be imagined that a person stands at the instrument, represented by *DC*, Figure 254, and speaks to the diaphragm *D*. **The sound waves of the voice cause the diaphragm *D* to vibrate, and each vibration of the diaphragm causes a redistribution of the lines of force spreading from the adjacent pole of the bar magnet.** The moving lines of force give rise to induced electromotive forces in the coil *C*. Induced currents will therefore flow from this instrument through the lines *LL* to the instrument at the other end of the circuit, and passing through the coil *C* in the distant instrument will alter the magnetism of the magnet *N′S′*. But any alteration in the magnetic field due to *N′S′* will cause the diaphragm *D′* to change its position, since, if the field is increased, the diaphragm will be more strongly attracted; while if the field is weakened, the diaphragm will be less strongly attracted. Evidently, therefore, the diaphragm *D′* will respond to every motion of the diaphragm *D*. But the vibrations of *D′* give rise to sound waves in the air in its neighborhood. Hence, sound waves falling upon *D* are reproduced by *D′*.

LONG DISTANCE TELEPHONE

382. The apparatus described in the last paragraph is not adapted to the transmission of speech to any great distance, since the currents developed in the manner indicated are not sufficiently strong. The arrangement of apparatus which is employed in modern long distance telephone systems is represented in Figure 255. It consists essentially of a transformer *PS* wound upon a bundle of straight iron wires. The primary of this transformer is connected to a battery *B* and the transmitter *T*. The essential parts of the transmitter are two plates between which there is placed a small quantity of granular carbon. This granular carbon forms a conductor of rather high resistance between the plates. **A motion of either plate in the**

transmitter will cause a variation in the resistance of the granular carbon conductor between the plates and therefore a variation in the current which is circulating about the coil *P* from the battery ***B*.** The mouthpiece of the transmitter is placed in front of one of the plates just mentioned. Evidently, sound waves

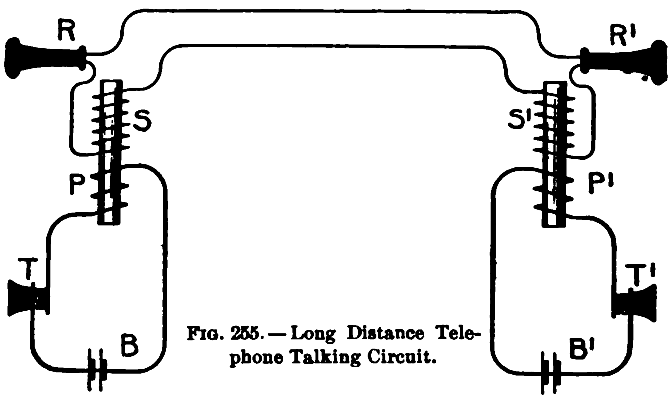

Fig. 255. — Long Distance Telephone Talking Circuit.

falling upon this plate will produce the effect mentioned above. Therefore, when a person speaks into the transmitter, the current in the primary circuit *PBT* fluctuates in accordance with the sound waves which fall upon the plate of the transmitter. This varying current in the coil *P* causes a variation in the magnetism of the core of the transformer *PS* and gives rise to induced currents in the coil *S* which is connected by way of the receiver to the lines running to the distant station. These induced currents traversing the line give rise to sound waves in the distant receiver as explained in the paragraph above. The apparatus at the distant end of the line is the exact duplicate of that installed at the near end.

In the arrangement shown in Figure 255 the apparatus employed for calling or attracting the attention of the person at the farther end of the line is not included. In the practical form of the instrument an automatic switch called the receiver hook is so arranged that when the receiver is hung up the circuit represented in Figure 255 is open and the line is connected to the call bell, and the calling device or magneto, which is, in fact, a small dynamo that may be employed for sending current over

the line to the distant bell. When the receivers at both ends of the line are off the hooks, the connections are as shown in Figure 255.

CENTRAL ENERGY SYSTEMS

383. In the arrangement of apparatus described in the last section, a battery is required at each telephone. There must also be provided a magneto, that is, a small hand dynamo for sending a signal to the distant station in "calling up." The central energy system is now quite commonly employed. It possesses, among others, the following advantages: (*a*) the batteries used for operating the telephones are all located at the central station and hence are more conveniently cared for; and (*b*) the subscriber to "call up" Central has only to lift the

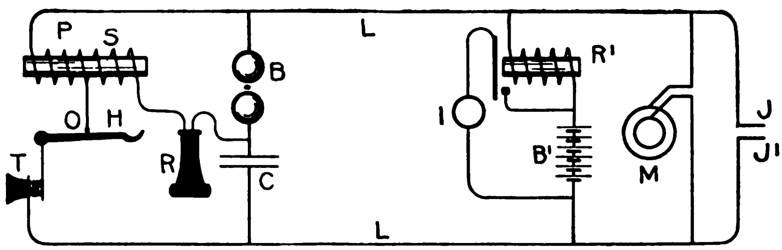

FIG. 256. — Central Energy Circuit.

receiver from the hook. In Figure 256, one of the more simple central energy circuits is shown. The subscriber's station, containing a transmitter *T*, a hook switch *H*, a transformer *PS*, a receiver *R*, and a bell *B*, is represented at the left. The central station, containing a battery *B'*, a magneto *M*, a relay *R'*, and an incandescent lamp *I*, is represented at the right. The two stations are connected by the lines *LL*.

The operation of this system is as follows: When the receiver *R* is hanging upon the hook switch *H*, the main circuit from the transmitter *T* through the primary of the transformer *P* is broken at *O* so that no current from the battery *B* can flow through this circuit. *C* is a condenser placed in series with the bell *B*, which, of course, prevents the flow of current through the bell circuit. When Central wishes to call the sta-

tion represented, an alternating current is sent over the lines *LL* from the magneto *M* or other suitable source. This alternating current, surging into and out of the condenser *C*, will ring the bell *B*. If it is desired to call Central from the station represented, it is only necessary to lift the receiver *R* from the hook *H*. When this is done, a spring raises the hook and closes the circuit at *O*. This allows current to flow from the battery at the station over the line *L* through the transmitter *T* and by way of the hook switch *H* and the primary coil *P* of the transformer, thence back over the line *L* to the central station. The current which flows in this circuit energizes the relay *R′*, which closes the circuit of the incandescent lamp *I*. The lamp is then lighted and constitutes the signal to Central. When the subscriber talks into the transmitter *T*, the current through the primary *P* of the transformer fluctuates as described above, and this gives rise to induced currents in the secondary *S*, which travel over the lines *LL* to the central station or beyond, according to connections. To connect this station with that of any other subscriber, Central makes connections at the points *JJ′*. From this arrangement of apparatus, it is evident that the "talking current" and the primary current flow over the same lines. It is found in practice that this does not interfere with the transmission.

ELECTROMAGNETIC WAVES

CHAPTER XXXII

MAXWELL'S THEORY

384. Reference has already been made in the study of heat and electrostatics to the universal medium called the ether which is supposed to fill all space. We have seen that the ether in the neighborhood of a charged body is in a state of strain. It is also assumed that the ether in a magnetic field is in a strained condition, the strain under these circumstances being of a different nature from that produced by the electrostatic charge. It follows that upon the sudden discharge of a charged body or the sudden demagnetization of a magnet, a disturbance of the ether will take place. This is simply another way of saying that the ether tends to relieve itself of the strain to which it is subjected while in the presence of a charged body or the magnet producing the magnetic field. One can imagine that this disturbance spreads through all space very much as a disturbance spreads in all directions over the surface of a pond of still water when a stone is dropped into it. Such a disturbance of the ether is known as an **electromagnetic wave**, since both the magnetic and the electrostatic effects are present.

In 1864 Maxwell explained on a purely mathematical basis that it ought to be possible to establish electric waves in the ether. In 1888 Hertz succeeded in carrying out Maxwell's suggestion, and not only produced electromagnetic waves by apparatus designed by himself, but succeeded in detecting their presence at some considerable distance from the apparatus from which they were caused to spread.

HERTZ'S APPARATUS

385. The apparatus used by Hertz in his investigations consisted of an **oscillator** and a **resonator**. The oscillator is repre-

sented in Figure 257. It consists of two plates of metal *A* and *B* to which are attached small rods terminating in knobs as indicated in the figure. In the use of this apparatus the rods are connected to the secondary terminals of an induction coil as indicated in the diagram. When the coil is set in operation, the plates *A* and *B* become charged, and a difference of potential is established between the knobs which, when a certain value is reached, causes a spark discharge between the knobs. If this were a simple discharge, the disturbance of the ether resulting would consist of a single pulse which would move forward through space as a single ripple moves across the surface of a still pond. It is found, however, that with the arrangement of apparatus indicated in the figure, **the discharge is oscillatory** in character, so that, instead of a single ether pulse, there will be a succession of pulses constituting a train of waves somewhat like the series of waves which travels over the surface of a still pond when a stick is moved up and down in the water several times in succession.

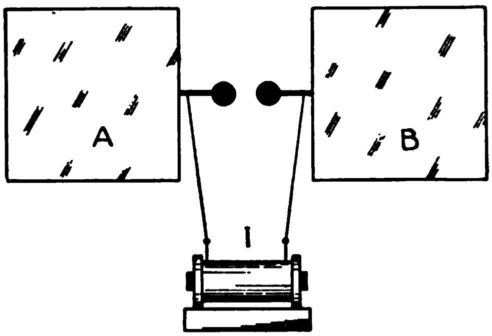

FIG. 257. — Hertz Oscillator.

The resonator employed by Hertz is shown in Figure 258. It consists of a circular loop of wire *AB* terminating in small polished knobs *C*. In the actual form of the apparatus, arrangement is made for adjusting the distance between the knobs *C* by means of a micrometer screw. Hertz found that when this resonator was placed in certain positions opposite the oscillator, a discharge would take place

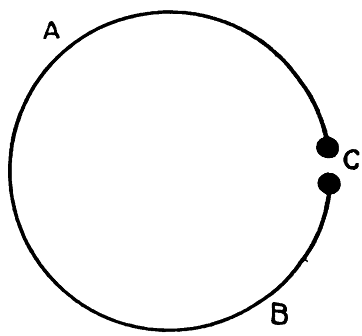

FIG. 258. — Simple Resonator.

between the knobs C whenever a discharge occurred in the oscillator. That is to say, ether waves set up by the oscillator, falling upon the resonator AB, were able to impart to the resonator a certain amount of energy which appeared at the knobs C in the form of the electric discharge.

Hertz found in his experiments that the position of the resonator with respect to the oscillator had much to do with the magnitude of the effect produced in the resonator. He found that it was necessary to place the resonator in such position that the knobs of the resonator would become charged by electrostatic induction from the charges on the plates of the oscillator, or in such position that the lines of force set up by the discharging current between the plates of the oscillator would cut the conductor of the resonator in such way as to produce a difference of potential between its knobs. The maximum effect obtained was when the resonator was so placed that these two effects were combined.

Hertz also showed the resonator might be made up in the form of the oscillator shown in Figure 257. When this form of resonator is employed, it is placed with its axis AB parallel to that of the oscillator.

THE OSCILLATORY DISCHARGE OF THE OSCILLATOR

386. The largest effects in the transmission of energy by means of the electromagnetic waves are secured when the discharge of the oscillator is oscillatory in character. The reason for this is at once apparent when it is understood that for each oscillation a wave or pulse passes out through the ether, each producing its own effect. **Now if the receiver or resonator upon which the waves fall is of such character that it vibrates electrically at the same rate that the oscillator does, then the effect of each wave falling upon the resonator will be added to that of all of the others,** so that by a succession or train of waves an effect may be secured which is many times as great as that which would be given under the same conditions by a single wave or pulse. The rate at which the oscillations take place in an oscillator is found to depend upon the electrostatic capacity and the self-induction of the oscillator. By adjusting the values

of these quantities the oscillations may be given any period desired.

RESONANCE

387. As pointed out in the last paragraph, **the effect upon the resonator is greatest when the dimensions of the resonator are such as to cause the charge to vibrate upon it at the same rate that the charge vibrates upon the oscillator.** When this condition is secured, the resonator is said to be in the condition of resonance. The resonator may be "tuned" (brought into the condition of resonance) by altering its dimensions, that is to say, by changing either its inductance or its capacity, since, as in the case of the oscillator, the rate at which the charge oscillates is dependent upon these two quantities.

LODGE'S EXPERIMENT

388. The effect of resonance may be very convincingly shown by the following experiment, which is due to Lodge. Let *ABCD*, Figure 259, represent a rectangular circuit containing a condenser *J* and a spark gap *S*. Let it be assumed that the terminals of the condenser *J* are connected to the secondary terminals of an induction coil *I* as indicated in the figure. When the jar is sufficiently charged by the e. m. f. of the induction coil, a spark will pass at *S*. This discharge of the jar through the circuit *ABCD* will be oscillatory, and electromagnetic waves will spread out into space from this discharge circuit. Let *EFGH* be a second circuit of approximately the same form and dimensions, having its plane parallel to that of the circuit *ABCD*. The side *FG* of

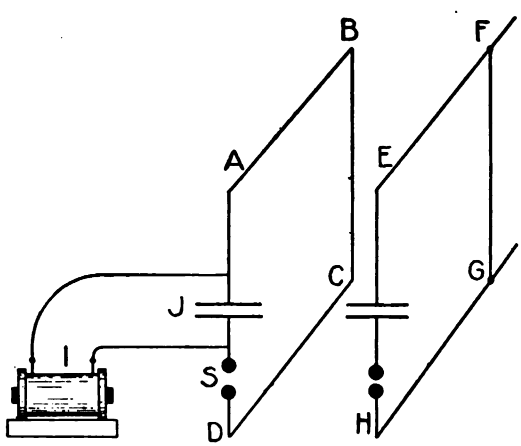

Fig. 259.

this circuit is supposed to be free to slide upon the conductors *EF* and *HG* so that the size of the rectangle *EFGH* may be varied at will. It is found that when *FG* is in a certain position so as to make the rectangle *EFGH* of approximately the same size as *ABCD*, sparks will pass across the spark gap of the circuit *EFGH* when the discharge occurs at *S*. In other words, under these circumstances, *ABCD* is serving as an oscillator, *EFGH* as a resonator. If, now, the size of the resonator rectangle is altered by moving the side *FG*, all else remaining the same, the discharge of this circuit will cease, indicating that the resonator is no longer responding to the electric waves which fall upon it. A very slight motion of the side *FG* is sufficient to throw the resonator out of "tune."

WIRELESS TELEGRAPHY

389. In wireless telegraphy a practical application is made of the principles enunciated in the foregoing paragraphs. Electromagnetic waves, generated by an oscillator, may be detected at great distances, providing a resonator is used which is tuned to resonance with the oscillator at the sending station. If, therefore, a succession of signals is sent out according to some prearranged code, messages may be transmitted by means of these electromagnetic waves. The form of oscillator commonly employed in wireless telegraphy is that of a more or less nearly vertical wire, or group of wires, attached to suitable supports. This system of wires is called the aërial. For long distance work the aërial is sometimes 150 or 200 feet in height. One form of aërial is that shown in Figure 260. It consists of a fan-shaped group of wires *A*, supported on a mast or tower (not shown in the figure), communicating with one knob of the spark gap *G*. The other knob is connected to the earth as indicated. The terminals of the induction coil are connected above and below the spark gap. The discharge which occurs between the knobs of the spark

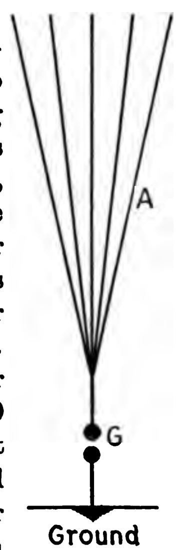

FIG. 260.

gap is oscillatory in character, and gives rise to a train of electromagnetic waves.

The receiving apparatus may consist of a similar aërial upon which the electric waves are allowed to impinge. But in order that the receiving apparatus may be operated at great distances from the sending station, it is necessary to substitute for the spark gap some device which is more sensitive in its indications. Various devices have been used for this purpose. One of these is known as the **coherer.**

DETECTORS

390. The coherer consists essentially of a small glass tube *ACB*, Figure 261, and two metal rods which reach into the tube, leaving a small opening *C* at the center which is filled

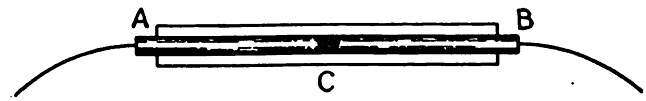

FIG. 261.

with metal filings. The resistance of this device between the points *A* and *B* is found to be quite high, owing to the loose contact between the filings at *C*. When the coherer is connected in a receiving circuit upon which electric waves are falling, the discharge which takes place through the metal filings **tends to cause them to cohere in such manner that the resistance of the device from *A* to *B* is very much decreased.**

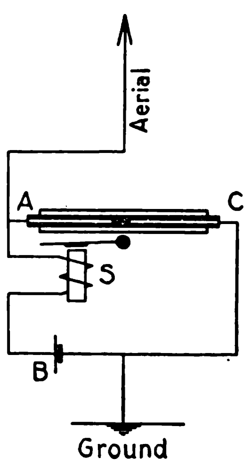

FIG. 262.

A simple arrangement of apparatus illustrating the use of the coherer is represented in Figure 262. *AC* is the coherer which takes the place of the spark gap in the aërial. A local circuit, containing a sounder *S*, or some equivalent device, and a battery *B*, is completed through the coherer *AC* as indicated. Under ordinary conditions no appreciable current flows in this local circuit, because of the high resistance of the coherer. When electromagnetic waves fall upon

the aërial, and oscillations take place in the system of conductors connected to the coherer, its resistance is decreased, as pointed out above, and a current of sufficient strength to operate the sounder *S* flows in the local circuit. The armature of the sounder is arranged to vibrate when current is flowing in the local circuit, and is placed close to the coherer so that it strikes the coherer when vibrating. In this manner the filings of the coherer are caused to "decohere," and the apparatus is made ready for the succeeding signals.

The magnetic detector of Marconi is a detector which has recently come into extensive use in wireless telegraphy. The principle upon which it depends for its action is as follows. Let Figure 263 represent the aërial of a receiving station. In place of the usual spark gap, it contains at *G* a coil of wire of comparatively few turns. Over this is a second coil connected to an ordinary telephone receiver *T*. *WW* is a fine iron wire supported by suitable clockwork by which it is made to travel slowly in the direction of its own length through the two coils. A magnet *NS* is fixed permanently near the moving wire *WW*, as indicated in the figure. This magnet tends to magnetize the iron wire, which retains a certain amount of this magnetism as it passes into the coil *G*. So long as this magnetic condition in the wire *WW* is constant, no effect is produced by the moving magnetized wire upon the coil connected to the receiver; but any change in the magnetic condition of the wire is followed at once by induced electromotive forces in the coil referred to, and results in sounds in the receiver. When electric oscillations take place in the aërial, the magnetic condition of the wire *WW* is altered by the surging currents in the aërial. Hence oscillations in the aërial are always accompanied by sounds in the receiver, and by these sounds the oscillation may be recognized.

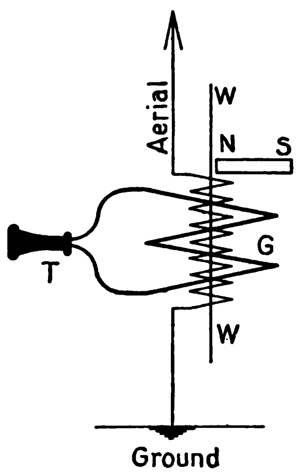

FIG. 263.

ELECTRIC DISCHARGE

CHAPTER XXXIII

POINT DISCHARGE

391. The simplest kind of electric discharge is that which takes place from a sharp point; this is called **point discharge.** It is found that under ordinary atmospheric conditions a charge is retained by a conductor for a limited time only, even though the conductor is carefully insulated. The discharge which takes place under these circumstances was formerly attributed to the presence of moisture and dust particles in the air. Careful experiment has shown, however, that this discharge takes place when moisture and dust both have been carefully removed from the air.

The modern theory attributes this discharge to what is known as the **ionization** of the air.

IONIZATION

392. A gas is said to be ionized when it contains free electrons or free positive atoms. The simplest way of ionizing a gas is by heating it. Thus the gases in and about a flame or near an incandescent solid are ionized. It has been shown that a very hot body loses its charge more quickly than a cold one. The explanation is that in the more violent vibratory motion of the molecules, corresponding to the higher temperatures (Section 198), the electrons become separated from the atoms.

When electrons or free positive atoms are caused to move with high velocity through a gas, they collide violently from time to time with molecules of the gas. As a result of such collisions the gas may become more completely ionized.

THEORY OF THE POINT DISCHARGE

393. Let it be imagined that in the neighborhood of a negatively charged body there are a few free positive atoms. These atoms, being attracted by the negative charge, move toward the charged body. The nearer they opproach the more rapid their motion and hence the more violent the collisions when they occur near the charged body. **The effect of these collisions is to ionize the air in the immediate neighborhood of the charged body.** The electrons and positive atoms thus set free start into rapid motion, the electrons are repelled, and the positive atoms attracted to the charged body. The positive atoms thus falling upon the body gradually neutralize the charge.

A point facilitates this process since, as we have seen (Section 250), the surface density of charge on a point is greater than at any other part of the conductor, and ionization will therefore take place more rapidly in that vicinity.

A positively charged body is discharged in a similar manner. The electrons in this case are the attracted bodies, which by their rapid motion and collisions with the neutral portions of the air effect its ionization.

THE BRUSH DISCHARGE

394. The point discharge is accompanied by a faint bluish glow which extends to a short distance from the point. When the difference of potential between the charged body and its surroundings is very great a "brush" is formed near the point. This consists of a large number of faint sparks or streamers which radiate from the point to a distance, it may be, of several inches. This discharge is very beautiful, but is only faintly luminous and can only be seen in the dark. It is to be thought of as a modified point discharge taking place in essentially the same manner.

THE DISRUPTIVE DISCHARGE

395. If the terminals of an electric machine in operation are brought sufficiently near together, sparks will be observed to pass between them. This is known as the **disruptive discharge.** This discharge is accompanied by the development of heat,

light, and sound. In general the energy of the charge is converted into these three forms of energy. The sound and the light developed are at once apparent. The heating effect is readily proven by placing in the path of the discharge some inflammable material, for example, ether, or by causing the discharge to take place along a very fine wire, in which case, providing the charge is sufficiently large, the wire will be fused in consequence of the heating action of the discharge.

THE EFFECT OF PRESSURE UPON THE DISCHARGE

396. We are already familiar with the characteristic features of the electric discharge when it takes place in air at ordinary pressures. We have seen that the "spark" is narrow and, except when the terminals between which the spark occurs are very close together, its path is zigzag and oftentimes forked. This form of discharge is accompanied by heat, light, and sound.

If the electric discharge is caused to take place in a region in which the pressure is somewhat less than atmospheric pressure, very marked changes take place in the character of the discharge. When the pressure of the air in which the discharge takes place has been reduced to about one thousandth of an atmosphere, the discharge is known as the Geissler discharge.

THE GEISSLER EFFECT

397. The changes in the character of the electric discharge in regions of low pressure were studied quite extensively by Geissler. In his investigations he employed a tube like that represented in Figure 264. BC is a glass tube sealed at both ends and having platinum wires A and K sealed through the walls of the glass. By connecting the tube to an air pump by means of a side connection E the air may be gradually exhausted from the tube and the corresponding changes in the character of the discharge observed. Such a tube is called a Geissler tube. When the air has been exhausted until the pressure within the tube is about one thousandth that of the atmosphere, the effect within the tube is known as the Geissler

effect. The characteristics of the discharge under these conditions may be briefly described as follows; Let A represent the wire by which the discharge enters the tube, *i.e.* the anode, and K the cathode. When the pressure conditions are as indicated above, the cathode K is surrounded by a luminous layer known as the **negative** glow, which extends from the surface of the cathode to a comparatively short distance. Next to this negative glow is a **dark space** which is apparently devoid of any luminous discharge. Beyond this and reaching from the dark space to the anode is the **positive column**. This consists of a peach blossom colored luminosity which apparently fills the entire tube.

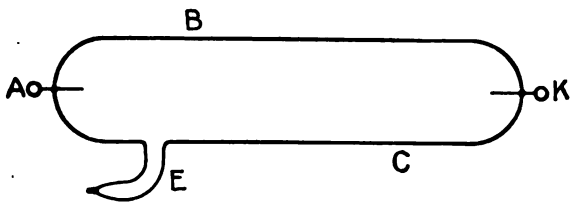

FIG. 264. — Vacuum Tube for showing the Effect of Pressure upon the Character of the Electric Discharge.

If the pressure of the air within the Geissler tube is still further diminished, it will be observed that the dark space gradually increases in length until it occupies the entire length of the tube. At this point in the exhaustion a new set of phenomena appear.

THE CROOKES EFFECT

398. When the exhaustion of the Geissler tube has been carried forward as indicated in the last paragraph until the dark space occupies the entire length of the tube which takes place at a pressure of about one millionth of an atmosphere, the walls of the tube become brilliantly fluorescent. The luminous effect at this stage of the exhaustion seems to be limited almost entirely to the walls of the tube. The luminous discharge in the gas itself which is so prominent in the Geissler effect is at this stage of the exhaustion almost, if not quite, entirely absent. This effect is known as the Crookes effect, and a tube which has been exhausted to this degree is known as a Crookes tube.

CATHODE RAYS

399. The fluorescence of the walls of a Crookes tube is caused by the bombardment of the walls by matter which is projected from the cathode. It can be shown experimentally that there is such a projection of material particles from the cathode and that they are projected at enormous velocities; further, that they move in straight lines. The following experiment is designed to demonstrate the rectilinear propagation of cathode rays. Let Figure 265 represent a Geissler tube more or less nearly spherical in form. Let K be a cathode having a concave surface. Let it be assumed that the terminal A is used as the anode. It will then be observed that the luminous discharge within the spherical bulb tends to bend in a curved path between A and K as represented by the dotted line. When B is used as an anode, the discharge curves between B and K; and when C is used as an anode, the discharge curves as indicated by the dotted line running from C to K.

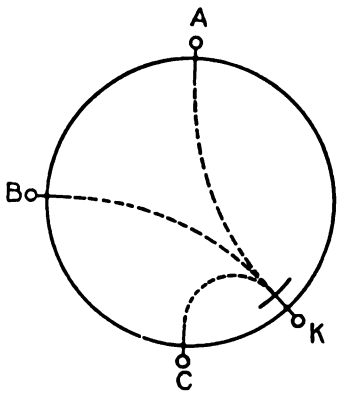

FIG. 265. — Geissler Tube.

If now the exhaustion is carried forward until a Crookes vacuum is reached, the character of the discharge will be that shown in Figure 266, in which the cathode rays passing from the terminal K are concentrated at the center of curvature of the concave cathode. Beyond this point they again diverge and fall upon a portion of the opposite wall of the tube D, which is rendered highly fluorescent by their impact. **The**

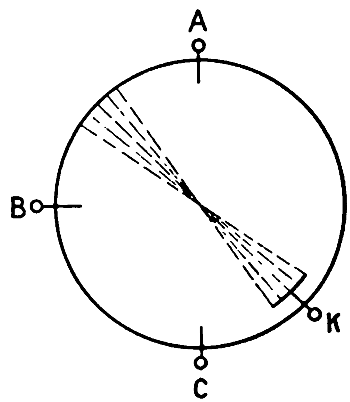

FIG. 266. — Crookes Tube.

path of the cathode rays as represented in Figure 266 is independent of the terminal which is used at the anode. It will thus be seen that the cathode rays travel in straight lines and that their direction of motion is not at all determined by the position of the anode.

PROPERTIES OF THE CATHODE RAYS

400. The cathode rays are characterized by three effects, the mechanical effect, the heating effect, and the production of X-rays.

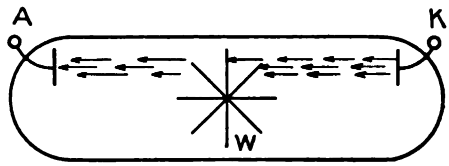

FIG. 267. — Mechanical Effect of Cathode Rays.

The mechanical effect of the cathode rays is very readily shown by the apparatus represented in Figure 267. Let *A* represent the anode and *K* the cathode of a Crookes tube of the form shown in the diagram. The cathode rays will then stream across from *K* toward *A* as indicated by the arrows. If a wheel having light vanes is mounted as indicated in the diagram so that the cathode rays impinge upon these vanes, the wheel will be set in rapid rotation, thus demonstrating the mechanical effect of the cathode rays.

The heating effect of the cathode rays is shown by means of a tube like that represented in Figure 268. *A* is the anode and *K* a concave cathode. *P* is conveniently a piece of platinum supported at the point at which the cathode rays are concentrated. When such a tube is connected to a powerful induction coil, the piece of platinum *P* becomes strongly heated. It may be even melted by the action of the cathode rays.

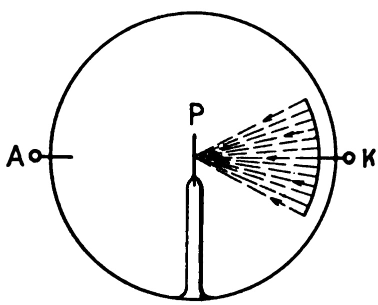

FIG. 268. — Heating Effect of Cathode Rays.

The production of X-rays is demonstrated conveniently by the use of a tube like that shown in Figure 269, which is known as a focus tube. The

bulb of this tube is nearly spherical, having extensions on opposite sides through which the anode and cathode are introduced. In the tube represented the anode A is supposed to terminate at the center of the bulb in a small sheet of platinum which is placed at an angle of 45° to the axis of the tube.

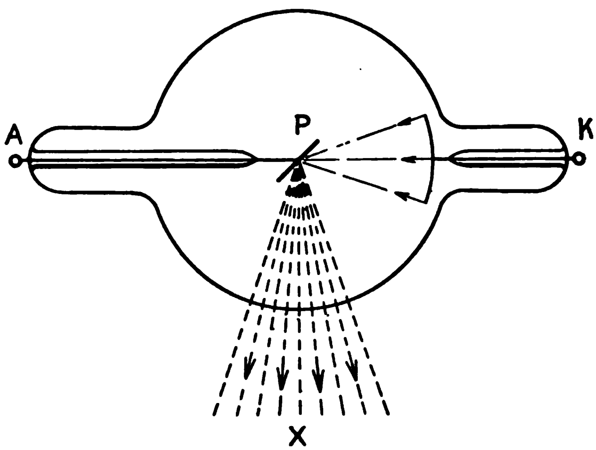

FIG. 269. — X-ray Tube.

The cathode rays are concentrated upon this piece of platinum when the discharge is passing. With this arrangement and a Crookes vacuum in the tube, it is found that, when the discharge passes, P becomes the source of what is known as the X-rays or Roentgen rays. The X-rays under the assumed conditions are given off in greatest abundance in the directions indicated by the arrows X.

OTHER CHARACTERISTICS OF THE CATHODE RAYS

401. In addition to the characteristics mentioned above it has been demonstrated that the velocity of the cathode rays is about one tenth the velocity of light (2.8×10^9 centimeters per second). It has also been demonstrated that the mass of the charged particles which are assumed to make up the cathode rays is very small as compared even with the atom of hydrogen, being equal in mass to about one-thousandth that of the hydrogen atom. The demonstration of this fact has led to the

belief that the atom, which hitherto had been regarded as the smallest portion of matter, is, in reality, made up of a great many small particles. These smaller parts of the atom are called **corpuscles** or **electrons.**

A stream of cathode rays may be deflected by a magnet. If the tube represented in Figure 267 is placed in a strong magnetic field of such direction that the lines of force pass perpendicular to the paper in the diagram, the cathode rays, instead of passing in straight lines from K toward A, will be curved toward the top of the tube or toward the bottom of the tube, depending upon the positive direction of the magnetic field.

CANAL RAYS

402. If a perforated cathode is used in a highly exhausted tube, luminous streams may be observed to emerge from the perforations, and pass in a direction opposite to that of the cathode rays. These rays have been called **canal rays.** They are capable of producing phosphorescence, and may be deflected by a magnetic or an electric field. They have been shown to consist of positively charged particles, their masses being of the same order of magnitude as that of the hydrogen atom.

X-RAYS

403. The principal effects which characterize the X-rays are the following: They do not seem to be regularly reflected or refracted. They cannot be focused by a lens. In these respects they seem to be different from ordinary light. They cannot be deflected by a magnet. In this respect X-rays differ from cathode rays. They will affect a photographic plate. In this respect they are like cathode rays and ordinary light. They are capable of penetrating considerable thicknesses of solid matter, opaque to ordinary light. Thus the X-rays will pass through a piece of wood several centimeters in thickness. They will also pass through comparatively thick layers of aluminum and ebonite. They will also pass through thin layers of such metals as zinc and iron and lead, layers which are quite opaque to ordinary light waves. They will pass

through flesh and bone. They give rise in certain bodies to very strong fluorescence. In this particular they are also like the cathode rays. X-rays also produce strong ionizing effects.

The real nature of X-rays is not clearly understood, but most of the phenomena to which they give rise may be explained on the assumption that they consist of irregular pulses in the ether.

THE SKIAGRAPH

404. If a layer of material which fluoresces strongly under the action of the X-rays is placed in some such position as *X*, Figure 269, and the hand is placed between the focus tube and the layer of fluorescing material, a "shadow picture" of the hand will be produced upon this fluorescent layer, since the flesh of the hand will intercept, in part at least, the X-rays which would otherwise fall upon the fluorescent layer. The bones of the hand, being more opaque than the flesh, will cast deeper shadows and may be seen clearly outlined upon the fluorescent screen.

A substance which lends itself very readily to use as a fluorescent material is the chemical compound known as the platinocyanide of barium. The tungstate of calcium is also employed for this purpose.

If a sensitive plate such as is used in photography be used in place of the fluorescing screen, a permanent shadow picture or "skiagraph" may be obtained by developing the plate in the usual manner, after an exposure before the X-ray tube.

RADIOACTIVITY

CHAPTER XXXIV

BECQUEREL'S DISCOVERY

405. In 1896 Becquerel discovered that a certain compound of uranium emitted a radiation which produced an effect upon a photographic plate similar to that produced by X-rays. Further study showed that this property was possessed by other uranium compounds and by the element itself. It was also found that the action upon the plate was independent of the nature of the uranium compound employed, and was determined solely by the quantity of uranium present. Becquerel also showed that this radiation was capable of discharging electrified bodies.

It was later shown by Rutherford that this discharging of electrified bodies was due to the ionizing action of the radiation. The following experiment may be performed for showing the ionization due to the radiation from uranium: Let *B*, Figure 270, represent a battery, one terminal of which is connected to a metal plate *a*, the other terminal being connected to ground. Another metal plate *b* is placed a few centimeters above *a* and is connected to ground through an electrometer or sensitive instrument for detecting electrostatic charge. Under ordinary conditions, *b* will not acquire a permanent charge, since it is insulated from *a* and the connections in *E* are such as to insulate it from the

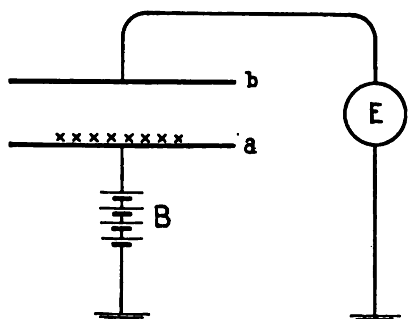

FIG. 270.

ground. **If a small quantity of uranium compound is sprinkled upon *a*, *b* will acquire a charge of the same sign as that on *a*.** That is, if *a* is connected to the positive pole of the battery, *b* will acquire a positive charge. If *a* is connected to the negative pole of the battery, *b* becomes negatively charged. This is exactly that which would be expected if the air between the plates were ionized.

RADIOACTIVE SUBSTANCES

406. The study of uranium compounds led to the belief that this radiating property is characteristic of uranium, and that its radiation is not due to any outside cause but is emitted spontaneously. This property of emitting radiations of the character described above is called radioactivity. A substance capable of emitting such radiations is called a **radioactive substance.**

The discovery of the radioactivity of **uranium** led investigators to examine other substances for the same property. It was found that **thorium** and its compounds also possess this property, though perhaps in smaller degree. Later it was found by M. and Mme. Curie that certain specimens of pitchblende showed a higher degree of radioactivity than uranium itself. This led to a more careful study of pitchblende, as a result of which two new substances, **polonium** and **radium,** were discovered. Polonium was found to differ from uranium in this important respect, that while the radioactivity of uranium is constant, that of polonium gradually becomes less as time passes.

Radium, the most active of the known radioactive substances, is an extremely rare element, although found in minute quantities in various minerals from different parts of the world. The chief source of radium is the pitchblende of Bohemia. Several tons are required even of this substance to yield a small fraction of a gram of radium. It is usually prepared in the form of radium bromide. In this compound it is highly active. It is phosphorescent and causes other substances like calcium tungstate, platino-barium cyanide, and willemite to phosphoresce.

Another radioactive substance, **actinium**, was discovered by Debierne. Pitchblende is the source of this substance also. It is similar to thorium in its chemical nature, but is much more active.

Radium is by far the most active of all the radioactive substances. Its radioactivity is at least a million times as great as that of uranium. A few milligrams of radium bromide will produce powerful photographic and ionization effects and will render a fluorescent screen brilliantly luminous. Its radioactivity is so great that it is really dangerous to handle because of its painful effects upon the skin when the exposure lasts for an appreciable length of time.

THREE KINDS OF RADIATION

407. The radiation emitted by radium is complex and may be separated into three distinct parts. These parts are called the α-rays, the β-rays, and the γ-rays. If a slender stream of radiation from radium is passed through a strong electric field, a portion will pass on unchanged in direction (γ-rays), two other portions will be deflected (α and β-rays), the one being deflected in a direction exactly opposite to that of the other. It was this fact that led to the separation of the α- and β-rays and their identification. The arrangement of the apparatus for this experiment is shown in Figure 271. A quantity of radioactive material R is placed beneath a heavy plate of lead having a small hole H at its center. By this means a slender stream of rays passing vertically upward above the plate is secured. Metal plates P and N are arranged as shown. P is strongly charged positively and N negatively. pp is a photographic plate arranged to receive the rays and show their separation. When the experiment is performed, the developed

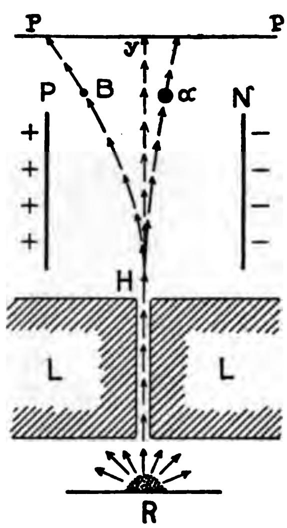

FIG. 271.

plate shows that a portion of the rays are undeviated. This portion consists of γ-rays. Another portion is strongly deviated towards the positive plate *P*. This portion consists of β-rays. A third part is slightly deviated towards the negative plate *N*. This part is composed of α-rays. The rays may also be separated by a strong magnetic field. This may be effected by an arrangement of apparatus like that of Figure 272, which is similar to that shown in Figure 271 except that strong magnet poles *N* and *S* are substituted for the charged plates. In this experiment the deflection of the rays is at right angles to the magnetic field, that is, perpendicular to the plane of the paper. The α-rays are deflected away from the β-rays toward the reader.

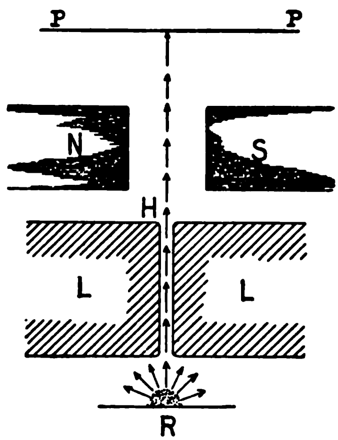

FIG. 272.

THE α-RAYS

408. The α-rays consist of positively charged atoms (α-particles) of matter. They are projected from the radioactive substance at a velocity of something like 20,000 miles per second. Their principle characteristics are their powerful ionizing action and their inability to penetrate (that is, carry their ionizing action) to a distance of more than two or three inches in air. They are completely absorbed by a sheet of aluminum 0.01 centimeter thick. The mass of the α-particle is about twice that of the hydrogen atom.

THE β-RAYS

409. The β-rays consist of electrons (β-particles) and carry, of course, negative charges. They leave the radioactive substance with a velocity nearly as great as that of light (186,000 miles per second). They produce ionizing effects, but are very much less powerful in this respect than the α-particles. Their penetration is much greater than that of the α-particles, their ionizing action being lost only after they have travelled several

feet in the air. A layer of aluminum 0.5 centimeter thick will absorb most of the β-particles. The mass of the β-particle is about $\frac{1}{800}$ that of the hydrogen atom.

THE γ-RAYS

410. The γ-rays consist in all probability of very abrupt waves in the ether. They seem to be identical in character with Roentgen rays. They are characterized by the fact that they cannot be deviated by an electric or magnetic field and do not carry charge. They have the property of ionizing a gas and a penetration exceeding that of the most penetrating X-rays.

If an X-ray tube is operated at low pressure (high vacuum) it gives very penetrating X-rays, sometimes called "hard" X-rays. These hard X-rays and β-rays are much alike. It is known that X-rays are produced by the bombardment of a solid by cathode rays, that is, by moving electrons. In other words, X-rays are produced by the sudden stopping of moving electrons. It seems reasonable to suppose that they would also be produced by the sudden starting of electrons. Now in a radioactive substance atomic explosions are supposed to be continually taking place, as a result of which α- and β-particles are projected from the substance. The resulting ether disturbances may be thought of as being somewhat like those produced by suddenly bringing moving electrons to rest, and the γ-rays as the resulting wave disturbance in the ether. This view is strengthened by the fact that γ-rays always occur in conjunction with β-rays.

MASS AND VELOCITY OF α- AND β-PARTICLES

411. The method used in determining the mass and velocity of an α- or β-particle will be understood from the following analogy: Consider the case of a cannon ball fired horizontally. The conditions are, an initial velocity v in a horizontal direction and a constant force (the weight of the ball) acting at right angles to v. The ball describes a curve AB, Figure 273. That is, while traveling a distance D horizontally it falls a distance d. Let t be the time of flight from A to B. Then $D = vt$ and

$d = \frac{1}{2}gt^2$. Combining these expressions and eliminating t, we have

$$v^2 = \frac{gD^2}{2d} \tag{106}$$

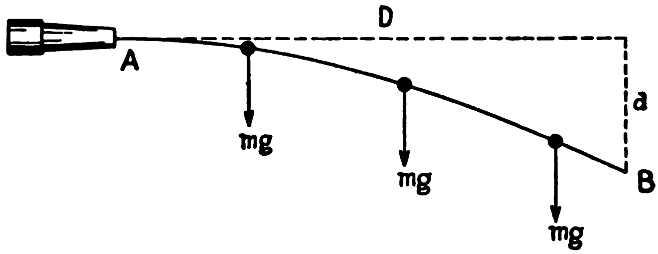

FIG. 273.

from which v may be obtained, if corresponding values of D and d are known.

Now consider an α- or β-particle projected at right angles to an electric field. Let P and N, Figure 274, represent two metal plates charged respectively with positive and negative electricity. Let the dotted line represent the direction of the velocity v with which the charged particle enters the electric field between P and N. The particle will be acted upon by a constant force F at right angles to the field, which is equal to the product of the charge q carried by the particle and e, the intensity of the electric field. Under these circumstances the particle will describe a curve similar to that described by the cannon ball in the case above considered.

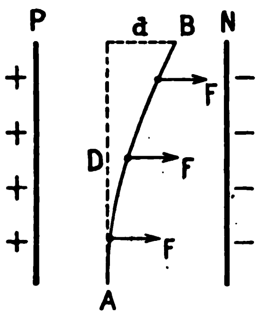

FIG. 274.

In a given time t the particle will travel a vertical distance D, such that $D = vt$. In the same interval it will travel a distance d horizontally, such that $d = \frac{1}{2}at^2$, in which a is the uniform acceleration in the direction of the electric field produced by the action of the field upon the charged body. From these relations, we obtain, as above,

$$v^2 = \frac{aD^2}{2d} \tag{106 bis}$$

Now the force acting on the charged particle is $F = qe$, as pointed out above. Hence (since $F = ma$), we have

$$a = \frac{qe}{m}$$

Substituting in the expression for v^2, we obtain,

$$v^2 = \frac{qeD^2}{2\,dm} \tag{107}$$

from which we may find the value of v, if q, e, and m are known and D and d are observed.

This relation may also be written in the form

$$\frac{m}{q} = \frac{eD^2}{2\,dv^2} \tag{108}$$

from which we may find the value of $\frac{m}{q}$ when e and v are known and d and D are observed.

In the case above considered, the charged particle describes a parabola. This form of path is due to the fact that the deflecting force is constant in magnitude and direction. If the charged particle is deflected by a magnetic field, it describes a circular path, since in this case the deflecting force is constant in magnitude **but continually changing in direction.** A moving charge is equivalent to an electric current, and the deflecting force is perpendicular to the field and to the path of the moving charge. Let the broken line, Figure 275, represent the direction in which the charged particle is moving as it enters the magnetic field. Assume that the magnetic field is perpendicular to the paper, the lines of force being represented by the dots in the figure. The path of the moving charge is then circular, the force F being constant in direction and always perpendicular to the path. The value of F is qvf, in which q is the charge, v its velocity, and f the intensity of the magnetic field. The acceleration of the particle in the

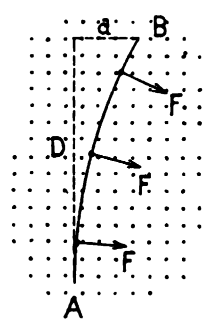

FIG. 275.

direction F is, therefore, $\frac{qvf}{m}$. But in uniform circular motion the radial acceleration is $\frac{v^2}{r}$. Hence,

$$\frac{qvf}{m} = \frac{v^2}{r}$$

But

$$r = \frac{D^2}{2\,d},$$

since AB is a circular arc, of which d is the sagitta and D the half-chord.

Therefore,

$$\frac{qvf}{m} = \frac{2\,dv^2}{D^2}$$

whence,

$$\frac{m}{q} = \frac{fD^2}{2\,dv} \tag{109}$$

Now let it be imagined that a moving charged particle is acted upon by an electric field and a magnetic field at the same time. Let it be assumed that the fields are at right angles to one another and so related that **the deflecting force of the magnetic field is equal to that of the electric field and opposite in direction.** Under these conditions the charged particle will pass through the electric and magnetic fields **without** deviation.

For this case, we have

$$qvf = qe$$

or,

$$v = \frac{e}{f} \tag{110}$$

That is, the velocity of the moving particle is given by the ratio of the intensities of the electric and magnetic fields, when these intensities are so related that they produce no deflection of the moving particle.

It has been found possible to carry out an experiment of this kind, and by this means to determine v, the velocity of the moving particle. When v has been determined, its value may be used in Equations (108) or (109), and the value of $\frac{m}{q}$ may be obtained. Finally, there is reason to believe that the charge q carried by an electron is the same as that carried by an hydrogen ion in electrolysis, **and this charge is known.**

It, therefore, becomes possible to determine from $\frac{m}{q}$ the value of m, the mass of the charged particle. The mass and velocity of the α- and β-particles given above have been determined in in this manner.

URANIUM X AND THORIUM X

412. It has been shown by Crookes that by a chemical process there can be separated from uranium a substance which is much more active photographically than the uranium from which it is derived, and that when the separation is made it leaves the uranium without photographic activity. The active substance separated from uranium in this manner is called uranium X.

Uranium is active photographically because of the β-rays which it gives off. The uranium left behind when uranium X is separated is active electrically but inactive photographically, because it gives off α-rays only.

Now it is found that after the separation is made the uranium X gradually loses its activity, while that of the uranium is gradually regained. Further, the rate of decay in uranium X exactly equals that of recovery in the remaining uranium. In all discussions of decay or recovery in radioactive substance, the time required for one half the change to be completed is called the period. Thus the period of decay for uranium X is equal to that of recovery for the remaining uranium. This fact leads to the conclusion that they are intimately associated. It points, in fact, to the probability that uranium X is continually being formed in uranium, the constancy of the radiation of the unseparated substance being accounted for by the fact that the quantity of uranium X present is such that its rate of decay just equals its rate of formation. If this view is correct, it ought to be possible after the remaining uranium has stood for a time to again separate from it uranium X. This is found to be the case.

Other experimenters have succeeded in separating from thorium a substance similar to uranium X which has been called thorium X. The period of thorium X is about 4 days,

while that of uranium X is about 22 days. Thus within uranium and thorium a process of transformation is continually going on accompanied by the formation of uranium X or thorium X. Similar processes have been shown to be continually taking place in radium and actinium.

EMANATION

413. Radium, thorium, and actinium have been found to give off a kind of gas which is called **emanation.** It is not affected by an electric field, hence does not consist of charged particles. It differs from an ionized gas in that it does not lose its ions in those processes which remove the ions from an ordinary gas. Its gaseous nature, however, is proved by the fact that it can be condensed. It condenses at a temperature of $-150°$ C.

When emanation is separated from thorium, the activity of the emanation dies away, while that of the thorium rises, much as in the case of uranium and uranium X. The period of thorium emanation is about 54 seconds. That of radium emanation is about 3.7 days.

RADIOACTIVE TRANSFORMATIONS

414. Many facts developed by experiment, of which those mentioned in the discussions of uranium X, thorium X, and emanation are illustrations, have led to the formulation of the **theory of successive changes.** This theory states that radioactive substances are gradually undergoing a process of transformation by which they are changing from one product to another. These changes are spontaneous and independent of all outside influences.

A large number of radioactive products are now known. The transformations of these products have been studied and their relations definitely established. Below is given a table of the radioactive products of uranium, together with their transformation periods and the rays emitted by them. Similar series have been developed for thorium and actinium.

Radioactive Products	Transformation Period	Nature of Rays Emitted
Uranium	5×10^9 years	α
Uranium X	22 days	β and γ
Ionium	?	α
Radium	2000 years	α
Emanation	3.7 days	α
Radium A	3 minutes	α
Radium B	26 minutes	α
Radium C	19 minutes	α, β, and γ
Radium D	40 years	No rays
Radium E	6 days	No rays
Radium F	45 days	β and γ
Polonium	140 days	α

The list of substances given in the first column of the table are to be understood as radioactive products of uranium. Thus uranium changes to uranium X, uranium X to ionium, ionium to radium, etc. Radium and polonium are thus seen to be evolved from uranium and appear at certain definite stages of its radioactive transformation. Actinium and thorium, with their transformation products, are apparently to be regarded as distinct families of radioactive substances. Recent experiments have shown, however, that there is some relation between actinium and uranium, although neither actinium nor any of its radioactive products appear in the direct line of descent set forth in the above table. Actinium and its products may perhaps be regarded as an offshoot of the uranium family. No such connection has thus far been established for thorium, so that from the standpoint of our present knowledge, this element with its products must be regarded as a distinct and independent radioactive group.

PART IV

SOUND

SOUND

CHAPTER XXXV

WAVES

415. The general topics of sound and light, which are discussed in the following pages, have to deal with phenomena of wave motion. Sound in the general sense consists of air waves or waves in some other material substance. In the same way, light is a wave phenomenon, the medium in which the waves are formed in this case being the ether. In view of these facts, it is convenient before taking up these subjects in detail to make a general study of wave phenomena and particularly of the laws governing the production and propagation of waves. Such a study is useful, not only because of its bearing on the subjects of sound and light, but also because of the knowledge it gives of other phenomena which partake of the nature of wave motion.

WATER WAVES

416. Every one knows that the surface of a body of water exposed to the wind is covered with waves. It is also generally known that such waves are sometimes large and sometimes small, and that they travel over the surface of the water at varying speeds. Water waves may also be produced by dropping a stone into a body of still water, or by moving an object rhythmically up and down at the water surface.

However they may be formed, we observe that certain effects always accompany water waves. We know, for example, that at the crest of the wave a certain mass of water is lifted above the normal level, while at a trough the surface is lower than normal. This means that energy is involved in such wave motion, and the important fact suggested is that wave motion affords a means of transferring energy.

A system of waves involves not only the potential energy suggested above, but kinetic energy as well. Consider the water waves represented in Figure 276. The vertical displacements at the crests *C* and the troughs *T* constitute the potential

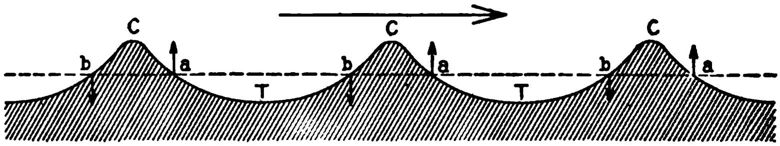

FIG. 276. — Water Waves.

energy effects. Now, in front of each crest, at such points as *a*, the water is rising. Behind the crest, at such points as *b*, the water is falling. Furthermore, at crest and trough there is horizontal motion. It will be evident, therefore, that all these portions possess kinetic energy.

In such waves the **disturbance** consists in the elevation of certain portions of the surface water above the normal level and the depression of certain other portions. Gravity tends to restore equilibrium (normal level). But the rising and falling masses, because of their inertia, pass and repass their equilibrium positions, and thus the wave motion is maintained. Nevertheless, as the motion continues, energy is absorbed by the water, due to friction (viscosity) effects, and, if the supply of energy is withdrawn, the wave motion gradually ceases. The gradual dying away of waves under such circumstances is called damping, and the gradually decreasing waves are said to be damped.

ENERGY OF SOUND AND LIGHT WAVES

417. As in the case of water waves, light waves and sound waves involve energy and afford a means of energy transfer. Sound waves consist of alternate compressions and rarefactions in the transmitting medium. These compressed and stretched portions of the medium have potential energy much like that possessed by a compressed or stretched spring. Those parts of the medium which lie between the compressions and rarefactions are in motion, and therefore have kinetic energy. Light waves also represent energy. A light wave, as it travels through the

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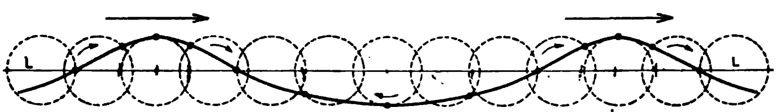

FIG. 277.

In shallow water the particles move in ellipses which become flatter and flatter as one approaches the bottom.

THE RELATION BETWEEN VELOCITY, WAVE LENGTH AND FREQUENCY

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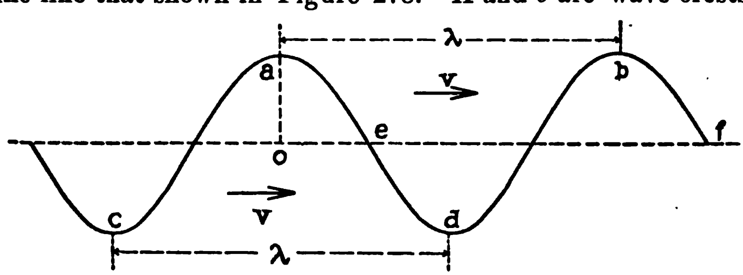

FIG. 278.

c and *d* are troughs. Evidently, a complete wave contains a crest and a trough. **The *wave length* of a wave is the distance from any point on a wave to the corresponding point on the next wave.** For example, it is the distance from *a* to *b*, from *c* to *d*, or from *e* to *f* in the figure. The **amplitude** of a wave is the distance between the crest or trough and the mean position of the wave. *Oa* is the amplitude of the wave shown in Figure 278.

Let it be imagined that the wave represented in Figure 276 is traveling toward the right. A given particle which rises upon the successive crests and sinks into the successive troughs, as the system of waves passes it, evidently makes a complete to and fro excursion for every complete wave that passes the particle. Suppose that n complete waves pass the given particle in one second. The **frequency** of the disturbance is then said to be n. Evidently, the time required for one wave to pass the particle in question will be the nth part of a second. This is called the **period** of the disturbance. Let T represent this period. Then,

$$T = \frac{1}{n}$$

The distance through which the disturbance (wave) moves toward the right in the time T is evidently the length of one wave. Call the wave length λ. It follows at once that

$$v = \frac{\lambda}{T}$$

or,

$$v = n\lambda \qquad (111)$$

RIPPLES

420. Water waves having a length of a few millimeters only are called ripples. Surface tension has a great deal to do with the formation and propagation of ripples. A moment's consideration will show that surface tension acts in much the same way that gravity does to restore equilibrium when the surface of a liquid is broken into waves. For long waves, the gravity effect predominates, while for ripples, surface tension produces

the larger effect. The propagation of waves a few centimeters in length depends partly on gravity and partly on surface tension.

THE VELOCITY OF WATER WAVES

421. The velocity of propagation of a water wave can be shown to be

$$v = \sqrt{\frac{g\lambda}{2\pi} + \frac{2\pi T}{\lambda\rho}} \tag{112}$$

where g is the acceleration of gravity, λ the wave length, T the surface tension, and ρ the density of the liquid. Evidently, for very long waves, the second term under the radical becomes negligible in comparison with the first. This equation is based upon the assumption that the depth of the water is great as compared with λ. In shallow water the velocity of a water wave is less than in deep water.

STATIONARY WAVES

422. An important effect is produced by **two waves of the same wave length and amplitude which are traveling in opposite**

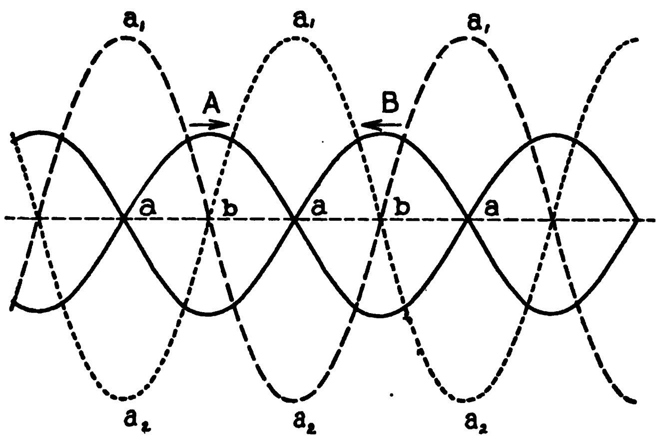

FIG. 279.

directions through the same region. The effect of such a combination of waves will be understood by reference to Figure 279. Let A represent a wave traveling toward the right, B a

similar wave traveling toward the left. Consider the combined effect of these two waves upon the particle *a*. Evidently this particle under the conditions represented in the figure is rising on the crest of each wave, that is to say, it is being lifted by the crest of the wave *A*, which is approaching it, and it tends at the same time to rise on the crest *B*, which is approaching from the opposite direction. Since, supposedly, these waves are traveling with the same velocity, the crests *A* and *B* will reach the particle *a* at the same instant. Since the particle *a* responds to each wave, independent of the presence of the other, it will be elevated to a greater height than it would be if moving under the influence of either wave alone. At the instant, therefore, that the crests *A* and *B* reach the particle *a* it will occupy some such position as a_1. Again, when two troughs reach the particle *a* it will sink to some such point as a_2.

Considering now the particle *b*, it will be evident that at the moment in which the conditions represented in the diagram are supposed to exist, the particle *b* will remain in its mean position. That is to say, it will be displaced neither up nor down, since the effect of the wave *A* would be to raise it onto a crest, while the effect of the wave *B* is to sink it into a trough. Under the combined influence of the two waves, the particle *b* therefore remains at rest. A moment's consideration will show that this particle is at all times at rest, since at any instant it is elevated by the one wave to exactly the same distance that it is lowered by the other. In other words, **the particles *b* remain stationary at all times, while the particles *a* are displaced through amplitudes much greater than those of the individual waves. Particles lying between *a* and *b* vibrate to and fro through amplitudes which are small for those particles lying near *b*, and large for those particles lying near *a*. The dotted lines in the figure represent the stationary wave which results from the combination of the waves *A* and *B*.** The stationary points *b* are called **nodes**, and the regions midway between the points *b* are called **loops**.

The conditions necessary for the production of **stationary** waves are:

1. The component waves must be of the same wave length.

2. They must have the same amplitude.
3. They must be traveling in opposite directions.

A simple illustration of stationary waves is the following. Let AB, Figure 280, represent a rubber tube or a slender spiral spring, the end A being fastened to a hook in the vertical wall, as indicated in the diagram, the end B being grasped in the hand. If the end B is now moved up and down with a regular motion,

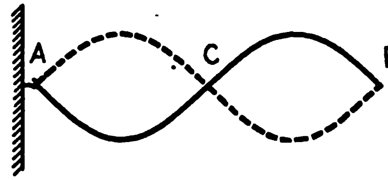

FIG. 280.

properly timed with respect to the length of the tube AB, stationary waves will be established. In one of the simplest cases there will be a stationary point C at the center of the tube in addition to the points A and B, which are to be regarded as stationary, since B moves through a small distance only. The portions between A and C, and C and B will vibrate so that the tube momentarily assumes the form indicated by the heavy line ACB, and a moment later, the form indicated by the dotted line ACB. Evidently, this wave is a stationary wave, since the crests of the wave always appear at the same place, and do not travel in the direction A to B, or B to A. The two component waves which are necessary to make up this stationary wave are the wave sent out from the hand toward the wall, and the reflected wave returning toward the hand from the fixed point A. By making the to and fro motion of the hand more rapid, it is possible to cause the tube AB to vibrate in three segments. In this case there will be four stationary points, including A and B. If the to and fro motion is made still more rapid, the tube may be caused to vibrate in four segments, in which case there will be five stationary points, including A and B, etc.

Stationary waves may also be produced in a trough of water. Let Figure 281 represent a long narrow glass vessel or trough filled with water to the line ef. Any disturbance of the level will result in waves which will travel to the ends of the trough and will there be reflected. By disturbing the water in the proper manner it will be found possible to establish stationary

water waves in the vessel. Let it be assumed, for example, that the water is so disturbed that it rises at the center of the trough and falls simultaneously at both ends, so that the surface of the water assumes the form indicated by the broken line. If now left to itself, the water which is heaped up at the center of the tank will flow right and left toward the ends, the tendency being toward a restoration of the general level *ef*. The inertia of the water carries it beyond the position of equilibrium so that it rises above *e* and *f* at the ends of the trough, and sinks below the level *ef* at the center, assuming momentarily the form indicated by the dotted line. A return current will now set in, water flowing from both ends of the trough toward the center. The surface of the water will again assume the form shown by the broken line. Evidently this motion of the surface of the water constitutes a stationary wave, since there are two points *a* and *b* for which there is no vertical motion of the water. These are the nodes of the stationary wave. The regions *e*, *f*, and *g* constitute vibrating segments. It is interesting to note in this connection that *a* and *b* are regions of maximum horizontal motion, while the regions *e*, *f*, and *g* have no horizontal motion. This stationary wave in the trough is very readily maintained by dipping a paddle vertically into the water at *a* or *b* and imparting to it a horizontal vibratory motion in the direction *ef*, the period of the vibratory motion being, of course, determined by the length of the vibrating body.

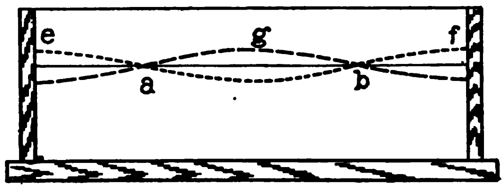

FIG. 281.

REFLECTION

423. When a system of waves strikes a rigid obstacle, it is turned back or reflected from the obstacle. If the reflected train of waves is traveling in a direction exactly opposed to that of the incident wave train, stationary waves will result.

This would be the case when the incident wave train is moving at right angles to the reflecting surface of the obstacle. If the angle of incidence (angle between the wave front and the surface of the obstacle) is something other than 0°, the reflected train will move off as a new and independent train.

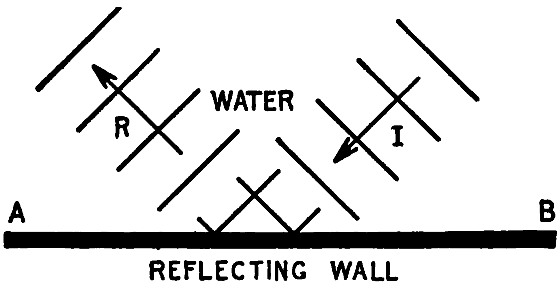

FIG. 282.

Figure 282 shows the manner in which a train of water waves is reflected by a rigid wall. The direction in which the incident waves are traveling is shown by the arrow *I*. The reflected wave train is moving in the direction given by the arrow *R*. *AB* is the reflecting wall. The angle between a wave front of the incident wave train and the wall (angle of incidence) is equal to the angle between a wave front of the reflected train and the wall (angle of reflection).

REFRACTION

424. When a wave train passes into a medium in which its velocity is changed, it is said to be refracted. Thus a system of water waves is refracted when it passes from deep to shallow water. If the angle of approach to the shallow water is such that one edge of the wave enters before the other, the wave will change its direction as it passes into the

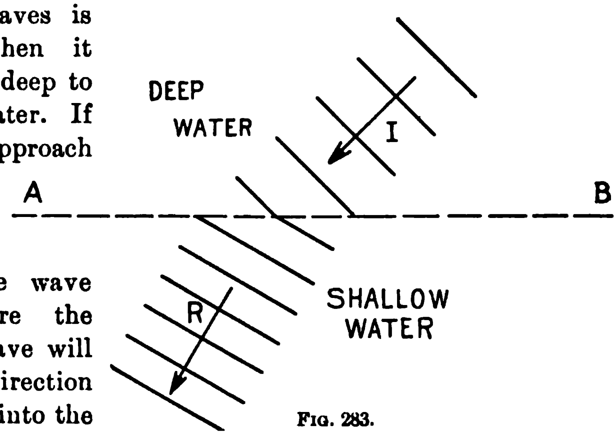

FIG. 283.

PART IV

SOUND

SOUND

CHAPTER XXXV

WAVES

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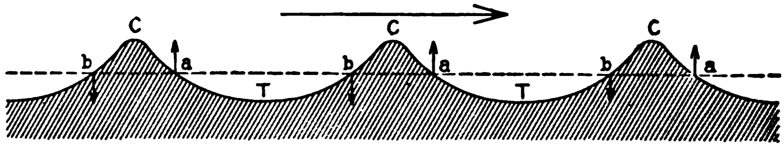

Fig. 276. — Water Waves.

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ENERGY OF SOUND AND LIGHT WAVES

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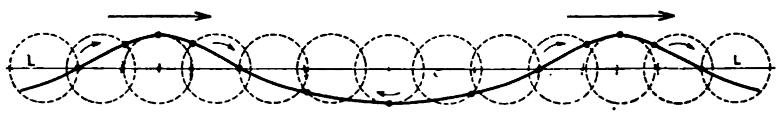

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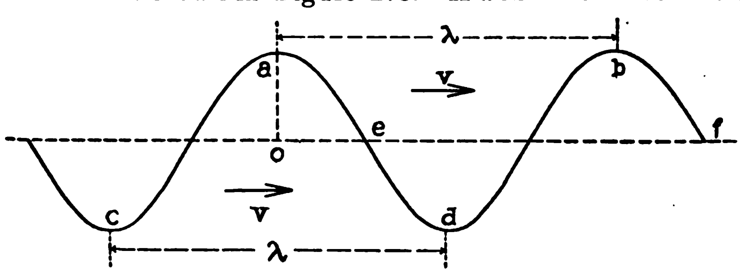

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$$T = \frac{1}{n}$$

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RIPPLES

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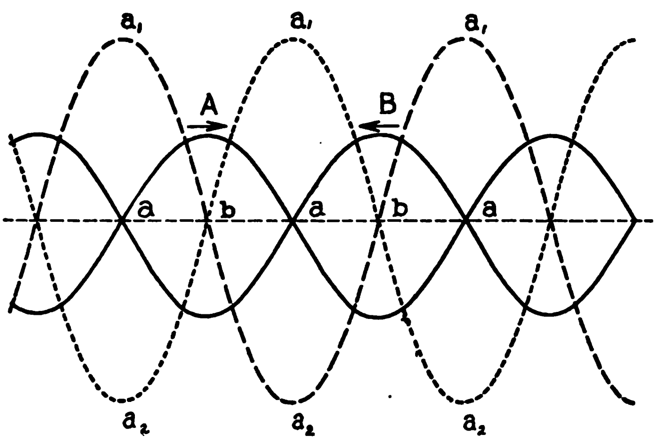

Fig. 279.

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The conditions necessary for the production of **stationary** waves are:

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A simple illustration of stationary waves is the following. Let AB, Figure 280, represent a rubber tube or a slender spiral spring, the end A being fastened to a hook in the vertical wall, as indicated in the diagram, the end B being grasped in the hand. If the end B is now moved up and down with a regular motion, properly timed with respect to the length of the tube AB, stationary waves will be established. In one of the simplest cases there will be a stationary point C at the center of the tube in addition to the points A and B, which are to be regarded as stationary, since B moves through a small distance only. The portions between A and C, and C and B will vibrate so that the tube momentarily assumes the form indicated by the heavy line ACB, and a moment later, the form indicated by the dotted line ACB. Evidently, this wave is a stationary wave, since the crests of the wave always appear at the same place, and do not travel in the direction A to B, or B to A. The two component waves which are necessary to make up this stationary wave are the wave sent out from the hand toward the wall, and the reflected wave returning toward the hand from the fixed point A. By making the to and fro motion of the hand more rapid, it is possible to cause the tube AB to vibrate in three segments. In this case there will be four stationary points, including A and B. If the to and fro motion is made still more rapid, the tube may be caused to vibrate in four segments, in which case there will be five stationary points, including A and B, etc.

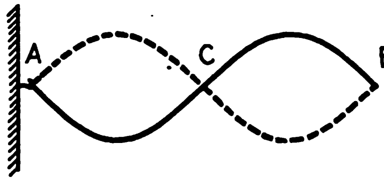


FIG. 280.

Stationary waves may also be produced in a trough of water. Let Figure 281 represent a long narrow glass vessel or trough filled with water to the line ef. Any disturbance of the level will result in waves which will travel to the ends of the trough and will there be reflected. By disturbing the water in the proper manner it will be found possible to establish stationary

water waves in the vessel. Let it be assumed, for example, that the water is so disturbed that it rises at the center of the trough and falls simultaneously at both ends, so that the surface of the water assumes the form indicated by the broken line. If now left to itself, the water which is heaped up at the center of the tank will flow right and left toward the ends, the tendency being toward a restoration of the general level *ef*. The inertia of the water carries it beyond the position of equilibrium so that it rises above *e* and *f* at the ends of the trough, and sinks below the level *ef* at the center, assuming momentarily the form indicated by the dotted line. A return current will now set in, water flowing from both ends of the trough toward the center. The surface of the water will again assume the form shown by the broken line. Evidently this motion of the surface of the water constitutes a stationary wave, since there are two points *a* and *b* for which there is no vertical motion of the water. These are the nodes of the stationary wave. The regions *e*, *f*, and *g* constitute vibrating segments. It is interesting to note in this connection that *a* and *b* are regions of maximum horizontal motion, while the regions *e*, *f*, and *g* have no horizontal motion. This stationary wave in the trough is very readily maintained by dipping a paddle vertically into the water at *a* or *b* and imparting to it a horizontal vibratory motion in the direction *ef*, the period of the vibratory motion being, of course, determined by the length of the vibrating body.

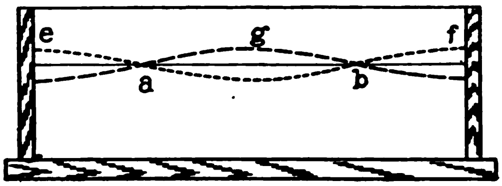


FIG. 281.

REFLECTION

423. When a system of waves strikes a rigid obstacle, it is turned back or reflected from the obstacle. If the reflected train of waves is traveling in a direction exactly opposed to that of the incident wave train, stationary waves will result.

This would be the case when the incident wave train is moving at right angles to the reflecting surface of the obstacle. If the angle of incidence (angle between the wave front and the surface of the obstacle) is something other than 0°, the reflected train will move off as a new and independent train.

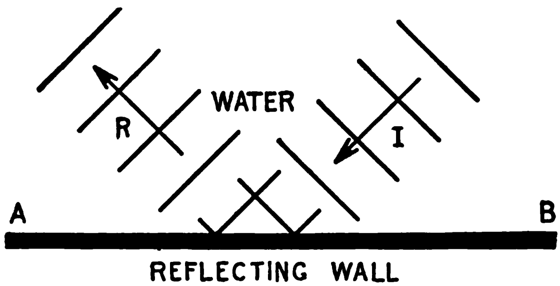


FIG. 282.

Figure 282 shows the manner in which a train of water waves is reflected by a rigid wall. The direction in which the incident waves are traveling is shown by the arrow *I*. The reflected wave train is moving in the direction given by the arrow *R*. *AB* is the reflecting wall. The angle between a wave front of the incident wave train and the wall (angle of incidence) is equal to the angle between a wave front of the reflected train and the wall (angle of reflection).

REFRACTION

424. When a wave train passes into a medium in which its velocity is changed, it is said to be refracted. Thus a system of water waves is refracted when it passes from deep to shallow water. If the angle of approach to the shallow water is such that one edge of the wave enters before the other, the wave will change its direction as it passes into the

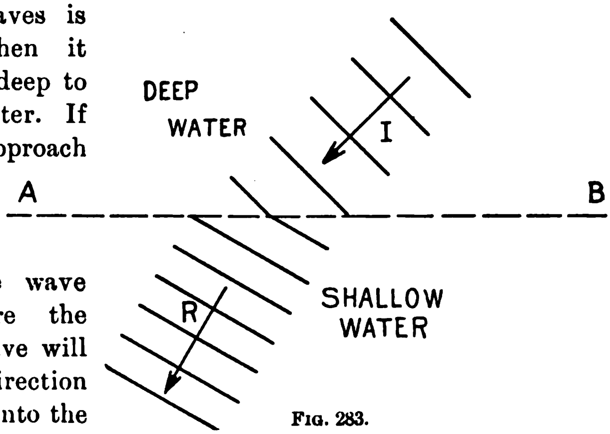


FIG. 283.

shallow water. This effect is due to the fact that the first edge to enter the shallow water is retarded, while the other edge is still traveling with its deep water velocity, and only after the first edge has fallen a certain distance behind, and the entire wave front is in shallow water, will all parts of the wave again have the same velocity.

The change of direction which a train of water waves undergoes as it passes from deep to shallow water is shown in Figure 283. The angle between a wave front of the incident train I and the line AB is the angle of incidence. The angle of refraction is the angle between a wave front in the refracted train R and the line AB. In the case illustrated the angle of refraction is less than the angle of incidence.

Problems

1. Two stakes stand 5 m. apart in a pond. A stone is dropped into the water in line with the stakes. At a certain instant it is observed that there is a crest at each stake and 5 crests between them. What is the wave length of the disturbance?

2. A crest travels from one stake of problem 1 to the other in 14.3 sec. How many times will the water at each stake rise and fall in this interval? What is the velocity of the waves?

3. The velocity of deep water waves is $v = \sqrt{\frac{g\lambda}{2\pi}}$. What is the velocity of water waves 50 cm. long on a deep pond where $g = 980$?

4. A system of water waves is moving across a pond. $\lambda = 200$ cm., $v = 56$ cm./sec. At a certain instant a crest is at a stake standing in the water. A, B, C, and D are points between the stake and the source of the waves. Their distances from the stake are 2, 5, 8.5, and 11.5 m., respectively. Is the surface of the water above or below the normal level at these points? Is it moving up or down?

5. A point moves uniformly in a circle of 10 cm. radius. The angular velocity ω of the point is 10 rad./sec. ωt is the angular space swept over by the moving radius vector of the point. Plot a curve showing the relation between corresponding values of ωt and $r \sin \omega t$, using values of ωt as abscissæ and values of $r \sin \omega t$ as ordinates.

6. What is the equation to the curve found in problem 5?

7. Plot the curve $y = 6 \sin \frac{2\pi}{3} t$.

8. Plot the curve $y = 3 \sin \frac{2\pi}{6} t$.

9. Plot the curve $y = 6 \sin \frac{2\pi}{3} t + 3 \sin \frac{2\pi}{6} t$.

NATURE OF SOUND

CHAPTER XXXVI

NOISES AND MUSICAL SOUNDS

425. The word "sound" is used in two distinct senses. It is sometimes applied to that sensation which is peculiar to the organ of hearing, and sometimes to a disturbance in the air or other elastic medium which is capable of producing that sensation. The latter or physical sense is the one in which we shall use the term in the following discussions. **Hence, sound is a disturbance in an elastic medium, of such nature that when it falls upon the ear it is capable of exciting the auditory nerve.**

There are two kinds of sound, namely, musical tones and noises. **A musical tone is defined as a regular succession of similar disturbances, or a series of disturbances occurring at regular intervals.** A musical tone is produced, for example, by the to and fro vibrations of the prong of a tuning fork. **All sounds other than musical tones are called noises.** A noise may consist of a single disturbance or pulse, such as accompanies the firing of a gun, or a series of irregular disturbances, such as that caused by the tearing of cloth, or the scraping of a foot upon the pavement.

THE MANNER IN WHICH SOUND IS PRODUCED

426. Experience teaches that musical tones proceed from vibrating bodies, and that vibratory motion in a body is necessary in order that it may produce such a sound. If a violin string is examined while sounding, it will appear blurred at the center and spread out into a spindle-like form which indicates that the center of the string is swinging rapidly to and fro. If the prong of a tuning fork which is sounding is closely examined, it will be observed that the end appears blurred which is occasioned by the rapid to and fro motion of the prong.

This vibratory motion of a sounding body, being imparted to an elastic medium, the air, for example, sets up a wave motion in that medium. This motion, traveling forward through the medium and falling upon the auditory nerve, produces the sensation of sound. The manner in which this wave motion is established will be readily understood from the various analogies with which we are already familiar. Consider, for example, the wave motion on the surface of a pond of water. If a stone is dropped into the still surface of the water, a depression is formed in this surface. This depression is surrounded by an elevation. Gravity tends to restore the equilibrium, or what amounts to the same thing, a uniform level of the liquid, but the effect of gravity in restoring the level at the center of disturbance results in a disturbance of the level at points a little farther out. These again, in being restored to their original positions, disturb those still farther out, and so on. That is to say, a disturbance passes in all directions over the surface of the still water from the center of disturbance, that is, the point at which the stone falls into the water. In a similar manner a disturbance may be created in the air. This disturbance, because of the tendency of the medium to restore itself to its original condition of equilibrium, travels outward in all directions. Similarly, the ether may be disturbed at a given point and a wave motion or pulsation spread in all directions from the point of disturbance, owing to the tendency of the medium to return to the unstrained condition, or condition of equilibrium, at the point at which the disturbance occurs.

As a matter of fact, the three kinds of wave motion just mentioned differ from one another in very important respects; but the reason for the spreading of the disturbance, that is, the propagation of the wave motion, is the same in each case.

THE MEDIUM OF PROPAGATION OF SOUND WAVES

427. A material medium is necessary to the transmission of sound waves. The ether transmits light waves, heat waves, and electric waves, as has already been pointed out. The ether does not transmit sound waves. That a material medium is necessary to the propagation of sound waves may be demon-

strated in the following manner. If a small music box is placed under the receiver of an air pump, it will be observed that so long as the receiver is filled with air the musical sounds proceeding from the music box may be distinctly heard. Upon pumping the air from the receiver the sounds grow fainter and and fainter until all of the air is removed. If it were possible to completely remove the air from the receiver, the sounds of the music box would be no longer audible. Evidently from the statements heretofore made with reference to the ether, this medium fills the exhausted receiver; and if it were possible for this medium to transmit sound waves, we should still be able to hear musical sounds proceeding from the box even after the air had been pumped away.

If now, into such an exhausted receiver, oxygen, hydrogen, or any other gas is admitted, the sounds at once become audible again. In this way it might be demonstrated for each of the several gaseous bodies that each one is capable of transmitting sound.

The transmission of sound by a liquid is easily demonstrated in a variety of different ways. A person diving in the water can hear the voices of people standing on the shore. If a bell is sounded beneath the surface of the water in a lake, it may be heard by divers at great distances. A simple illustration of the principles of transmission of sound waves by liquids is the following: let *AB*, Figure 284, represent a tuning fork, to the bottom of which is attached a thin disk of cardboard *CD*. *EF* is a dish of water resting upon a table. Let it be imagined that the fork is held between the thumb and finger and set vibrating. The sound will be scarcely audible. If, however, the fork

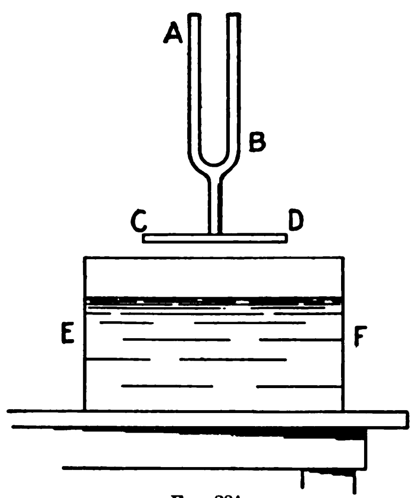


FIG. 284.

is lowered until *CD* comes into contact with the water in the vessel *EF*, the sound will be decidedly increased in loudness. If now the tuning fork, instead of being placed in contact with the water in the vessel *EF*, is placed upon the table, it will be found that the effect is the same as that produced by placing the cardboard disk *CD* in contact with the water. It is evident, therefore, that it is the contact between the fork and the table which is responsible for the increased loudness, and that, therefore, in the first experiment the water constitutes a good medium of connection between the fork and table, or in other words, that the disturbance is readily propagated through the liquid.

The propagation of sound through solids may be illustrated by the following experiment: Let *AB*, Figure 285, represent a suitable sounding board, for example, a door or a table top. Let *CD* represent a long slender bar of wood. If the bar of

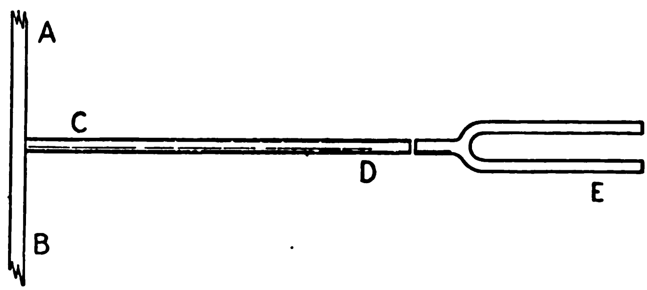


FIG. 285.

wood is placed in the position shown, with one end resting against the sounding board, and the tuning fork *E* is brought into contact with the other end of the bar, the effect is the same as if the tuning fork were brought directly into contact with the sounding board *AB*, that is, the tone will be materially strengthened or increased in loudness. This demonstrates that the disturbance is readily transmitted by the material of which the bar is made. Trying various solid materials, they will be found to serve equally well in the experiment.

THE VELOCITY OF SOUND

428. The velocity with which sound travels depends both upon the character and the condition of the medium in which the propagation takes place. The velocity of sound in air at 0° C. and under standard conditions as to pressure is 331 meters per second (1089 feet per second). The velocity of sound in air varies with the temperature. The velocity of sound at $t°$ C. is given by the following equation:

$$v = 331\sqrt{1 + 0.003665\,t} \qquad (113)$$

The velocity of sound in some of the more common media is given in the following table:

Substances	Temperature	Velocity in Meters per Sec.
Air	0° C.	331
Hydrogen	0° C.	1286
Water	8° C.	1435
Iron	20° C.	5130
Glass	20° C.	6000

THE GENERAL CHARACTER OF A SOUND WAVE

429. The general character of a sound wave may be understood from the following considerations: Let AB, Figure 286,

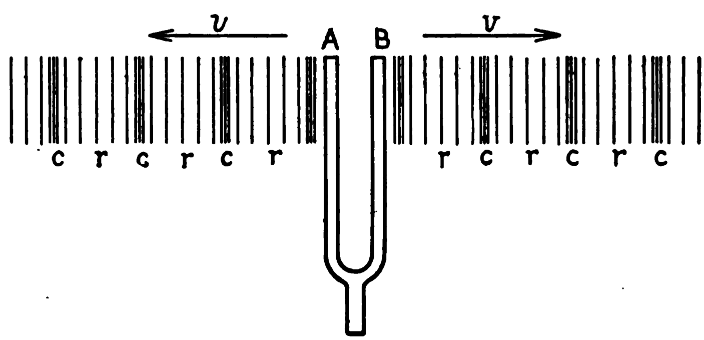


Fig. 286.

represent the prongs of a vibrating tuning fork. When the prong B moves toward the right, it compresses the air in front of it. The compressed region, tending to return to its original

condition of equilibrium, compresses those regions **which are** a little farther from the vibrating prong. These in **turn transmit** the disturbance to portions still farther removed. Thus a **compression** spreads out into space in front of the prong indefinitely. When the prong B moves toward the left, the air at the right of the prong is rarefied. The surrounding medium tends to restore the equilibrium of pressures in the immediate neighborhood of the prong, which results in a rarefaction a little farther out. These portions in turn hand on the disturbance to more widely removed portions of the medium. In this manner a **rarefaction** spreads into the space surrounding the prong of the tuning fork. The same thing is occurring in the neighborhood of the prong A. A similar effect takes place in the neighborhood of any sounding body. **This succession of compressions and rarefactions which is sent out by the tuning fork constitutes sound waves.** Thus a sound wave is made up of a compression and rarefaction. The different particles (portions) of the air, through which the disturbance passes, vibrate to and fro in much the same manner as the prong B of the tuning fork itself, **this vibratory motion being parallel to the direction in which the sound waves are traveling.** A wave motion of this character is called a **longitudinal wave motion.**

GRAPHICAL REPRESENTATION OF A SOUND WAVE

430. Let the series of small circles MM, Figure 287, represent a "chain" of undisturbed air particles. If a sound wave is made to travel along this chain, the particles in certain

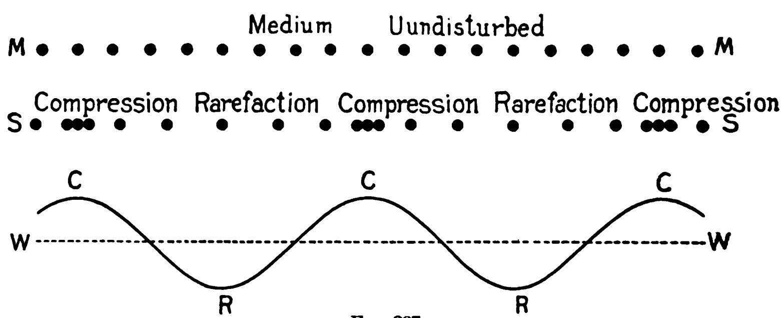


FIG. 287.

portions of the chain will **at a given instant** be crowded together (compression) and in certain other portions drawn apart (rarefaction) as pointed out in the last section. The grouping of the particles under these conditions will be something like that shown in the chain *SS*. **This disturbance of the chain of particles may be completely represented by the curve *CRCR*, the higher portions *CC* representing the compressions and the lower portions *RR* representing the rarefactions.** In the use of such a diagram to represent a sound wave it must be remembered that **the actual motion of the individual particles is in the direction *WW*, and not at right angles to this direction as might be inferred.**

COMBINATIONS OF SOUND WAVES

431. The smooth curve shown in Figure 287 represents a **simple wave**, that is to say, such a sound wave as is developed by a tuning fork. If two tuning forks are caused to sound side by side, the adjacent portions of the air are called upon to transmit two sound waves at the same time. If three sounding bodies are vibrating side by side, certain portions of the surrounding medium will be called upon to transmit three waves at the same time. Generally speaking, under these circumstances, the individual waves are transmitted by those regions of air through which they are passing with the same fidelity as that with which each would be handed on if it existed alone. The **complex** wave motion which the air particles undergo, under these circumstances, could not be represented by a simple curve like that shown in Figure 287.

Let it be assumed that two tuning forks are sounding side by side, the one making twice as many vibrations per second as the other. The waves given off by these two forks will be different in length, the one being twice as long as the other. This follows at once from Equation (111), since v the velocity of sound is the same for all frequencies, the product $n\lambda$ is constant, therefore λ varies inversely as n, that is, the higher the frequency the shorter the wave length and *vice versa*. If the curves A and B, Figure 288, represent the individual waves given off by the two vibrating bodies the actual motion of the

air particles transmitting these two waves will be that shown by curve C, the ordinates of which are found by taking the sums of the ordinates of the component curves A and B. Thus the actual vibratory motion of the air particles which are transmitting several different simple waves at the same time may be of a more or less complex nature, depending upon the manner in which the simple component wave motions are related.

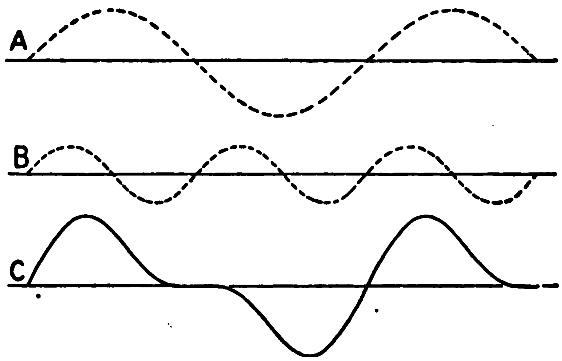


FIG. 288.

INTERFERENCE

432. An important case of the combination of two waves is the following. Let A and B, Figure 289, represent **two waves of the same wave length and amplitude traveling through the same region in the same direction.** Let it be assumed that they

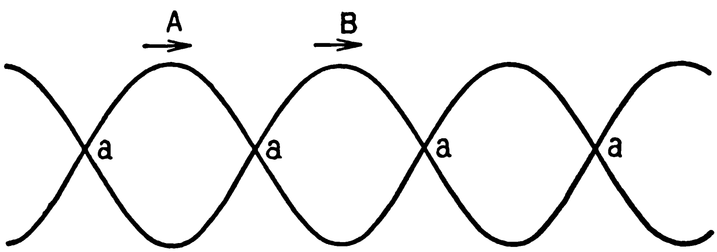


FIG. 289.

are so related to one another that the crest of the one wave falls exactly over the trough of the other as indicated in the figure. Consider the effect of the combination of waves upon any particle, for example, a. Evidently this particle tends to rise on to the crest of the wave A as the wave progresses toward

the right and at the same time it tends to sink into the trough of *B*. But since the form of the wave *B* is identical with that of *A* and the waves are traveling with the same velocity, it will be evident that the rate with which *a* tends to rise on *A* is exactly equal to the rate with which it tends to sink under the influence of *B*. The result is that **the particle *a* remains stationary**. Considering any other particle in the medium through which the disturbances *A* and *B* are passing, it will be evident that it is conditioned exactly as *a* is conditioned; that is, it will remain stationary under the combined influences of the two waves. This effect is known as **interference**. Under these circumstances the wave *B* completely neutralizes the effect of the wave *A* so far as the displacement of the different parts of the medium through which the disturbance is passing is concerned.

A restatement of the conditions under which this effect takes place is desirable:

1. The waves must be of equal wave length.
2. They must have equal amplitudes.
3. They must be traveling in the same direction.
4. They must be so related that the crest of the one is exactly opposite the trough of the other.

BEATS

433. If the two waves represented in Figure 289 are related as described in the preceding paragraph except that **the wave**

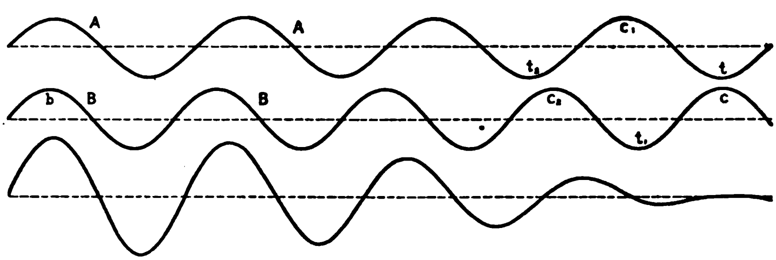


FIG. 290.

length of one of the disturbances is slightly greater than that of the other, the combination will produce the effect known as **beats**. An understanding of this effect may be obtained from

a consideration of Figure 290. Let A and B represent the component waves as before. Let it be assumed that A has the greater wave length as indicated in the diagram. Let it be further assumed that the conditions represented at the right of the figure are those required for interference. A combination of the crest c of B with the trough t of A produces almost complete interference. The interference is not quite complete, since the half waves considered are not quite of the same length. Considering the trough t_1 of B and the crest c_1 of A, it is evident that the interference is less complete. Considering the crest c_2 and the trough t_2, evidently the interference at this point is still less complete. Considering the effects of the successive half waves toward the left evidently at the point b, the waves, instead of interfering, combine in their effects, since at this point crest coincides with crest (approximately) and trough with trough. Evidently at this point the interference effect is almost entirely absent. At a point e, still farther to the right, the waves become once more so related as to bring about the interference effect. In other words, at certain points in the space through which these two waves are traveling, their effect is that of almost complete interference. At intermediate points the effects of the individual waves are added, and the individual waves conspire to produce a large effect. It will be readily understood, therefore, that if two sound waves related to one another as described above are falling upon the ear, the effect is that of a pulsating sound, since, when the waves fall upon the ear, in the condition represented at b, a loud sound will be heard; and when a moment later the waves as they fall upon the ear are related as represented at c, the effect will be almost zero. Such waves are said to produce "beats."

This effect may be illustrated very simply by sounding at the same time two tuning forks which give off sound waves of slightly different length. If two tuning forks are at hand which vibrate at the same rate or make the same number of vibrations per second, they may be put into condition for producing beats by loading the prongs of one fork with small pieces of wax. This weighting of the prongs tends to make the fork vibrate more slowly, that is to say, give off longer sound waves. By

adjusting the size of the pieces of wax used, the change in the wave length may be made anything desired.

The conditions necessary for the production of beats are :

1. The waves must be nearly equal in wave length.
2. They must have equal amplitudes (for maximum effect).
3. They must be traveling in the same direction.

THE REFLECTION OF SOUND WAVES

434. Experiment, as well as every day experience, teaches that sound waves may be reflected. The manner in which this is accomplished will be understood from the following considerations: Imagine a compression in a sound wave in air coming in contact, say, with a brick wall. A small part of the compression effect which has been handed on from point to point through the air will be taken up by the particles of the brick wall and transmitted. Since, however, the mass of these particles is comparatively great, they are set in motion only with difficulty. The compression, reaching the surface of the wall and meeting with difficulty in its advance, relieves itself in the reverse direction. A large part of the sound wave is turned back, the compression again traveling through the same medium as that through which it has just passed in its approach to the wall. In the same manner a rarefaction is reflected. Generally speaking, when a sound wave encounters the boundary between media of different density, a part of the disturbance will pass into the second medium, another part will be reflected. When the second medium is dense as compared with the first, the larger part of the disturbance will be reflected. When the second medium is less dense than the first, the disturbance will be largely handed on to the second medium.

The **echo** is a result of the reflection of sound. One of the important effects in the echo is the apparent change of the position of the source from which the sound waves are proceeding. This effect will be readily understood from a consideration of the fact that the sound waves after reflection are advancing from the obstacle at which reflection took place. The sound, therefore, appears to come from this obstacle or from some point as far beyond the obstacle as the source is in front.

A **multiple echo** is secured when there are two or more reflecting surfaces. Consider, for example, the effect of firing a gun in a cañon or long hall. If the ear upon which the sound falls is at some distance down the cañon, a succession of sounds will reach the ear, since a part of the disturbance will be propagated directly from the source to the ear, while the other portions of the disturbance will reach the ear only after one or more reflections from the walls of the cañon. Let AB and CD, Figure 291, represent parallel walls of a cañon. Imagine that a gun is fired at S. Several reports will be heard by an ear placed at E. The disturbance which first reaches the ear travels directly along the path SE. The second effect travels along the path SFE, being reflected once at the point F. A third disturbance will reach the ear E along the path $SGHE$, having been twice reflected, etc. Thus a succession of sounds reaches the ear at E and the resulting effect is a roar instead of a single sharp report, such as would be received by the ear in the absence of the walls AB and CD.

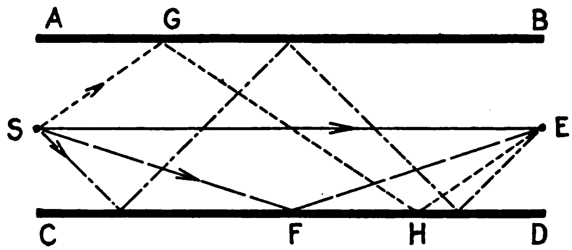


FIG. 291. — Reflection of Sound Waves in a Cañon.

THE REFRACTION OF SOUND

435. As stated in the last paragraph, when a sound wave falls upon the boundary surface between media of different density, a part of the disturbance enters the second medium. The direction of motion of that portion of the disturbance which enters the second medium is in general changed. This is evident from the fact that in different media sound waves travel with different velocities. Except, therefore, in that particular case in which every part of the sound wave enters the second medium at the same instant, one portion of the sound wave will be retarded with respect to the other portions.

That is, while the sound wave is passing from the one medium to the other there is, for a short time, a part of the wave in one medium and a part in the other. That portion which is in the medium in which sound travels with the greatest velocity will get ahead of the other portion. This effect is known as **refraction.**

The distinctness with which sounds are heard at night as compared with that with which they are heard in the daytime is accounted for largely by the effect of refraction. Consider, for example, the condition of affairs represented in Figure 292.

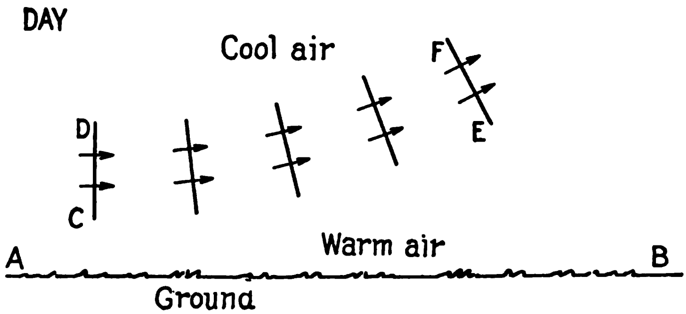


FIG. 292.

Let AB represent the surface of the ground. Let CD represent a sound wave traveling from the point O toward the right, as indicated by the small arrows. **In the daytime this wave tends to leave the surface of the ground because of the effect of refraction.** In the daytime those layers of air which lie close to the ground are more strongly heated than those at higher levels, and since sound travels more rapidly in warm air than in cold, the lower layers of the air transmit the disturbance more rapidly than the higher layers. Thus, the lower portion of the wave outruns the upper portion and the direction of the wave becomes like that at EF. In this position of the wave the direction of motion of the disturbance is that shown by small arrows drawn across the line EF. It is, therefore, evident that under these circumstances the disturbance tends to leave the surface of the ground.

In the evening, when the lower layers of the atmosphere be-

come chilled, it may happen that the upper layers are at a higher temperature than those in immediate contact with the surface of the earth. Under these circumstances an effect

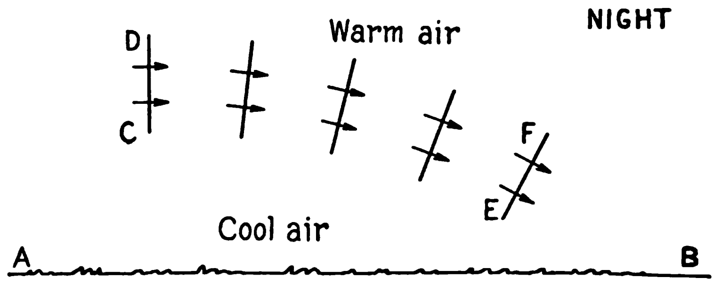


FIG. 293.

takes place which is just the reverse of that represented in Figure 292. **Thus sounds travel to greater distances along the surface of the earth at night than they do in the daytime.** (See Figure 293.)

CHARACTERISTICS OF A MUSICAL SOUND

436. There are three things which characterize a musical sound; namely, pitch, loudness, and quality.

The **pitch** of a musical sound is defined as the number of vibrations per second. Thus if the prong of a tunning fork makes 256 complete vibrations per second, the pitch of the musical sound which the tuning fork gives off is 256.

The loudness of a musical sound refers to the amount of energy carried by the sound wave. Thus, if a tuning fork is thrown into violent vibratory motion it gives off a loud sound. If it is but gently excited, it gives off a weaker sound. The energy transmitted to the medium surrounding the fork in each case is altogether different, being greatest of course in case of the loud sound.

The quality of a musical sound is that which enables one to distinguish between the sounds coming from different musical instruments. For example, if a violin string is set vibrating in the usual manner, the sound is recognized as that of a violin. If a piano string is set vibrating in such manner as to give a

sound of the same pitch and loudness, we are enabled nevertheless to recognize it instantly as a piano tone and to distinguish it from the violin tone. That property of a tone which characterizes it and enables one to distinguish it from any other tone is its quality.

It is true of most sounding bodies that they may vibrate in two or more ways at once, that is to say, it is possible for them to give off sound waves of different wave length. For example, a violin string may be vibrating as a whole, thus giving off what is called its **fundamental** tone. At the same time it may be vibrating in halves. Under these circumstances each half of the string constitutes a sounding body giving off a sound wave one half as long as the wave corresponding to the fundamental tone of the string. In the same way the string may be vibrating in thirds, and so on. The tones given off by these vibrating segments are called **overtones**. A sounding body when vibrat-

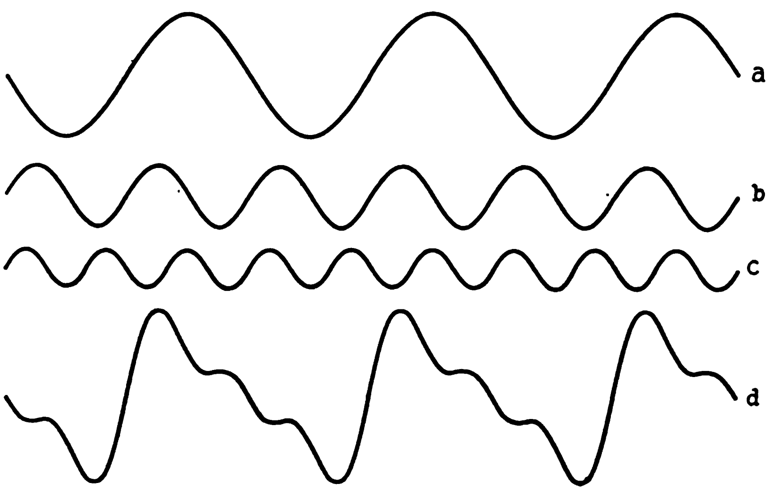


FIG. 294.

ing in this manner is said to give off a complex sound wave. **The quality of a complex sound wave is determined by the overtones which are present.** In Figure 294 is shown the character of the wave obtained when two overtones are present, one of which has twice the frequency, and the other three times the

frequency, of the fundamental. Let it be assumed that these sound waves are proceeding from a vibrating string. *a* represents the sound wave given off by the string vibrating as a whole, *b* the sound wave given off by the string vibrating in halves, and *c* the sound wave given off by the string when vibrating in thirds. ***d* represents the character of the disturbance, that is, the wave motion in the air in the immediate neighborhood of the vibrating string when the three sound waves, *a*, *b*, and *c*, are given off simultaneously.** Figure 295 gives the

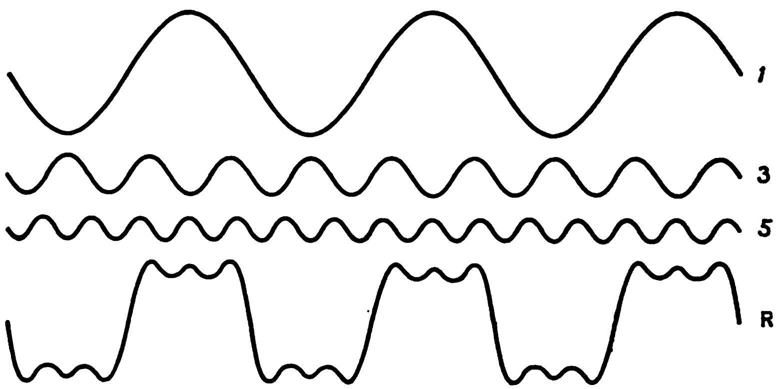


FIG. 295.

results of combining three sound waves whose frequencies are to each other in the ratio of one, three, and five. *R* is the resultant wave, that is, ***R* represents the character of the disturbance experienced by the air in the neighborhood of a string which is giving off simultaneously its fundamental tone and two overtones whose frequencies are three and five times as great as that of the fundamental.** The sound waves represented by *d*, Figure 294, and *R*, Figure 295, may have the same fundamental pitch and may have the same loudness, but they differ in quality, as is evident from the difference in the forms of the curves.

THE MEASUREMENT OF PITCH

437. The pitch of a sounding body may be measured in a number of different ways. The most direct method is to compare the number of vibrations which the body makes per second

with that of a standard, the pitch of which is known. One of the methods employed is that in which the siren is used. A siren consists of a cylindrical air chamber which, when the instrument is in use, is supplied with compressed air, the air being allowed to escape at regular intervals in puffs which give rise to a musical tone of definite pitch. The construction of the instrument will be understood from the following description: Let *C*, Figure 296, represent a cylindrical vessel which is supplied with compressed air through the tube *T*. The top of this hollow cylindrical vessel is perforated, having a number of holes uniformly spaced in a circle. Upon this face of the cylinder rests a plate *P* perforated in the same manner so that when properly placed the holes in the plate *P* coincide with the holes in the top of the cylinder. If the plate *P* is allowed to remain stationary in this position, streams of air will escape through the several holes. If the plate *P* is caused to rotate about the axis *A*, evidently the air will escape from the cylinder in puffs. The number of puffs occurring in each revolution is equal to the number of holes in the plate *P*. When the plate *P* is caused to rotate with sufficient rapidity, these puffs combine their effects in such manner as to produce a musical tone, the pitch of which is determined by the speed with which the plate *P* rotates. Evidently the pitch of the musical note produced by the instrument would be precisely the same if the plate *P* had but one hole in it. The loudness of the tone would, however, be decidedly less than that given off by the siren as usually constructed, since with the arrangement described, there are as many puffs given off simultaneously as there are holes in the plate *P*, and these several puffs combine to produce a more powerful disturbance in the surrounding air than would be produced by a puff escaping from

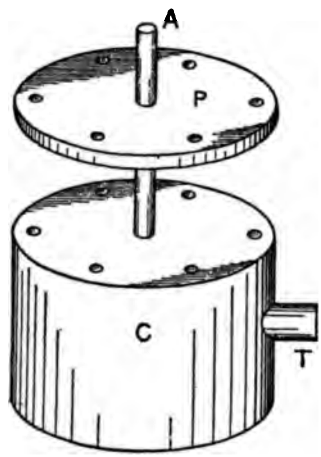


FIG. 296.

a single opening. By changing the speed of the plate *P*, the pitch of the sound given off may be changed at will.

The instrument is used in the following manner: Let it be desired, for example, to determine the pitch of a tuning fork. The tuning fork is caused to vibrate and the siren is put in motion, its speed being increased until the sound which is given off is of the same pitch as that of the tuning fork. By noting the speed of the plate when this result has been secured, we have at once the means of obtaining the pitch of the tuning fork. Evidently, **the pitch of the tone given off by the siren is obtained by multiplying the revolutions per second by the number of holes in the plate.**

THE LIMITS OF AUDITION

438. The human ear is sensitive to sound waves between certain limits only. If a sound is too low in pitch it is inaudible. If it is too high in pitch it is also inaudible. A sound having a pitch of less than 32 vibrations per second, or greater than 32,000 vibrations per second, is inaudible to the average human ear. It will be understood, of course, that these limits are not absolutely fixed. They differ with different individuals. These values may be taken as representing the limits for the average ear. The musical tones ordinarily employed in music vary in pitch between the limits of 40 and 4000 vibrations per second.

DOPPLER'S PRINCIPLE

439. When a sounding body moves toward or away from the observer, the pitch of the body is apparently altered. When the sounding body is moving away from the observer, the pitch is lowered. When it is approaching the observer, the pitch is raised. This effect is observed when one stands near a railway and listens to the bell of a locomotive as it passes. The effect is explained as follows: Let *e*, Figure 297, represent a sounding body moving toward the right with a velocity *V*. Let it be imagined that an observer is stationed at *A* and a second observer is stationed at *B*. Then the pitch of the sound given off by the body *e* will apparently be higher as heard by the

observer at B and lower as heard by the observer at A than the real pitch of the sound. Let it be imagined that the sounding body started from the position a, that in the interval during which it made one vibration it traveled to the position b. After completing another vibration it had reached the position c, and so on, so that upon reaching the position e it had com-

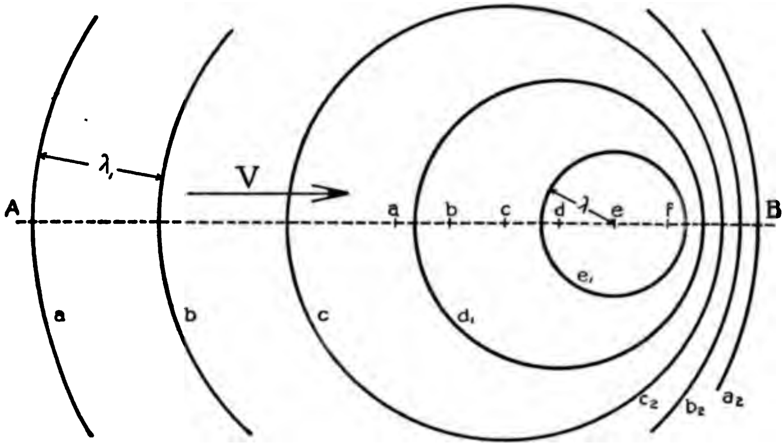


FIG. 297.—Illustrating Doppler's Principle.

pleted four vibrations. Let aa_1 represent the distance to which the disturbance has traveled in the medium while the sounding body has made these four complete vibrations. Then the disturbance which started from the sounding body when the sounding body was at a will have reached the position a_1 on the left and a_2 on the right when the sounding body has reached the position e. In the same way the disturbance which started from the body when it was at the position b will have reached the position b_1 on the left and b_2 on the right when the sounding body has come to the position e, etc. Evidently the successive disturbances a_1, b_1, c_1, etc., are more widely separated than the corresponding portions of the same disturbances which have passed off toward the right. In effect, therefore, **the waves at the left are longer and the waves at the right are shorter than would be the case if the body were stationary.**

The change in the wave length is obtained as follows. Let

λ_1 represent the distance a_1b_1 in the figure, that is to say, the apparent wave length of the sound wave at the left of the sounding body. Let V represent the velocity of the sounding body. Let v represent the velocity of sound and n represent the number of vibrations made by the sounding body per second. It is evident that λ_1 is equal to λ, the length of the sound wave which would be given off by the body if it were stationary + the distance through which the sounding body moves while making one vibration. But the distance through which the body moves while making one vibration is evidently $V \div n$. We have therefore,

$$\lambda_1 = \lambda + \frac{V}{n}$$

But $v = n\lambda$ (see Equation 111). Substituting the value of n from this equation in the equation above, we have,

$$\lambda_1 = \lambda + \frac{V\lambda}{v} = \lambda\left(1 + \frac{V}{v}\right)$$

or,

$$\lambda_1 = \lambda\left(\frac{v + V}{v}\right)$$

Or, since the lengths of sound waves vary inversely as their frequencies,

$$n_1 = n \cdot \frac{v}{v + V} \tag{114}$$

In the same way the pitch of the sound as received at B is found to be

$$n_2 = n \cdot \frac{v}{v - V} \tag{115}$$

If the velocity of the sounding body is greater than that of the sound waves, the result is something like that shown in Figure 298; a, b, c, etc., are the successive positions assumed by the sounding body which is supposed to be traveling to the right with the velocity V. V being greater than v, the velocity with which the waves travel in the surrounding medium, it is evident that the disturbances originating at the successive positions of the vibrating body a, b, c, etc., will all be tangent to the same straight line as indicated in the diagram. This will be evident from the following considerations: Let t represent the time occupied by the moving body in traveling the distance

ab. When the moving body has reached the point *g*, the disturbance which started from *f* when the body was in that position will evidently have been traveling *t* seconds. That which started from *e* when the body was in that position will have been traveling 2 *t* seconds. That which started from *d* will have been traveling 3 *t* seconds, etc. But the lengths of the perpendiculars dropped from the points *f*, *e*, *d*, etc., upon the line *gh*, are to each other in the ratio of 1, 2, 3, etc. Therefore

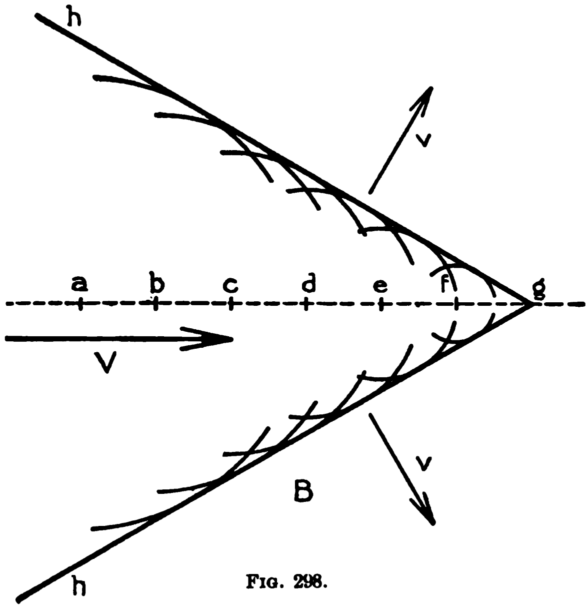


FIG. 298.

the disturbances which started successively from the points *a*, *b*, *c*, *d*, etc., will all arrive at the line *gh* at the same moment. It will therefore be evident that **a V-shaped wave enveloping the sounding body will move forward with it.** Under these circumstances the sounding body will have passed any point such as *B* located a short distance from the line in which the sounding body is traveling before the sound waves reach that point. The point of the V-shaped wave is traveling toward the right with the velocity of the sounding body. The V-shaped wave is traveling perpendicular to itself with the velocity of *v*, that is, the velocity of sound.

This effect is observed in other forms of wave motion. The V-shaped wave which accompanies a boat moving rapidly through the water and which spreads right and left from the prow is a familiar example. Of course the formation of this V-shaped wave is a question of relative motion between the boat and the surrounding medium. If a current of water is flowing past a stationary boat with a velocity greater than that of a wave in that medium, the same effect will be observed. A twig hanging in a swiftly moving stream gives rise to a wave of this character, which of course remains stationary as referred to the banks of the stream.

RESONANCE

440. A vibrating body capable of giving off a sound wave tends to vibrate at a definite frequency which is determined by the dimensions of that body and the conditions by which it is surrounded. **When a body is disturbed or set vibrating, it always tends to vibrate according to its own proper frequency.** To set a body vibrating in this manner requires but small expenditure of energy as compared with that which must be expended upon the body if it is forced to vibrate in any other way. Furthermore, it is possible to impart a motion of wide amplitude to any such body by a series of slight impulses, providing the impulses are so timed as to conform to the natural frequency of the vibrating body. This holds true, not only for such bodies as are capable of giving off sound waves, but for all vibrating bodies. A familiar illustration of this principle is that afforded by the swing. In order that a swing may be set vibrating through a wide angle by a series of slight impulses, it is necessary to time those impulses to conform to the proper period of the vibrating swing. The proper period of the swing is, of course, the period in which it vibrates when left to itself to swing as a pendulum. **When a sounding body is set in vibration by a series of slight impulses so timed as to conform to the proper period of the vibrating body, the effect produced is known as *resonance*.**

A good example is the following: Two tuning forks A and B, Figure 299, are supported upon the same base CD. If the

tuning forks have the same proper period or frequency, it will be found that when A is caused to vibrate, B will be set into "sympathetic" vibration. In other words, the resonance effect will be present. This is explained as follows: The vibrations of A are transmitted through the base CD as a series of very slight vibrations to the fork B. The first impulse causes a slight vibration in B, the second impulse adds its effect to that of the first, the third adds its effect to those of the first and second impulses, and so on; so that the effect is cumulative and the amplitude of the vibration in B is thereby gradually increased. Under suitable circumstances this effect is so marked that after a moment the vibrations in A may be stopped and B will continue to vibrate, giving off a very appreciable volume of sound.

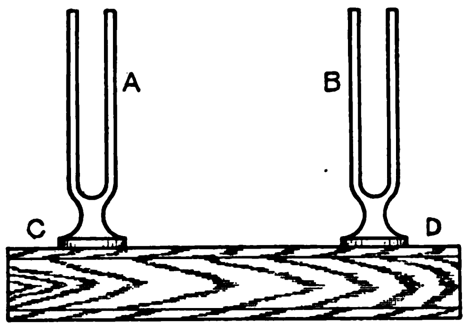


FIG. 299.

Another illustration is afforded by the reënforcement of the volume of sound given off by a tuning fork when it is caused to vibrate over an air column of suitable dimensions. The simplest way of obtaining the effect is as follows. Let F, Figure 300, represent a vibrating tuning fork and AB an air column inclosed by a cylindrical jar. In the general case the sound of the tuning fork is not reënforced by bringing the fork into the presence of the air column. But if the air column, which is capable of being set into vibration and giving off sound waves, is of such length that its frequency or proper period of vibration is the same as that of the fork, then it will be found that upon the approach of the tuning fork the air column will start into sympathetic vibration and will reënforce the sound given off by the tuning fork.

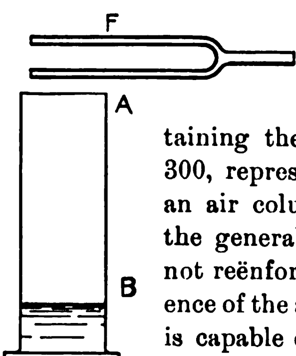


FIG. 300.

Problems

1. What is the velocity of sound in dry air at 30° C.?

2. What is the velocity of sound in air at − 20° C.?

3. A bell makes 100 vibrations (sends out 100 waves) per second. What is the wave length of the disturbance in air at 30° C.? At − 20° C.?

4. A tuning fork is found to give off waves 130 cm. in length in dry air at 0° C. How many vibrations does the fork make per second?

5. A man is observed chopping wood. He makes 25 strokes per minute. The sound of each stroke reaches an observer as the ax strikes the wood in the following stroke. Temperature of the air is −10° C. What is the distance of the chopper from the observer?

6. A flash of lightning is seen and 5 sec. later the first sound of the thunder is heard. What is the approximate distance of the nearest point of the discharge? Why is it impossible to determine the distance accurately by this means?

7. A man fires a rifle at a target 1000 ft. away. Velocity of the bullet = 1200 ft. / sec. At what point must an observer stand on a line drawn through the target perpendicular to the line joining gun and target, in order that the sound of the rifle and the impact of the bullet may reach him at the same instant? Assume velocity of sound = 1120 ft./sec.

8. A siren has a disk containing 16 holes. What is the pitch of the tone it gives when it makes 50 revolutions per second?

9. The pitch of the whistle of a locomotive drops a half tone in passing an observer. If the velocity of sound is 1100 ft./sec, what is the speed of the locomotive in miles per hour?

10. Draw a diagram of the wave system accompanying a body moving over the surface of a pond of water with a velocity $\frac{10}{9}$ that of the waves generated by the moving body?

THE MUSICAL SCALE

CHAPTER XXXVII

MUSICAL INTERVALS

441. The musical interval between two tones is the ratio of their frequencies, the frequency of the higher tone being taken as the numerator of the fraction. Thus two musical tones are said to be in unison when their frequencies are as 1 : 1, that is, when the ratio of their frequencies is unity. The interval between two musical tones is called an octave when the ratio of their frequencies is 2. The principal intervals employed in music are given in the following table:

Name of Interval	Ratio of Frequencies
Unison	$\frac{1}{1}$
Octave	$\frac{2}{1}$
Fifth	$\frac{3}{2}$
Fourth	$\frac{4}{3}$
Third	$\frac{5}{4}$
Minor Third	$\frac{6}{5}$
Minor Sixth	$\frac{8}{5}$

The value of a musical interval does not depend upon the absolute pitch of its components. For example, the interval between two tones whose frequencies are 60 and 120 is the octave. The interval between two tones whose frequencies are 256 and 512 is also the octave. That is, a musical interval depends only upon the ratio of the frequencies of the tones which bound it and is independent of absolute pitch.

CONSONANCE AND DISSONANCE

442. Two tones sounded together produce a pleasing effect when the ratio of their frequencies can be expressed by small numbers. Thus, aside from unison, the most pleasing interval

is the octave, the next the fifth, etc. **The pleasing effect of tones sounded together is called consonance.**

When two tones are sounded together the ratio of whose frequencies can be expressed only in large numbers, an unpleasant effect is produced. This effect is known as *dissonance*.

ADDITION AND SUBTRACTION OF MUSICAL INTERVALS

443. From the definition for the musical interval between two tones it is evident that the sum of two intervals is found by the process of multiplication rather than by the process of addition. Consider, for example, the musical interval a to c. Assume that this interval is made up of the two intervals a to b and b to c. Since the musical interval is defined as the ratio of the number of vibrations which the upper note makes to that made by the lower note, evidently the interval a to b is $\frac{b}{a}$. The interval between b and c determined in the same manner is $\frac{c}{b}$. Now the product of $\frac{c}{b}$ and $\frac{b}{a}$ is $\frac{c}{a}$ which is the musical interval c to a. **Thus the sum of the two intervals is obtained by multiplying their values together.**

In the same way the difference between two musical intervals is obtained by dividing the one by the other. Let it be required, for example, to find the difference between the **octave** and the **fifth**. Dividing the octave interval ($\frac{2}{1}$) by the fifth ($\frac{3}{2}$), we have

$$\text{Octave} - \text{fifth} = \frac{\frac{2}{1}}{\frac{3}{2}} = \frac{4}{3} = \text{fourth}.$$

That is, the difference between an octave and a **fifth** is a fourth, or the sum of a fifth and a fourth is an octave.

THE MAJOR CHORD

444. The combination of three tones whose frequencies are to each other as 4, 5, and 6 produces an effect upon the ear which is especially pleasing. The combination tone is rich, full, and satisfying. Such a combination is known as the **major chord**. The major chord is of importance in the

present study, since upon it is based what is known as the major scale.

THE MAJOR SCALE

445. Let three tones c′, e′, and g′ be taken, whose frequencies are to each other as 4, 5, and 6. Let a fourth tone c″ be taken, which is the octave of c′. **This major chord forms the skeleton of the major scale.** c′ is known as the **tonic** of the scale and g′ as the **dominant.** In order that we may have a specific example before us, let it be assumed that the frequency of c′ is 256. Then the frequency of e′ is $\frac{5}{4} \times 256 = 320$, and the frequency of g′ is $\frac{6}{4} \times 256$ or 384, and the frequency of c″ is 2×256 or 512, since the frequency of the octave of any tone is twice the frequency of the tone.

Let now g′ be taken as a basis for the formation of a new major chord. **To form a major chord upon g′, it is only necessary to place in combination with it two tones whose frequencies are to the frequency of g′ as 5 : 4 and as 6 : 4 respectively.**

That is to say, in any major chord the interval between the first and second tones is the major third. The interval between the first and third tone is the major fifth. Let b′ represent the second tone of the new chord formed upon g′. Since the frequency of b′ must be to the frequency of g′ as 5 : 4, therefore the ratio of the frequency of b′ to that of c′ must be $\frac{5}{4} \times \frac{6}{4} = \frac{15}{8}$, since the interval c′ to g′ is $\frac{6}{4}$. The absolute frequency of b′ is therefore $\frac{5}{4} \times 384$ or 480. In the same manner the frequency of the third tone, which, combined with b′ and g′, will form a major chord, is determined. Call this third tone d″. Then the frequency of d″ must be to the frequency of g′ as 3 : 2, and therefore the ratio of the frequency of d″ to that of c′. $= \frac{3}{2} \times \frac{3}{2} = \frac{9}{4}$, since the ratio of the frequency of g′ to that of c′ is equal to $\frac{3}{2}$. The absolute frequency of d″ in this scale is given by $\frac{3}{2} \times 384 = 576$. **If now a third major chord is formed upon f′ as fundamental, the major scale will be complete.** Comparing the frequencies of c″ and f′, we find the interval to be a fifth, hence c″ is the third tone on a major chord on f′. To determine a′, the additional tone necessary, we have simply to remember that the frequencies of f′ and a′ must be such that a′ : f′ as 5 : 4.

Thus, in the scale above mentioned, the frequency of f′ is $\frac{4}{3} \times 256 = 341$ (approximately), and the absolute frequency of $a' = \frac{5}{4} \times 341 = 427$ (approximately). The several tones whose frequencies have been determined in this manner may now be written as follows:

c′	d′	e′	f′	g′	a′	b′	c″	(d″)
256	288	320	341	384	427	480	512	(576)

The numbers written below the letters indicate the absolute frequencies of the corresponding tones. The frequency of d′ is obtained from that of d″, which is its octave, therefore the frequency of d′ is 288.

There are several different ways of representing the successive notes of the major scale, as follows:

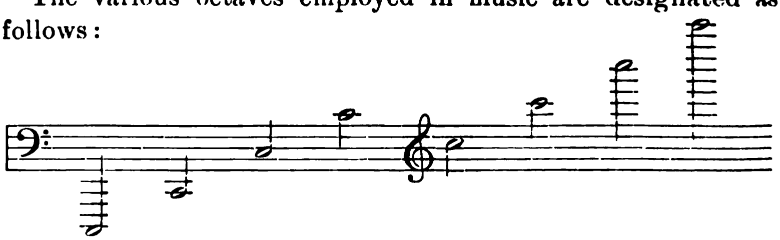
256	288	320	341	384	427	480	512
Do	Re	Mi	Fa	Sol	La	Si	Do
c′	d′	e′	f′	g′	a′	b′	c″
$\frac{1}{1}$	$\frac{9}{8}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{15}{8}$	$\frac{2}{1}$
First	Second	Third	Fourth	Fifth	Sixth	Seventh	Octave

The first, second, and fourth methods represented determine the absolute pitch of the notes; the others refer only to their relative positions on the scale.

The scale which has been developed in the above discussion occurs near the middle of the keyboard of a piano. c′ is called middle c.

In the **International System** the absolute pitches of the tones in this middle c scale are higher than here indicated, **the pitch of a′ being fixed arbitrarily at 435.**

The various octaves employed in music are designated as follows:



32	64	128	256	512	1024	2048	4096

The numbers written below the notes on the musical staff indicate their frequencies. In the International System the pitches are higher, middle c being taken as 261 instead of 256, as stated above.

THE INTERVALS OF THE MAJOR SCALE

446. The intervals occurring in the major scale are as follows:

$$\begin{array}{ccccccccccccccc} c' & & d' & & e' & & f' & & g' & & a' & & b' & & c'' \\ & \frac{9}{8} & & \frac{10}{9} & & \frac{16}{15} & & \frac{9}{8} & & \frac{10}{9} & & \frac{9}{8} & & \frac{16}{15} & \end{array}$$

The interval $\frac{9}{8}$ is called a **major second**. The interval $\frac{10}{9}$ is called a **minor second**. The interval $\frac{16}{15}$ is called a **half tone**.

The smallest interval recognized in music is $\frac{81}{80}$. It is the difference between the major and minor seconds. It is called the **comma**.

The name **diesis** is applied to the difference between the major third and the minor third. This difference is evidently equal to $\frac{5}{4} \div \frac{6}{5} = \frac{25}{24}$. This interval is important in the building up of the modified scale, since it is the interval by which a tone is raised or lowered when it is sharpened or flattened as discussed in the following paragraph.

TRANSPOSITION

447. The scale which is discussed above is based upon c' as the tonic or keynote. It is often-times desirable to make use of a scale having some other keynote. **If we attempt, however, to make use of any of the other of the above tones as the keynote and build a major scale upon it, we find at once that the tones whose frequencies are determined by the above discussion will not answer for this new scale.** If, for example, we choose the second note in the above scale, that is, d', as the keynote, we see at once that e' will not answer as the second tone of the scale, since the interval between e' and d' is not $\frac{9}{8}$, the interval which is required between the first and second notes of the major scale. In the same way the interval between d' and f' is not the major third as it should be if f' is to be used as the third note in the new scale. Evidently the notes which would be required to form the second and third notes of this new scale

must have frequencies of 324 and 360 (that is, $\frac{9}{8} \times 288$ and $\frac{5}{4} \times 288$). **It is thus seen that the scale as written above is incomplete in that we are of necessity compelled to use c′ at all times as the keynote.** To obviate this difficulty and to reduce the intervals of the major scale, which are found for certain purposes of melody to be rather large, intermediate notes have been introduced. These intermediate notes are called **sharps** and **flats.** For example, in the interval between c′ and d′ two notes are desirable c′♯ (c′ sharp) and ♭d′ (d′ flat). The interval between c′ and c′♯ is $\frac{25}{24}$. The interval between ♭d′ and d′ is $\frac{25}{24}$. In other words, to sharpen a note its frequency is increased by the interval $\frac{25}{24}$. To flatten a note its frequency is decreased by the interval $\frac{25}{24}$.

By means of the sharps and flats which are introduced in this manner between each two notes of the major scale excepting 3 and 4 and 7 and 8, these intervals being already small, we obtain a scale of 18 notes as follows:

c′	c′♯	♭d′	d	d′♯	♭e	e	f′	f′♯	etc.
1	$\frac{25}{24}$	$\frac{27}{25}$	$\frac{9}{8}$	$\frac{75}{64}$	$\frac{6}{5}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{25}{18}$	etc.

With this series of notes it is possible to build scales upon other tones than c′ as fundamentals. Nevertheless, **the difficulties above referred to are not entirely done away with by the introduction of sharps and flats.** It will be found necessary in spite of the large number of notes in this modified scale to introduce certain other notes to make the intervals correct for scales based on other tones than *c′* as keynote. Thus the scale becomes unwieldy and entirely impracticable, especially as applied to musical instruments having fixed notes, like the piano, in which a separate key is required for each separate note.

To overcome this difficulty the various intervals appearing in the scale are adjusted so as to be approximately correct according to the laws laid down above, but at the same time are given such values that fewer notes are necessary. The first step in the direction of this adjustment is effected by introducing between *c′* and *d′* a single note, the frequency of which lies somewhere between $\frac{25}{24}$ and $\frac{27}{25}$, this note being used as either c′ sharp or d′ flat. In this same manner one note is made to suffice between

d′ and e′, serving as d′ sharp and e′ flat. Proceeding in this manner the number of notes in the modified scale is reduced to 13. **Another step in the direction of the proposed adjustment is made by making all of the 12 intervals in this scale equal in value.**

THE TEMPERED SCALE

448. If, as suggested in the last paragraph, the 12 intervals of the scale of 13 notes are made equal, we obtain what is known as the equally **tempered scale** or the scale of **equal temperament.** In this scale the successive intervals are equal, that is to say, the ratio of the frequency of any note to that of the next note below it is the same. Since musical intervals are added by multiplying them together, evidently this interval must be of such value that its 12th power is equal to 2. In other words,

$$X = \sqrt[12]{2} \qquad (116)$$
$$= 1.05946$$

in which X is the interval between any two successive tones of the tempered scale.

It follows as a matter of course that this process of tempering the scale results in a certain amount of dissonance. This is not so marked, however, that the tempered scale may not be used for practically all purposes of music. The change in pitch of the various notes of the scale, beginning on c′ as the keynote, due to the tempering of the scale, is shown below:

	c′	d′	e′	f′	g′	a′	b′	c″
True	256	288	320	341.3	384	426.7	480	512
Tempered	256	287.3	322.5	341.7	383.6	430.5	483.2	512

The first horizontal row of numbers, marked True, gives the frequencies of the eight tones forming the true major scale as determined in the above discussions. The second row gives the frequencies of the eight tones forming the major scale of equal temperament.

Problems

1. How many beats per second will be heard when two organ pipes making 200 and 204 vibrations per second respectively are sounded together?

2. A whistle having a pitch of 1800 is sounded in a wind of 50 mi. per hour. Would the pitch of this whistle be normal to an observer "down the wind"? "Up the wind"? In a direction at right angles to the wind? If not, how would it be changed?

3. What is the interval between the fundamental and the fifth above the octave?

4. What is the interval between the second and the seventh of the major scale? Between the second and the sixth?

5. The frequency of an organ pipe is 200. What pitch must a pipe have to form with this a major third? A major sixth?

6. The pitches of three tones, α, β, and γ, are 150, 500, and 650 respectively. What is the interval between α and β? Between β and γ? Between α and γ? What is the difference between the intervals $\alpha\beta$ and $\beta\gamma$?

7. Determine the frequencies of a series of notes forming a major scale on a note having a pitch of 96 as fundamental.

8. What tones will form the major chord ceg if the frequency of e is 1000?

9. What is the interval d′ to b′ on the true major scale? What is this interval on the tempered scale, assuming that c′ and c″ remain the same?

SONOROUS BODIES

CHAPTER XXXVIII

VIBRATING BODIES USED IN MUSICAL INSTRUMENTS

449. In musical instruments various sorts of sonorous bodies are used, of which the following examples may be given:

Air columns	Organ pipes, wind instruments.
Rods	Xylophone, Geneva music boxes.
Plates	Bells.
Strings	Piano, violin, etc.

Experiment shows that each of these classes of vibrating bodies is subject to certain simple laws which express the frequency or pitch of the tone given off in terms of the dimensions of the vibrating body from which it proceeds.

THE LAWS GOVERNING THE VIBRATIONS OF AIR COLUMNS

450. The laws governing the vibrations of air columns may be obtained from the following general consideration: Let it be imagined that a sound wave is sent down a long tube or pipe, see *AB*, Figure 301, in the direction of the arrow *D*. Let it be imagined that the end *A* of the tube is closed. This sound wave will be reflected at the closed end of the tube and, returning in the direction of the arrow *R* and combining with the oncoming wave, it will set up stationary waves (Section 422) in the air column which fills the tube *AB*. These stationary waves may be represented by the curved lines drawn in the figure. It will be borne in mind, however, that these curved lines do not represent the direction in which the particles are vibrating, since, as we have seen, in a sound wave the particles move to and fro in the direction in which the sound is traveling. That is to say, in the case under consideration, the air particles vibrate

in little orbits parallel to the axis of the tube. Nevertheless, the stationary waves established may be represented in a general way as pointed out above by the curved lines as drawn. Thus the points *b* are stationary points or **nodes.** The points *a* are the points of maximum displacement or disturbance, that is to say, the particles at *a* vibrate through greater distances than any other particles ; the particles at *b* remain stationary. The curves and symbols here used correspond to those of Figure 279. Under the conditions described above it will be evident that a solid diaphragm might be placed across the tube *AB* at any point *b* without disturbing the wave motion which is present in the tube. This is equivalent to saying that *bb* represents the shortest length of the tube in which this particular wave can

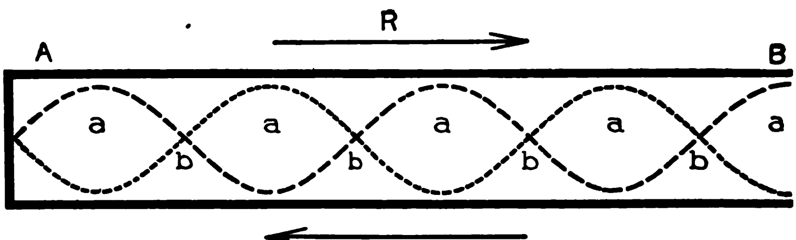


FIG. 301.

exist as a stationary wave. **Now *bb* is half a wave length of the disturbance which is assumed to be present.** We therefore arrive at the result that **a stationary wave may exist in a tube with closed ends, the length of which is one half that of the wave.** In a shorter tube this wave could not exist.

In the same manner we determine that *ab* is the shortest tube having one end closed and the other open in which the same wave might exist as a stationary wave. This will be evident from the fact that the particles in the neighborhood of *a* are the particles which are vibrating to and fro with the greatest freedom. This freedom of motion of the *a* particles would not be altered by cutting the tube across. We might, therefore, so far as the stationary wave between *b* and *a* is concerned, cut the tube across at *a* and put in a solid diaphragm at *b*, thus giving

us the length *ba*, which **is the shortest length of tube, having one end open and the other closed, in which the given stationary wave can exist.** A tube or a pipe having one end closed and the other open is called a **closed pipe.** A tube having both ends open is called an **open pipe.**

Evidently the shortest length of open tube which would accommodate this stationary wave is *aa*.

We therefore arrive by means of this discussion at the important result that **a closed pipe, in order to accommodate a given stationary wave, must have a length equal at least to one fourth that of the wave; and the shortest open pipe which will accommodate a stationary wave has a length equal to one half the length of the wave.**

THE FUNDAMENTAL TONE OF A VIBRATING AIR COLUMN

451. Let it be imagined that the air column contained in a closed pipe like that represented by *A*, Figure 302, is vibrating in such manner as to contain the longest possible stationary wave. From the discussion given above it is evident that the wave must have a node at the bottom of the pipe and an antinode or vibrating portion at the top of the pipe, as indicated by the curved dotted lines. Thus the longest wave which the pipe will accommodate is $4L$ where L is the length of the pipe. Next, considering the pipe represented by *B*, Figure 302, which is supposed to be open at both ends, it is evident that the longest wave which this pipe can accommodate is of such length that it has a node at the middle of the pipe and an antinode at either end. Thus the length of the pipe is one half that of the wave. In other words the longest wave which this pipe will accommodate is $2L$.

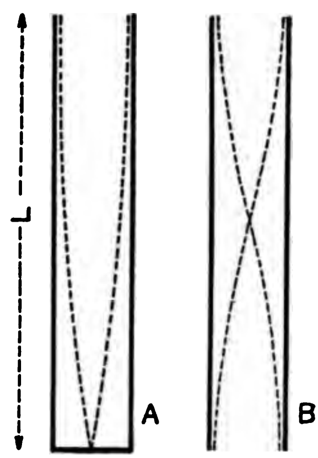


Fig. 302.

We have, therefore, for a closed pipe,

$$\lambda_1 = 4\,L \tag{117}$$

For an open pipe $\lambda_1 = 2\,L$, in which λ_1 is the length of the wave and L the length of the pipe. **If the same wave is to be accommodated in both an open and a closed pipe, the open pipe would require a length twice as great as that of the closed pipe.** It is for this reason that closed pipes are used for the deeper tones of the pipe organ.

THE OVERTONES OF AIR COLUMNS

452. While a closed pipe is capable of accommodating a wave whose length is four times that of the pipe, it will also accom-

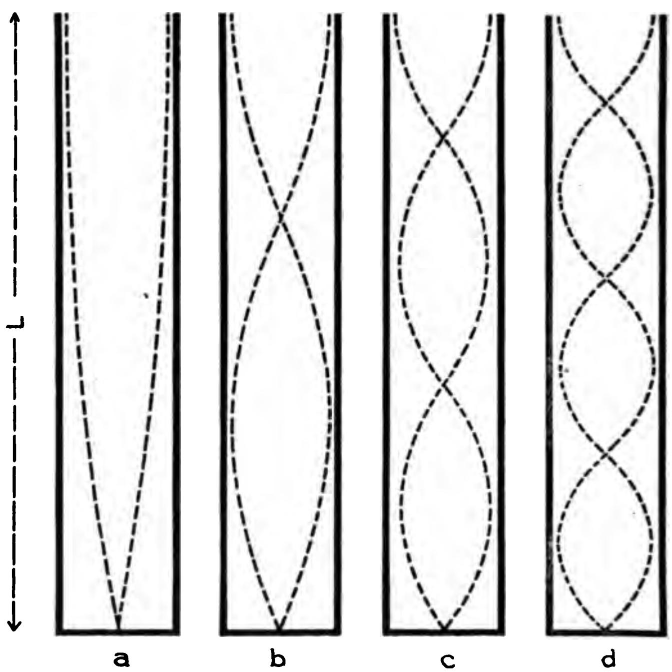


FIG. 303. — Overtones of a Closed Pipe.

modate shorter waves. The only conditions which the stationary waves must satisfy, in order that they may be accommodated by the closed pipe, that is to say, in order that they may exist in

the closed pipe, are that in every case there must be a node at the closed end of the pipe and an antinode at the open end. Thus, Figure 303 shows a closed pipe of length L with some of the longer wave lengths which it will accommodate. The wave length λ_1 present in (*a*) is $4\,L$. The wave length λ_2 present in (*b*) is evidently $\frac{4}{3}\,L$. When this wave length is present in the pipe, there are two nodes, one at the closed end of the pipe and the other at one third of the distance from the top. The wave

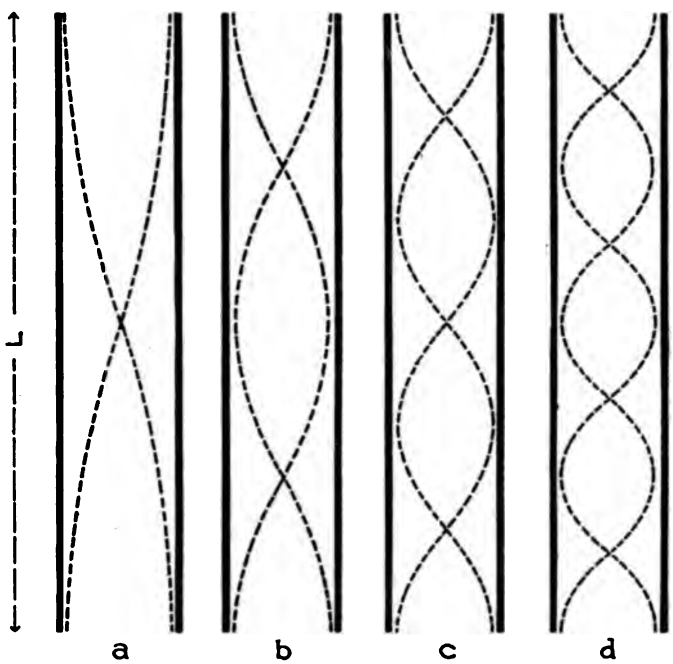


FIG. 304. — Overtones of an Open Pipe.

length λ_3 present in (*c*) is $\frac{4}{5}\,L$. The wave length λ_4 present in (*d*) is equal to $\frac{4}{7}\,L$. That is to say, the wave lengths of the various waves which may be accommodated by a closed pipe are to each other as 4, $\frac{4}{3}$, $\frac{4}{5}$, $\frac{4}{7}$, etc. Since the frequency of a tone varies inversely as its wave length, the corresponding frequencies are $\frac{1}{4}$, $\frac{3}{4}$, $\frac{5}{4}$, $\frac{7}{4}$, or, in other words, the several successive frequencies are in the order 1, 3, 5, 7, etc.

In Figure 304 is shown an open pipe with some of the longer

waves which it will accommodate, that is to say, which may exist as stationary waves within the pipe. Call the length of the pipe L. The length of the wave λ_1 present in the case represented in (*a*) is evidently $2L$. The length of the wave λ_2 present in the case represented in (*b*) is evidently L. λ_3 in the case represented in (*c*) is equal to $\frac{2}{3}L$, and in the case represented in (*d*) λ_4 is equal to $\frac{1}{2}L$. That is to say, the several wave lengths are to each other as the numbers, 2, 1, $\frac{2}{3}$, $\frac{1}{2}$. The relative frequencies corresponding are given by the numbers 1, 2, 3, 4, etc.

It is thus seen that the frequencies of the various tones which a closed pipe can give are to each other as the odd numbers, 1, 3, 5, 7, while those which the open pipe can give are to each other as the numbers 1, 2, 3, 4, a series containing both the even and the odd numbers. The tones corresponding to the waves present in cases *b*, *c*, *d*, etc., are called the overtones. Generally speaking, when an air column is thrown into vibration, it vibrates in several different ways at once, for example, as a whole, giving its fundamental note; in halves, giving the octave of this note; in thirds, giving the fifth above this octave; and so on.

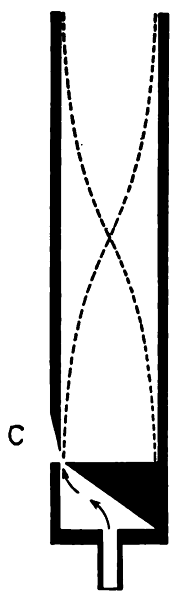


FIG. 305.

MANNER IN WHICH AN AIR COLUMN IS SET IN VIBRATION

453. Let Figure 305 represent an open organ pipe. In addition to the box containing the air column proper, there is a device at the bottom whereby a stream of air is directed against a sharp edge C in one of the walls of the pipe. The direction in which the air current moves is indicated by the arrow. When the stream of air first starts, a disturbance is created in the lower part of the column which travels to the top of the column, is there reflected and returns to the point C, and reacting upon C another disturbance travels up the tube in the same manner. Evidently the number of impulses which the stream of air at C

receives from the vibrating column is determined by the length of the column. Thus, the stream of air is started fluttering or vibrating to and fro at a frequency which is determined by the dimensions of the pipe. The vibrating column imparts its motion to the surrounding air, and thus a sound wave of a frequency corresponding to the dimensions of the air column is established. Since the pipe is open at *C*, evidently an antinode is present at this point. An antinode is present also at the upper end of the pipe. There is, therefore, a node at a point midway between these points. The wave given off by such a pipe is therefore twice as long as the pipe itself. Evidently, from the consideration shown above, if the pipe is closed at the top the fundamental wave length will be twice as great. The corresponding frequency will be $\frac{1}{2}$ as great, that is, one octave below that corresponding to the open pipe.

APPLICATION OF THE LAWS OF VIBRATING AIR COLUMNS

454. An interesting application of the law of vibrating air columns is the following: Let it be required to determine the pitch of a tuning fork. This may be done by finding the length of air column which will vibrate in unison with the fork. Thence by measuring the length of the column the wave length of the sound wave given off by the fork is at once known and from this the pitch may be calculated. Let Figure 306 represent a jar partly filled with water as indicated, above which is placed the vibrating tuning fork. Let it be imagined that water is gradually turned into the jar, thus shortening the air column between the surface of the water and the mouth of the jar until the resonance effect (Section 440) is secured. Let L represent the distance between the mouth of the jar and the surface of the water. When the condition of resonance is reached, a quarter wave length is present in this air column. That is to say, the sound wave given off by the tuning fork has a length such that $\lambda = 4\,L$. But $\lambda = v \div n$ (Equation 111) in which v is the velocity of

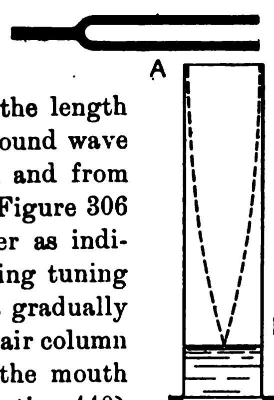


FIG. 306.

sound in air and n the frequency or pitch of the tuning fork. We have, therefore,

$$v/n = 4\,L$$

$$\therefore n = \frac{v}{4\,L} \tag{118}$$

KUNDT'S EXPERIMENT

455. The longitudinal vibrations of a rod of metal or of glass are like those of an air column. Imagine, for example, a glass rod clamped at the center and stroked endwise with a damp cloth. The rod will be set vibrating in very much the same manner that an open air column vibrates. The ends of the rod correspond to anti-nodes or vibrating segments. The center of the rod constitutes, of course, a node. In such a vibrating rod the same relation exists between the length of the rod and the length of the stationary wave which is present in it as that which holds for the air column inclosed by an open pipe, that is to say, the wave length of the stationary wave present is twice that of the rod.

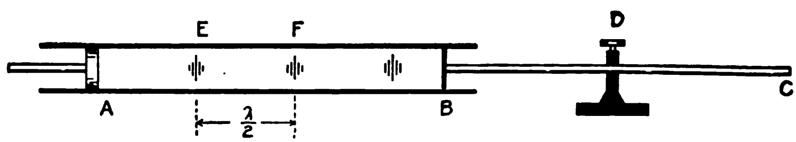


FIG. 307. — Kundt's Apparatus.

Application has been made of this and the foregoing principles to the determination of the relative velocities of sound in various solids and gases. The apparatus employed is that represented in Figure 307. AB is a glass tube, the bottom of which is sprinkled with a small quantity of cork dust or lycopodium powder.

BC is a rod, let us say, of brass, clamped at its center, as indicated at D. When the brass rod is stroked, it is thrown into longitudinal vibration as indicated above. These vibrations are communicated to the air column in AB by means of a disk of paper or wood attached to the end of the rod. The disk at A is moved backward and forward until the distance

AB is such as to accommodate the stationary wave train thus set up in the air column AB. When this adjustment is reached, it will be observed that the lycopodium is violently agitated at certain points within the tube, these being the points of greatest disturbance, of course. At other points the lycopodium remains at rest. These points are evidently nodes in the wave train. They are distinctly marked by the heaping up of the powder. The distance between two of these nodes is, as we have seen, $\lambda \div 2$, in which λ is the wave length in air of the sound proceeding from the rod. Thus the half wave length of the sound wave in air is determined. The corresponding half wave length in brass is, of course, BC. It follows, therefore, that the velocity of sound in brass is to the velocity of sound in air as BC is to EF. If the tube is now filled with another gas, the length of the wave will be found to be different, since the velocity of sound is different in different gases. But evidently the velocity of sound in the new gas is to the velocity of sound in air with which the tube was first filled as the corresponding distances between the nodes of the stationary wave trains.

TRANSVERSE VIBRATION OF STRINGS

456. Consider a string tightly stretched between two points as represented by AB, Figure 308. Imagine it to be displaced (drawn aside) and released. The disturbance (distortion of the string) will be propagated to both A and B, will be reflected at those points. These reflected disturbances will return to the center of the string, pass one another, and go on to the ends to be once more reflected, and so on. The actual vibrations of the string are therefore made up of two waves traveling to and fro along the string in opposite directions with equal velocities. These waves combine to form stationary waves in the string, and these stationary waves constitute the visible and effective vibrations of the string. Evidently the string makes one complete vibration (assuming that but two nodes are pres-

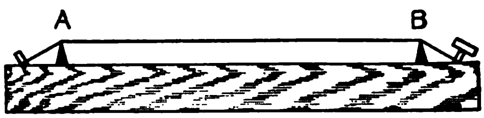


FIG. 308.

ent, namely, those at A and B) while one of the component disturbances travels from A to B and back again, a distance $2L$. Call the frequency of the vibrations n, and the velocity of the component disturbances up and down the string v. We have, then,

$$v = n\lambda$$

in which λ is the wave length of the component disturbances. But since the disturbance travels twice the length of the string for each vibration, therefore the wave length, $\lambda = 2L$. Hence,

$$v = n \cdot 2L$$

Now, experiment shows that the velocity of the component disturbances is given by the square root of the tension in the string, divided by the mass of the string per unit length. That is,

$$v = \sqrt{\frac{T}{M}}$$

in which T is the tension and M the mass per unit length. Combining these two expressions for v, we have,

$$n \cdot 2L = \sqrt{\frac{T}{M}}$$

or

$$n = \frac{1}{2L}\sqrt{\frac{T}{M}} \tag{119}$$

which is an expression for the frequency of the vibrating string in terms of its length and the tension and mass per unit length of the string.

This is known as the Law of Vibrating Strings. By means of this relation, the frequency with which a string will vibrate or the pitch of the sound which it will give off as it vibrates may be predicted. The law is exemplified in the stringing of a violin. The strings of a violin are all of the same length, so that the relative pitches of the strings are determined by the tensions to which they are subjected and their masses per unit length. The E string, the string of highest pitch on the violin, is given a high pitch by making it thin and light and by subjecting it to relatively high tension. The G string, or lowest

string, is given a low pitch by subjecting it to a relatively low tension, and by giving it a large mass per unit length. This is done by winding it with wire. The A and D strings, or strings of intermediate pitch, are thicker than the E string and are subjected to a higher tension than the G string. The law of vibrating strings is readily verified by means of the apparatus represented in Figure 309. Let AB represent that portion of

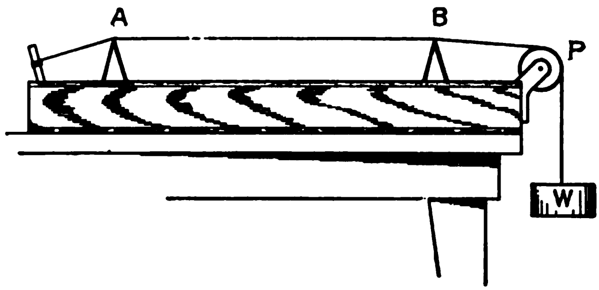


Fig. 309. — Sonometer.

the string which is caused to vibrate. W is a weight which is attached to the end of the string by a cord passing over the pulley P. By changing the value of this weight the tension in the string may be varied. Thus the dependence of pitch upon the tension may be investigated. By changing the position of the support B the length of the vibrating segment of the string may be varied at will and in this manner the dependence of the pitch upon the length of the string, other things remaining the same, may be determined. Finally, by taking strings of different weight or mass per unit length, subjecting them to the same tension, and making use of the same length of vibrating segment AB, the dependence of the frequency upon the mass per unit length may be determined.

TRANSVERSE VIBRATION OF RODS

457. It may be shown experimentally that the rate at which a given rod will vibrate transversely is given by the following expression:

$$n = \frac{Ad}{L^2} \cdot \sqrt{\frac{E}{\rho}} \tag{120}$$

in which E is the modulus of elasticity, ρ is the density, L is the length of the rod, d is its thickness in the direction of vibration, and A is a constant depending upon the manner in which the rod is fastened and upon the number of nodes present in the vibrating rod. The manner in which the rod vibrates is indicated in Figure 310, in which the heavy line represents the rod

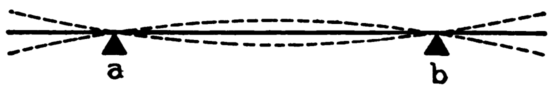


FIG. 310.

supported at two points a and b. If a rod supported in this manner is struck at its center with a mallet, it will be bent into the position shown by the dotted line. From this position it will vibrate to the reverse position as shown by the second dotted line, and so on, the points of support in this case constituting the nodes. By supporting rods of different length and thickness, the above law may be verified. For example, according to the above law, if two rods of the same material and the same length but having thicknesses in the ratio of 2 to 1 are employed, the thicker rod will give the octave of the note given by the thinner rod, and so on.

It should be noted in this connection that the width of the rod does not enter. In other words, the frequency is independent of the width of the rod.

LONGITUDINAL VIBRATION OF RODS AND STRINGS

458. Consider an elastic rod attached at one end to a rigid wall, Figure 311 (a). Let it be assumed that the rod is compressed by the application of a force at the outer end and then released. Under these conditions the rod will vibrate longitudinally and, if the vibrations are of proper frequency, will give a musical tone. The general character of the vibratory motion of the rod is shown at (a), (b), (c), etc.

As soon as the compressing force is removed, the compression in the rod begins to relieve itself. The first layer of particles at the free end of the rod moves to the right. This is followed by similar motion of the second, third, and other successive

layers. **Each moving layer continues in motion until all layers have been relieved of their compression and until it receives a pull to the left due to the stopping of the adjacent layer.** The condition of the rod shortly after the outward motion of the successive layers has begun is shown in (*b*). When the compression has been relieved throughout the rod, the condition is that represented in (*c*). If the time for one complete vibration of the rod is τ, then $\frac{1}{4}\tau$ is the time required for the rod to change from the condition (*a*) to the condition (*c*).

The condition represented in (*c*) exists for an instant only, since the layers adjacent to the wall are stretched by the outward motion of the rod. **The stretch in the rod near the wall spreads outward and the various layers are successively brought to rest.** When the last (outward) layer has been brought to rest the rod is in the condition shown at (*e*). The entire rod is now **stretched** and **stationary**. The time which has elapsed since the beginning of the vibration is $\frac{1}{2}\tau$. The condition of the rod for a time intermediate between $\frac{1}{4}\tau$ and $\frac{1}{2}\tau$ is shown at *d*.

The stretch in the rod (*e*) now begins to relieve itself. The end layers at the right are set in motion toward the left and the strain in the successive layers disappears. The condition shortly after the stretch begins to relieve itself is represented (*f*). When the stretch has been entirely relieved, all parts of the rod are moving to the left (*g*). The time elapsed is $\frac{3}{4}\tau$.

The motion of the rod is now stopped by a compression which sets in at the wall end of the rod. After this compression has started the conditions are like those shown at (*h*). When all layers have been brought to rest the rod is in its initial condition (*i*) and (*a*). The time elapsed is (τ). The rod will now begin a second vibration like that outlined above. These vibrations will continue until the energy of the vibrating rod has all been frittered away in friction effects or spread through the surrounding medium in wave motion.

It is interesting to note the energy transformation in the vibrating rod. At (*a*), (*e*) and (*i*) the energy of the rod is potential, like that of a stretched or compressed spring. At (*c*) and (*g*) the energy of the rod is kinetic. At intermediate conditions the energy is partly potential and partly kinetic.

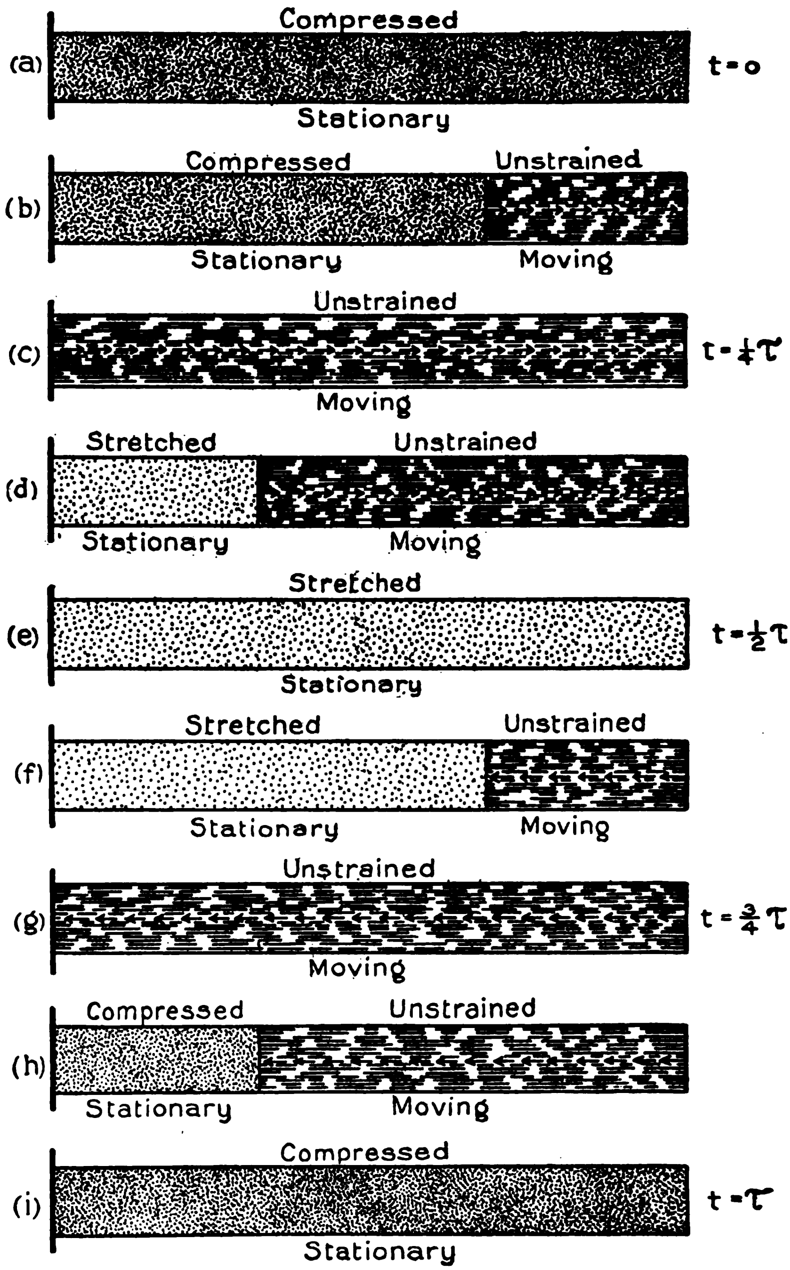

Compressed
(a)
t = o
Stationary
Compressed
Unstrained
(b)
Stationary
Moving
Unstrained
(c)
t = ¼τ
Moving
Stretched
Unstrained
(d)
Stationary
Moving
Stretched
(e)
t = ½τ
Stationary
Stretched
Unstrained
(f)
Stationary
Moving
Unstrained
(g)
t = ¾τ
Moving
Compressed
Unstrained
(h)
Stationary
Moving
Compressed
(i)
t = τ
Stationary

FIG. 311.

A rod clamped at its center, or a string stretched between two supports, may be set into vibratory motion analogous to that of the rod in Figure 311. The motion of a rod clamped at its center is such that its ends move in opposite directions, each half having a motion like that described above.

THE TUNING FORK

459. When a straight rod is caused to vibrate transversely, the nodal points *a* and *b*, Figure 312, are at a distance from the ends of about $\frac{1}{6}$ the total length of the rod. If the rod is curved as shown at *B*, Figure 312, it will be found that the nodal points are nearer together. If the rod is more sharply curved as at *C* and *D*, the nodal points will be found to be still closer together. Finally, if the rod is bent into the form shown at *E*, the vibrating segment at the center is quite short and especially if the rod is made thick in this region. A tuning fork consists of a steel rod bent into the shape represented at *E* and supplied with a stem at the center. The principal object in giving the tuning fork this form is to exclude the secondary or harmonic vibrations which are present in the straight bar. These secondary tones being excluded, the tuning fork gives a very pure musical tone.

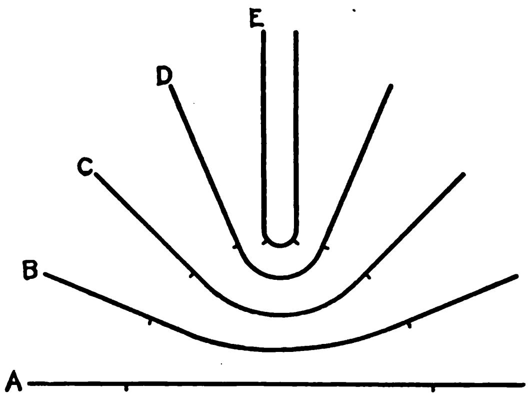


FIG. 312.

THE VIBRATION OF PLATES

460. The vibration of a plate may be examined by regarding it as a bar from each of two adjacent edges, and by considering the manner in which the vibrations in these two

directions are combined. For example, in Figure 313, let *ABCD* represent a vibrating plate. Let it be assumed that, as regarded from the edge *AB*, it is vibrating like the bar represented in Figure 310. Evidently there will be two nodal

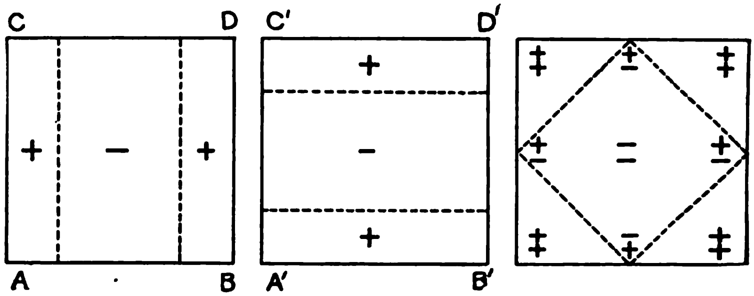


FIG. 313.

lines across the plate as represented by the dotted lines in the figure. Let it be assumed that at a given instant the edges of the plate are rising while the center of the plate is falling. This may be represented by the + and − signs as indicated in the figure. Let it be assumed that the plate regarded from the edge *AC* is also vibrating like the bar represented in Figure 310. Let it be further assumed that at the instant under consideration the plate in response to this vibratory motion is rising at the edges and falling in the center as indicated at $A'B'C'D'$. Now it will be evident that the actual motion of any part of the plate is the sum of the motions due to these two vibratory motions. For example, the center of the plate is falling under both vibratory motions, the four corners of the plate are rising under the influence of each of the two vibratory motions. Evidently there is a point at the center of each edge which is rising in response to one of the vibrations and falling in response to the other. These points will, therefore, remain stationary. Upon examination it is found that under these circumstances the nodal lines upon the plate are as shown by the dotted lines in the square at the right of Figure 313.

If it is assumed that the vibratory motion of the plate at the moment under consideration is the opposite of that shown at $A'B'C'D'$, then the result obtained is that represented in Fig-

ure 314. Under these circumstances the central part of the plate remains stationary, and the corners of the plate remain stationary, while the center points of the edges rise and fall under the influence of both of the component vibrations.

The nodal lines represented in Figures 313 and 314 are readily obtained by placing a vibrating plate in a horizontal position and sprinkling the surface with sand. The sand is driven away from the vibrating segments and collects along the nodal lines, outlining them in a very definite manner. The vibrations

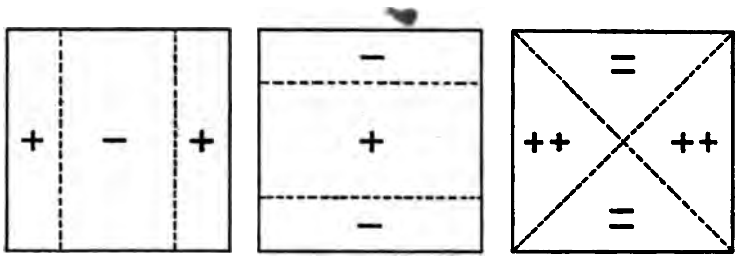


FIG. 314.

considered in the above discussions represent the more simple cases. Ordinarily a plate breaks up into a great many more vibrating segments than are indicated in these discussions. The vibrations represented in Figures 313 and 314 correspond to the lowest pitches which the vibrating or sounding body is capable of giving.

Problems

1. What is the pitch of the fundamental tone of an open pipe 100 cm. long? Temperature = 0° C.

2. What is the pitch of the first overtone of the pipe of problem 1? Of the second overtone?

3. What are the pitches of the fundamentals and first and second overtones of a closed pipe 100 cm. long? Temperature = 0° C.

4. An open pipe is giving its fundamental tone. A hole at the middle of its length is suddenly opened. What effect is produced?

5. If the pipe of problem 4 is giving its first overtone and the hole is opened, what effect will be produced?

6. What are the wave lengths of the sound waves given off by the pipe of problem 3?

7. A tuning fork making 256 vibrations per second is fixed in front of a tube the length of which is adjusted to resonance at 0° C. What change in the length of the tube will be necessary to secure resonance at 20° C.?

8. A closed tube is adjusted in length to give resonance with a tuning fork making 64 vibrations per second. Give the frequencies of three other forks of higher pitch to which this tube will also respond.

9. Three closed pipes containing air, oxygen, and hydrogen at 20° C. are of such length as to respond when giving their fundamental tones to a fork having a frequency of 1000. What are their lengths?

10. An organ pipe containing air at 20° C. gives a tone having a pitch of 500. What will be the pitch when the pipe is filled with hydrogen at the same temperature?

PART V

LIGHT

LIGHT

CHAPTER XXXIX

THE NATURE OF LIGHT

461. It was pointed out in the discussion of **heat waves** that a hot body gives off waves of different wave lengths. Those which produce heating effects are called heat waves, those which affect the optic nerve are called light waves. Aside from the question of wave length, however, these two kinds of waves are identical. They travel in the same medium (the ether) with the same velocity, and obey the same laws of reflection, refraction, etc. In the subject at hand, we shall confine our attention to a discussion of those wave lengths which are capable of affecting the optic nerve.

Various theories have been advanced in attempts to explain the manner in which a luminous body is capable of affecting the eye. One of the theories which held its ground for a long time was known as the **corpuscular theory**. This theory assumes that a luminous body is continually throwing off small particles of matter. These particles are repelled from the luminous body at very high velocities. Falling upon the surface of other bodies, they are reflected, thus rendering these bodies luminous. Falling directly upon the retina of the eye, they produce the sensation of light.

Although this theory was for a time popular, it was eventually displaced by the **wave theory**, which is the one at present universally accepted. This theory assumes that the disturbance known as **light consists of a wave motion in the medium known as the ether**. That the ether is the medium of propagation of these light waves is evidenced by various facts and phenomena; for example, the ordinary incandescent lamp bulb is exhausted of air and other gases, the vacuum being made

as nearly perfect as is possible with the best of air pumps. But the incandescent filament of such a lamp is capable of illuminating its surroundings in spite of the fact that the disturbance which travels outward from the filament must pass through a vacuum. The fact that the light of the sun reaches the earth is perhaps sufficient proof that light does not require for its transmission a material medium. This is evident from the fact that practically the entire space between the luminous surface of the sun and the earth is devoid of matter, at least in the ordinary sense, that is to say, it is a vacuum.

The real nature of this wave motion will be better understood after we have discussed some of the various phenomena which serve to prove that light consists of a wave motion. For the present it will suffice to state that this wave motion is a transverse wave motion, and that it travels through the medium of propagation, the ether, with a finite velocity. The manner in which the velocity of the disturbance is determined and the proofs for the statement that the waves are transverse are described below.

THE VELOCITY OF LIGHT

462. The velocity of light is so great that for a long time it was considered to be infinite. That light does require a definite length of time for traveling a given distance was first determined by **Roemer** in 1675. Roemer came upon this discovery in an accidental way while making astronomical observations upon one of the satellites of the planet Jupiter. He was endeavoring to determine the period of the satellite, that is to say, the time required by the satellite to make one complete revolution about the planet Jupiter. His observations extended over a period of many months. In comparing the results of his observations he found that, beginning at that time of year at which the earth was situated directly between Jupiter and the sun, for a period of half a year the successive observations taken upon Jupiter's satellite showed that the period was apparently increasing. Continuing his observations for another six months he determined that the period of the planet during this interval was apparently decreasing so

that at the end of a year the period of the planet was once more the same as that observed at the beginning. Roemer's explanation of this apparent variation in the period of Jupiter's satellite was as follows: Referring to Figure 315, let S represent the sun, EE' the orbit of the earth about the sun, JJ' the orbit of the planet Jupiter.

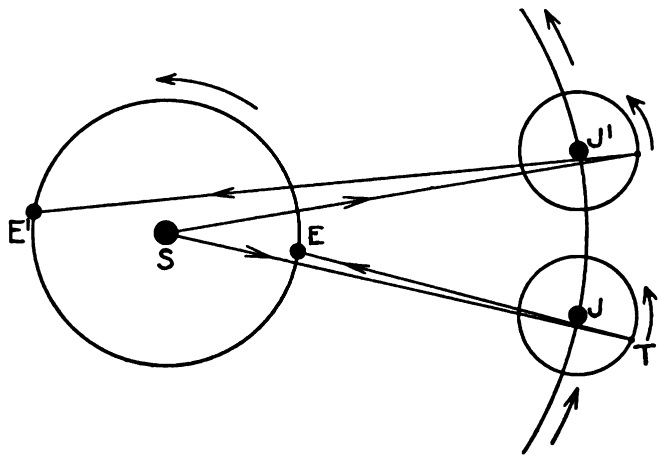


FIG. 315. — Illustrating Roemer's Method.

orbit of the planet Jupiter. The small circles drawn about J and J' represent the orbit of the satellite T. The curved arrows indicate the direction in which the various bodies are supposed to be moving in their orbits. When the earth is in the position E and Jupiter is in the position J, the distance between the earth and the planet is given by the **difference** between the radii of their orbits. When the earth is in the position E', six months later, the planet Jupiter will have passed to some such position as J' (it requires about 12 years for Jupiter to pass once around its orbit). Under these circumstances the distance between the earth and the planet is equal to the **sum** of the radii of their orbits. Thus, the distance between the earth and the planet when the earth is at E' is greater than the distance between the earth and the planet when the earth is at E by the distance EE', that is, by the diameter of the earth's orbit. Therefore a light signal passing from the planet Jupiter in its J' position would have to travel

farther to reach the earth by the distance EE' than it would have to travel when the planet is in the J position. If light travels with finite velocity, a definite interval would be required for the light to travel this extra distance. Roemer found in his observations that the light signal came apparently 1000 seconds too late when the earth is at E'. The mean diameter of the earth's orbit is 186,000,000 miles. Therefore, if this explanation of the observed facts is correct, it required 1000 seconds for light to travel 186,000,000 miles. In one second, therefore, it would travel the thousandth part of this distance or 186,000 miles. Roemer's explanation of the facts observed by him, while undoubtedly the true one, was not well received and in a short time was forgotten. Fifty years later a noted English astronomer, by the name of Bradley, determined the velocity of light by an entirely different astronomical method, obtaining practically the same result as that obtained by Roemer. This served to direct the attention of the scientific world to the work of Roemer, who was then given due credit for the discovery he had made.

BRADLEY'S DETERMINATION OF THE VELOCITY OF LIGHT

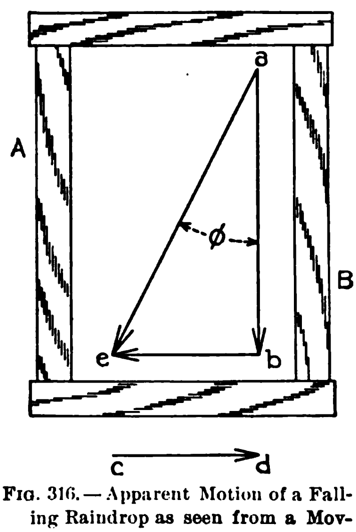


FIG. 316. — Apparent Motion of a Falling Raindrop as seen from a Moving Car.

463. Bradley's determination of the velocity of light was based upon the principle of **aberration.** This principle, briefly stated, is as follows: **The apparent velocity of one body as seen from a second body is given by the vector difference of the two individual velocities.** Consider, for example, the case of falling raindrops as viewed from a moving train. Let it be assumed that the rain is falling vertically. Let the magnitude and direction of this velocity be represented by the arrow *ab*, Figure 316. Let the arrow *cd* represent the velocity

of the car. Then, according to the principle given above, the apparent velocity of the raindrops is given by *ae*, the vector sum of *ab* and $-cd$, that is to say, the vector difference between *ab* and *cd* (Section 21). Let *AB* represent the frame of a car window. Then to a person within the car a raindrop appearing at *a* will travel across the window in the direction *ae*. This effect of the apparent change in the direction of motion of the one body (the raindrop) due to the motion of the body from which the observation is taken (the car) is called **aberration**. The angle ϕ is called the angle of aberration.

Just as the apparent motion of the raindrops is different from their real motion, so the apparent direction of a wave motion is altered by the motion of the observer. Let the dotted circle, Figure 317, represent the orbit of the earth, the curved arrows indicating the direction in which the earth moves in its orbit. When the earth is at *A*, it is moving toward the right; when at *B*, it is moving toward the left. Let it be imagined that from the earth as it travels about its orbit observations are being made upon a star located, for example, at *S*. It will be evident from the discussion given above that when the observation is taken from *A* the star will be apparently displaced to some such position as *S'*. When the observation is taken from *B*, the apparent displacement of the star will be in the opposite direction, the star appearing in some such position as *S''*. When the earth

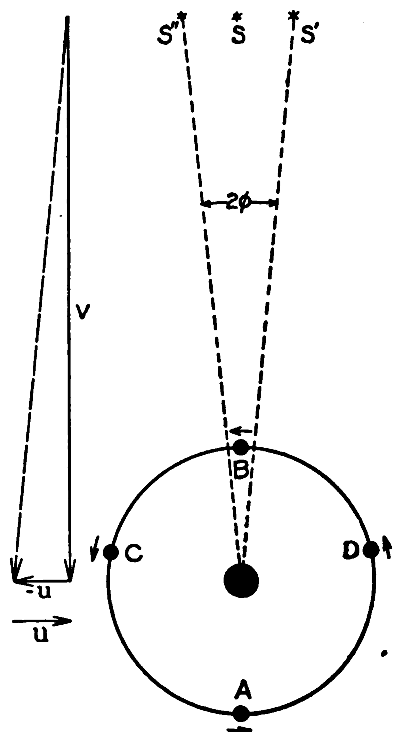


FIG 317.—Illustrating Bradley's Method.

reaches some such position as C or D, the star will be seen in its true position. By taking observations upon the star, while the earth makes one complete revolution in its orbit, the double displacement $S'S''$ of the star is readily determined. This double displacement is determined as an angle and is evidently equal to twice the angle of aberration. Call the velocity of light v. Let the velocity of the earth in its orbit be represented by u, we have, then,

$$\tan\phi = \frac{u}{v}$$

or,

$$v = \frac{u}{\tan\phi} \tag{121}$$

If u is known and the tangent of ϕ determined by measurement, we have at once the means of calculating the velocity of light.

FOUCAULT'S METHOD

464. In 1849 **Fizeau** devised a method for determining the velocity of light by measuring the time required for light to travel over a comparatively short distance on the surface of the earth. About one year later **Foucault** developed what might be called a laboratory method. The essential parts of Foucault's apparatus are shown in Figure 318. S is a source of light, M

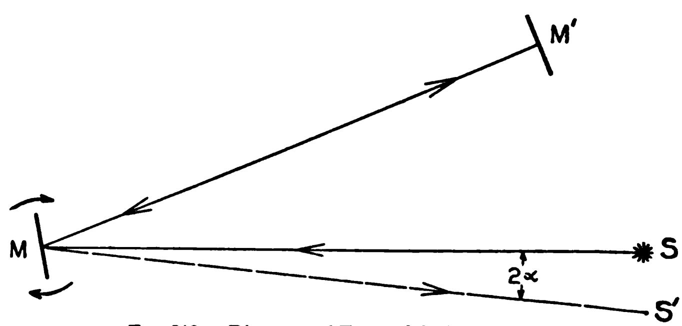


FIG. 318.—Diagram of Foucault's Apparatus.

is a plane mirror revolving at high velocity about an axis in its own plane and perpendicular to the paper as shown in the diagram, the direction of rotation being indicated by the curved arrow. The beam of light proceeding from the source S falls

upon the mirror M, and is thence reflected to a second mirror M', which is stationary. It falls upon this second mirror perpendicular to its surface and therefore returns along the same path to the revolving mirror M. If now M were stationary, the light would also retrace the path SM. Since, however, the disturbance requires an appreciable time to pass from M to M' and back again, the mirror M will have turned through a small angle before the disturbance again reaches it. Therefore, instead of being reflected back to S, it will be reflected to some such point as S'. The angle 2α is twice the angle through which the mirror has turned in the interval during which the disturbance travels from M to M' and back again. Let it be assumed that the mirror makes n revolutions per second. It will therefore require $\frac{1}{n}$ second for it to make one revolution. Evidently if it requires $\frac{1}{n}$ second to turn through the angle 2π, the time required for it to turn through the angle α is given by the following relation:

$$\frac{t}{\frac{1}{n}} = \frac{\alpha}{2\pi}.$$

or,

$$t = \frac{\alpha}{2\pi n}$$

where t is the time required for the mirror to turn through the angle α. This is also the time required for light to travel a distance $2MM'$. We have, therefore,

$$t = \frac{2MM'}{v}$$

Equating these two values of t, we obtain

$$v = \frac{4\pi n \cdot MM'}{\alpha} \qquad (122)$$

Hence, knowing the speed of the mirror M, the distance MM', and the angle α, the velocity of light is readily calculated from this formula. 2α is given by the ratio of the distance SS' to SM.

The best determinations of the velocity of light have been made with this form of apparatus.

The most recent determinations of the velocity of light indicate that its value is not far from

$$v = 3 \times 10^{10} \text{ centimeters per second}$$
$$= 186000, \text{ miles per second (approximately)}.$$

THE INDEX OF REFRACTION

465. One of the most important applications of the laboratory method for the determination of the velocity of light has been in the investigation of the value of the velocity of light in different media. A study of this kind is readily carried out, for example, with Foucault's apparatus by placing between the mirrors M and M' a long tube containing the medium under investigation. The tube is provided with glass ends and is so placed that the light in traversing the distance MM' and back again must travel lengthwise through the tube.

The ratio of the velocity of light in air to its velocity in a second medium is called the *index of refraction* of the second medium as referred to air. This ratio is found to be greater than unity for all substances whose densities are greater than that of air. This fact, when discovered, had a decided effect upon the displacement of the old corpuscular theory by the new wave theory, since under the old theory it was necessary to assume that light traveled more rapidly in a dense medium than in a rare medium.

THE RECTILINEAR PROPAGATION OF LIGHT

466. Light travels in straight lines in all directions from its source. This is evidenced by the fact that the light proceeding from any luminous body is capable of forming an image of that body. The formation of an image in this manner will be understood by reference to Figure 319. Let AB represent a box having a single small opening at A. Imagine a luminous object CD to be standing in front of this box. Each point on this luminous object may be thought of as a distinct and separate source of light. Consider the light which is being emitted by the point C at the upper extremity of this object. This disturbance travels in all directions. A very small part of this

disturbance will find its way through the opening A and fall upon some such point as B on the opposite wall of the box. **Thus B is a luminous point, in effect an image, of the point C.** In the same way the point D will produce an image of itself at

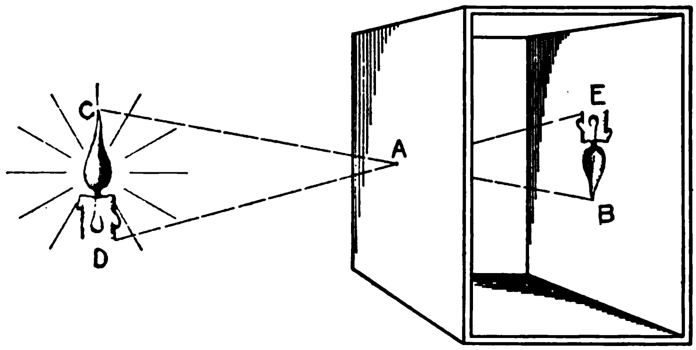


FIG. 319.

E and the light proceeding from points intermediate between C and D will fall upon corresponding points between B and E. Thus, **for every point on the object CD there is a corresponding point in the image BE.** The fact that CD is thus able to form an image of itself at BE is proof of the fact that light travels in straight lines.

WAVE FRONT AND RAY

467. The *wave front* of a wave is defined as a line drawn through all points on the wave which are in the same condition as regards displacement and direction of motion. For example, a line drawn along the crest of a water wave would be a wave front. A line drawn along the trough of a water wave or a line drawn along the side of a water wave joining particles which are equally displaced from their positions of rest, would also be a wave front.

The term *ray of light*, as it will be used in the following discussions, **is intended to indicate a line drawn perpendicular to the wave front.**

A wave front always moves perpendicular to itself; hence a ray is a line drawn through a wave front to indicate the direc-

tion in which the disturbance is traveling. For example, the lines *CB* and *DE*, Figure 319, are rays. These rays are not to be thought of as limiting in any sense the direction in which the disturbance is spreading, since we know that the disturbance proceeding from each point in the luminous object *CD* is radiated in all directions.

HUYGHENS' PRINCIPLE

468. In determining the position of a reflected or refracted wave the application of what is known as Huyghens' principle is found to be of service. This principle is as follows: **Each point on a wave may be regarded as a separate disturbance, and the combination of the secondary wavelets proceeding from these individual sources determines the position of the advancing wave.** The principle is made clear by the example represented in Figure 320. Let *S* represent a source of light and *W* a part of one of the spherical waves proceeding from it. Let it be required to find the position of this wave after the lapse of the time *t*. Let *WW'* represent the distance over which the disturbance travels in the given time. **With the successive points *A*, *B*, *C*, *D*, etc., on the wave *WW* as centers, draw circles each having a radius *WW'*.** Then the envelope of these secondary wavelets will be the wave front *W'W'* required. In this envelope the secondary wavelets conspire to produce a maximum disturbance. At other points they interfere in such manner that practically the entire disturbance *WW* is handed on, as indicated, to *W'W'*.

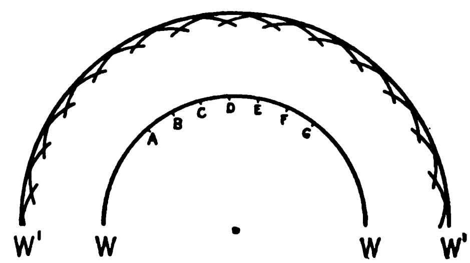


FIG. 320.

HUYGHENS' CONSTRUCTION FOR A REFLECTED WAVE

469. The application of Huyghens' principle will be understood by considering the following cases: Let it be required, for example, to determine the position of a plane wave after it

had been reflected by a plane mirror. **The application of Huyghens' principle to this case, and in fact to all cases of reflection, consists in considering each point of the approaching wave as it reaches the mirror a secondary source of disturbance.** Then, by combining the secondary wavelets which proceed from these several sources, the total reflected wave is found. In Figure 321 let *WW* represent a plane wave approaching the plane mirror *MM*. Let it be required to find the position of this

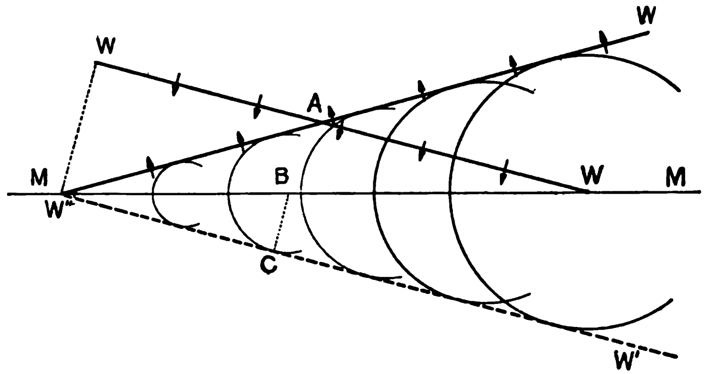


FIG. 321.

wave after reflection. **The first step in the application of Huyghens' principle is to draw** what is known as **the "dotted position"** of the **wave** *W' W''*. This is the position to which the wave would pass, in the length of time required for the disturbance to travel from *W* to *W''*, if the mirror were absent. In the presence of the mirror, however, the entire disturbance is turned back, so that after reflection it is traveling on the same side of the mirror *MM*. Consider that point of the wave *WW* which is in contact with the mirror at *M*. This part of the disturbance begins, at the instant corresponding to the position of the wave shown in the figure, to travel back in the medium above *MM*. Evidently it will travel as far above *MM* as it would have traveled below in the same length of time, that is, *WW'*. If, therefore, a circle is drawn about the point *M* with a radius equal to *WW'*, it will be evident that the reflected disturbance corresponding to the first point of the wave to come in contact with the mirror will have reached some

point on this circle above *MM*, when the other end of the wave has reached the mirror. In like manner, when the center point of the wave has reached the mirror at *B*, it is reflected and begins to travel back from *MM* and at the moment that *W* comes to *W''* the *A* disturbance will evidently have traveled from the mirror a distance equal to *BC*, that is, equal to the distance which it would have traveled forward in the same length of time had its course been unobstructed by the mirror. If, therefore, a circle is described about the point *B* as a center and having a radius equal to *CB*, it is evident the *A* disturbance will lie somewhere on this arc above *B*. **Thus, the disturbance which travels away from the mirror from each point on its surface is determined by drawing circles about the several points as centers tangent to the dotted position.** Evidently the reflected wave is the envelope of these circles on the opposite side from the dotted position. Thus in Figure 321 *W''AW* is the reflected wave.

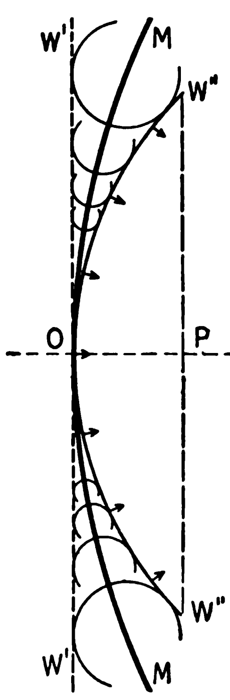


Fig. 322.

A PLANE WAVE REFLECTED FROM A CONCAVE MIRROR

470. The position of a plane wave after reflection at a curved surface is obtained in a similar manner. In Figure 322 let *P* represent a plane wave approaching the concave mirror *MM*. **Let *W'W'* represent the dotted position of the plane wave, that is, the position to which the plane wave would have passed in the absence of the mirror.** Taking the successive points on the mirror *MM* as centers, draw circles which are tangent to the dotted position. Then *W''W''*, the envelope of the small circles, is the reflected wave, its direction of motion being indicated by the small arrows. The distances *MW'* and *MW''* are equal, each being measured perpendicular to the corresponding wave front; but these two

wave fronts are parallel for points near O, and for such points the distances MW' and MW'' would be measured perpendicular to the line $W'W'$. The distance OP, the greatest distance between an arc and its chord, is called the **sagitta of the arc.** The above statement with reference to the distances MW' and MW'' is therefore equivalent to saying that **the sagitta of the reflected wave is twice that of the mirror, provided but a small portion of the wave in the neighborhood of O is considered.** This being the case, it is an easy matter to show the relation which exists between the radius of curvature of the reflected wave and the radius of curvature of the mirror.

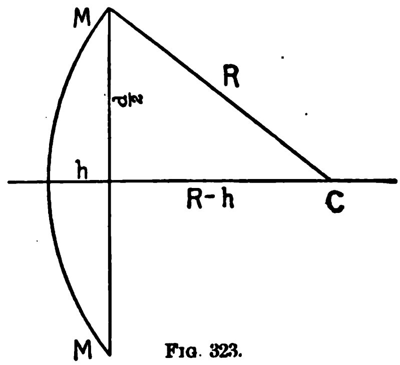


FIG. 323.

The relation between the sagitta of an arc and its radius is obtained as follows. Let MM, Figure 323, be the arc of a circle having its center at C. Let the straight line MM be the chord of this arc. Call the distance MC, that is, the radius of the arc, R. Call the sagitta h. Call the chord d. Then evidently the distance from the chord to the center of the circle is $R-h$. Therefore from the right-angle triangle, we have,

$$R^2 = (R-h)^2 + \frac{d^2}{4}$$

That is,

$$R^2 = R^2 - 2\,Rh + h^2 + \frac{d^2}{4}$$

Whence,

$$h = \frac{d^2}{8\,R} \tag{123}$$

providing h is so small that its square may be neglected, which is usually the case in the application of this formula to curved mirrors. In the example given in the last paragraph, let it be assumed that the sagitta of the mirror is h. Call the sagitta of the reflected wave K. From the relation given in Equation (123), we therefore have

$$h = \frac{d^2}{8\,R} \text{ and } K = 2\,h = \frac{d^2}{8\,b}$$

in which R is the radius of curvature of the mirror and b is the radius of curvature of the reflected wave. Combining these equations, we obtain,

$$b = \frac{R}{2}$$

In other words, the radius of curvature of the reflected wave under the assumed conditions would be one half the radius of curvature of the mirror.

CONVEX AND CONCAVE WAVES

471. **A wave front is said to be convex if its rays diverge. A concave wave front is one whose rays converge.** Thus WW,

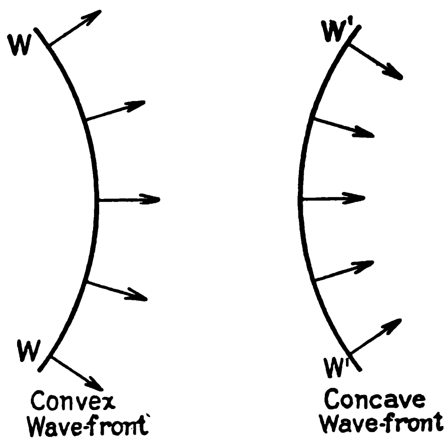


FIG. 324.

Figure 324, is a convex wave front. $W'W'$ is a concave wave front.

A CONVEX WAVE REFLECTED BY A CONCAVE MIRROR

472. The case of a convex wave reflected from a concave surface is of particular importance. In Figure 325, let MM represent the spherical mirror, WW' the dotted position of the incident wave, $W''W''$ the position of the reflected wave at the moment the last point on the wave leaves the mirror. By

construction cb is equal to cd. Call the sagitta of the mirror h, the sagitta of the approaching wave h', and the sagitta of the reflected wave h''. That is,

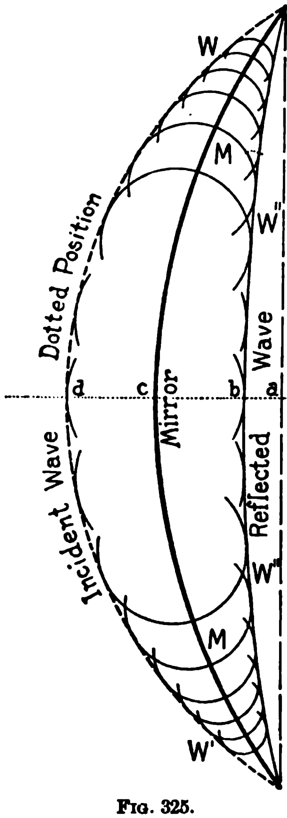


FIG. 325.

$$ac = h$$
$$ad = h'$$
and $$ab = h''$$
Then evidently, $$cd = h' - h$$
and $$bd = 2(h' - h)$$
But $$ab = ad - bd$$
$$\therefore h'' = h' - 2(h' - h)$$
or, $$h'' = 2h - h'$$

If we call the radius of curvature of the mirror R, the radius of curvature of the approaching wave a, and the radius of curvature of the reflected wave b, we have from the last section

$$h' = \frac{d^2}{8a},\ h = \frac{d^2}{8R},\ \text{and}\ h'' = \frac{d^2}{8b}$$

Substituting these values in the above equation, we obtain,

$$\frac{d^2}{8b} = \frac{2d^2}{8R} - \frac{d^2}{8a}$$

or finally, $$\frac{1}{a} + \frac{1}{b} = \frac{2}{R} \qquad (124)$$

THE FORMATION OF IMAGES BY MIRRORS

473. A real image of an object point is a point at which the light, proceeding from the object point, is concentrated or focused after being reflected by the mirror. A virtual image of an object point is a point from which the light proceeding from the object point appears to come after being reflected by the mirror.

The case discussed in the last section is again represented in Figure 326, in which MM is the curved mirror. The incident and reflected disturbances are represented by rays. O is sup-

posed to represent a source of light. **The spherical waves** proceeding from O toward the mirror are reflected and focused at I. $OP = a$, the radius of the incident waves; $IP = b$, the radius of the reflected waves; $CP = R$, the radius of the mirror MM. I, the point at which the disturbance proceeding from O is focused after reflection from the mirror M, is called

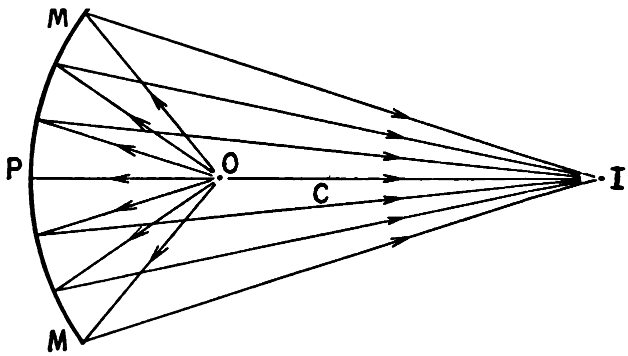


FIG. 326.—Conjugate Points.

the conjugate of O, and O and I are called **conjugate points.** I is also called a real image of the point O. Evidently in this case the effect of the mirror has been to change the form of the wave front from convex to concave. In the discussion of Section 472 it was assumed that the curvature of the incident wave was in the same direction as that of the mirror, that is, that the center of curvature of the mirror and the incident wave lay on the same side of the mirror. Under this heading there are certain special cases that it is worth while to note in particular.

CASE I. $R = \infty$. If the mirror is plane, then $R = \infty$. We have, therefore, from Equation (124),

$$\frac{1}{a} + \frac{1}{b} = 0$$

whence,

$$b = -a$$

That is to say, the curvature of a wave reflected from a plane mirror is the same as the curvature of the incident wave but opposite in direction. Hence a wave reflected from a plane mirror appears to come from a point as far behind the mirror

as the real object, or source of the wave, is in front of the mirror. See Figure 327.

CASE II. $a = \infty$. If the incident wave is plane, then $a = \infty$. Therefore from Equation 124, $b = \frac{R}{2}$. This case has already been fully discussed in Section 470. Evidently if $a = \frac{R}{2}$, then $b = \infty$. That is to say, a spherical wave originating at a point in front of a concave spherical mirror half way from the mirror to its center of curvature, will be plane after reflection. It will also be evident that a spherical wave having its origin at a point less than the distance $\frac{R}{2}$ from the concave mirror, will after reflection have a curvature opposite to that of mirror. This is indicated in the formula by the negative sign which appears before the expression for b.

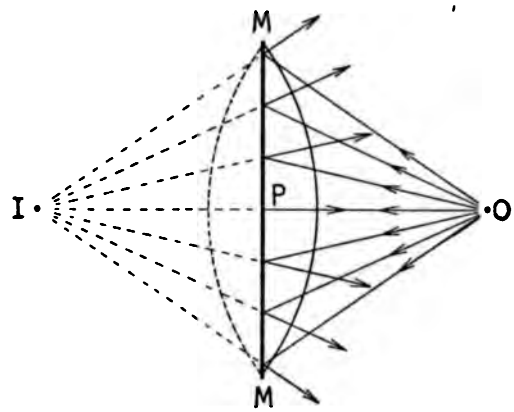


FIG. 327.

CASE III. $a = R$. If $a = R$, then from Equation (124) we have $b = R = a$. That is, a disturbance originating at the center of curvature of the mirror will return after reflection to the point from which it started. In other words, under these circumstances, the image and object coincide.

A CONVEX WAVE REFLECTED BY A CONVEX MIRROR

474. When the curvature of an incident wave is opposite in direction to that of the mirror, Equation (124) becomes

$$\frac{1}{b} - \frac{1}{a} = \frac{2}{R}$$

This will be evident from the following considerations: Let MM, Figure 328, represent a convex mirror of radius R, hav-

ing its center of curvature at the left. Let $W'W'$ be the dotted position of an incident wave having its center of curvature at the right. Then $W''W''$ is the reflected wave as determined by Huyghens' construction. In this case, evidently,

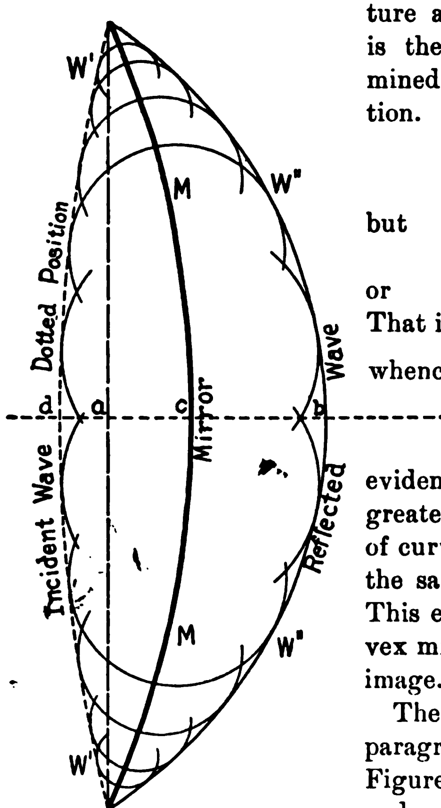


FIG. 328.

$$bd = 2\,cd$$
$$\therefore ab = 2\,cd - ad$$
but
$$cd = ac + ad$$
$$\therefore ab = 2(ac + ad) - ad$$
or
$$ab = 2\,ac + ad$$
That is,
$$h'' = 2\,h + h'$$
whence,
$$\frac{1}{b} - \frac{1}{a} = \frac{2}{R} \quad \text{(See equation 123)}$$

From the figure it is evident that h'' will always be greater than h and the direction of curvature of the reflected wave the same as that of the mirror. This explains the fact that a convex mirror always gives a virtual image.

The case discussed in the last paragraph is represented again in Figure 329, in which the incident and reflected disturbances are represented by rays. MM is the convex mirror having its center of curvature at C. O is the object or point from which light passes to the mirror. I is the virtual image of O, that is the center of curvature of the reflected wave or point from which the reflected disturbances apparently proceed.

REMARK. One important assumption has been made in the derivation of the above expressions; namely, that the reflected wave is spherical. This assumption is justified provided a limited portion of the wave only is considered. In other words **these laws will be found to hold only in those cases in which the**

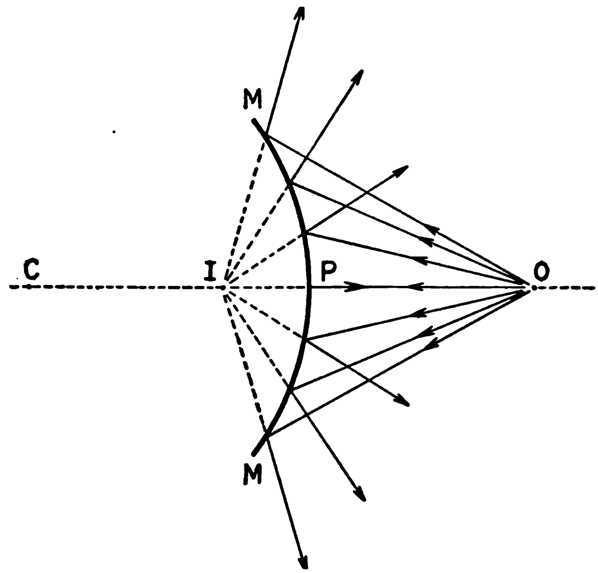


FIG. 329.

width of the mirror is small as compared to its radius of curvature. In Figures 325 and 328 the sagittas of the mirrors and the waves are greatly exaggerated for the sake of clearness.

Problems

1. In an arrangement of Foucault's apparatus the distance between mirrors is 5 km. What is the angular velocity of the rotating mirror that gives $\alpha = 3^\circ$?

2. Draw Huyghens' diagram for a plane wave reflected from a curved mirror.

3. Show by diagram that the image formed by a plane mirror appears to be as far behind the mirror as the object is in front.

4. If a wave after reflection at a plane mirror is to converge to a point, what must be its form before reflection?

5. Two mirrors are placed at an angle of 90°. A candle is placed between them. Locate the images.

6. Plane waves falling upon a concave mirror are focused at a point 15 in. from the mirror. What is the radius of curvature of the mirror?

7. A luminous object stands 30 in. in front of a concave mirror having a radius of curvature of 35 in. What is the distance of the image from the mirror?

8. A luminous object stands 20 cm. in front of a concave mirror. The radius of curvature of the mirror is 8 cm. Determine by diagram the position and size of the image.

9. Assume the curvature of the mirror in problem 8 to be reversed. Determine position and size of the image.

10. A concave mirror has a radius of curvature of 50 cm. Determine two pairs of conjugate foci.

REFRACTION

CHAPTER XL

THE BENDING OF A BEAM OF LIGHT

475. When a wave front passes from one medium into another of different density, it is said to be refracted. The refraction of a wave front is usually accompanied by a change in its direction. The simplest case of refraction is that of a plane wave passing from one medium to another, the surface separating the media being also plane. Consider the case represented in Figure 330, in which *MM'* represents the interface or surface

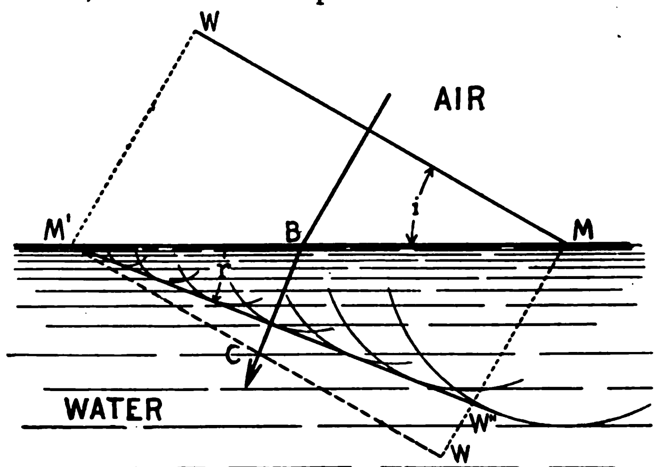


FIG. 330.

separating the two media, let us say, air and water. *MW* represents a plane wave approaching the surface as indicated. As in the case of a reflected wave **the first step in the application of Huyghens' principle is to draw the dotted position of the wave *M'CW*.** This dotted position, it will be remembered, is the position to which the wave would have gone had the second

medium, the water, been absent. In the presence of the water the disturbance at M will not travel as far in the time under consideration as it would have traveled in the same length of time in air. Since the velocity of light in water is about three fourths the velocity of light in air, evidently a disturbance entering the water at M will have traveled only three fourths as far as it would have traveled in air in the same length of time. If, therefore, a circle is described about the point M as a center with a radius equal to three fourths the distance MW, it is evident that the arc will mark the distance to which the disturbance has traveled in water while that part of the wave which is at W travels to M'. In the same way, if the center point B on the wave is considered, it is evident that instead of traveling the distance BC, which it would have traveled in air, it will travel but three fourths of the distance BC in water. Describing a circle about the point B with a radius equal to three fourths BC, evidently this arc will mark the distance to which the disturbance has traveled in the water when the disturbance W comes to M'. In like manner the disturbance in the water corresponding to any point on the approaching wave is determined. The refracted wave is then obtained by drawing the envelope of these several arcs.

It will be evident that the direction of motion of the wave front has been changed by its passing into the second medium as indicated by the large broken arrow. The amount of this change in direction is determined as follows: Call the angle between the approaching wave and the surface of the water, that is, the angle WMM', i. Call the angle between the refracted wave front and the surface of the water, that is, the angle $MM'W''$, r. Then evidently,

$$\sin i = \frac{WM'}{MM'}$$

and

$$\sin r = \frac{MW''}{MM'}$$

Dividing the first expression by the second, we obtain,

$$\frac{\sin i}{\sin r} = \frac{WM'}{MW''}$$

But $\frac{WM'}{MW''}$ **is the ratio of the distance which light travels in air to the distance which it travels in the same length of time in water.** Hence this ratio is equal to the ratio of the velocities of light in the two media. In other words, this ratio is the "index of refraction" of water as referred to air (Section 465). The symbol μ is commonly used to represent the index of refraction. We have, therefore, finally,

$$\frac{\sin i}{\sin r} = \mu \tag{125}$$

The angle i is usually known as the "angle of incidence." The angle r is called the "angle of refraction." It should be noticed that when the disturbance is traveling, as in the above case, from the rarer to the denser medium, that the wave front becomes more nearly parallel to the interface after refraction. It is evident that if the wave were traveling in the other direction so that $M'W''$ represented the wave approaching the interface, then MW would be the refracted wave. In this case the approaching wave is more nearly parallel to the interface than is the refracted wave.

THE SHALLOWING EFFECT IN WATER

476. An interesting result of refraction is the shallowing effect observed in water. Let *MM*, Figure 331, represent the surface of a shallow pond. Let *O* represent a luminous object, for example, a bright pebble lying upon the bottom. To an eye placed above *MM* this luminous object appears nearer the surface than it really is. This is known as the shallowing effect and is explained in the following manner: Consider the spherical waves which are proceeding towards the surface of the water from the point *O*. Let *WW* be the dotted position of one of the wave fronts, that is to say, the position to which the wave would have gone had it been traveling all the time in water. **Since, however, the central portion of the wave has been traveling for a certain length of time in air, it will have traveled to a position beyond the dotted position.** The actual position of the central portion of the wave when the edge

portions reach the surface is determined by a process analogous to that used in the last section. If CB represents the distance which the wave front would have traveled in water in a given length of time, then $\frac{4}{3}$ of CB will be the distance which it has traveled in air in the same length of time, since it travels in air with $\frac{4}{3}$ the velocity with which it travels in water. If, therefore, with C as a center a circle is described having a radius equal to $\frac{4}{3}$ CB, this arc will measure the distance to which the disturbance has actually traveled in air. In the same manner other arcs may be drawn about different points on the line MM as centers, each having a radius equal to $\frac{4}{3}$ the perpendicular distance from that point to the dotted position. The envelope of these several arcs determines the position of the refracted wave $W'W'$. Evidently the curvature of this wave has been increased by refraction. The directions in which the different parts of the refracted wave are proceeding are represented by the small arrows. **This wave therefore appears to come from some point O' above O.** In other words, the luminous object appears nearer the surface than it really is. From the discussion of the relation between the sagitta and the radius of curvature of an arc it will be evident that $O'C$ is equal to $\frac{3}{4}$ of OC. That is to say, since the sagitta of the wave front has been increased in the ratio of 4 to 3, its radius at the point B has been decreased in the ratio of 3 to 4. For points on the wave near M the change in curvature is greater and the shallowing effect more marked.

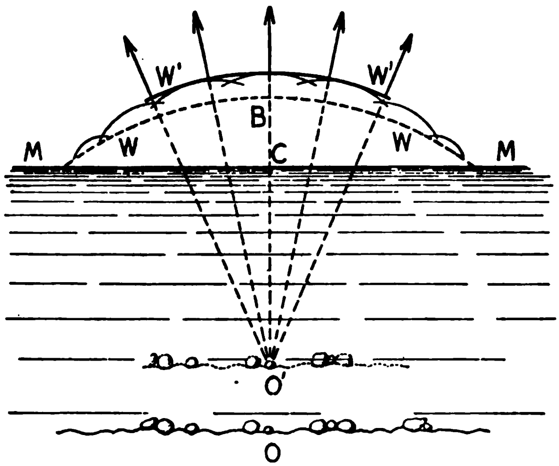


FIG. 331.—The Shallowing Effect in Water.

TOTAL REFLECTION

477. In general, when a wave front passes the surface separating media of different density a part of the disturbance is refracted and another part reflected. For example, in Figure 332, let *a* represent a ray of light falling upon the interface *MM* separating media of different density. A part *b* of the

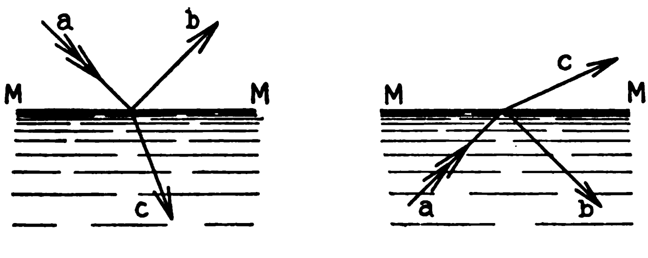


FIG. 332.

disturbance represented by the ray *a* is turned back into the same medium. A second part *c* is refracted into the second medium. Under certain conditions the refracted portion *c* is absent, that is to say, the disturbance is totally reflected. **The conditions necessary to secure this effect are: First, the wave must approach the interface from the side of the denser medium. Second, the angle of incidence must be greater than a certain angle known as the "critical angle."** The value of this angle depends upon the nature of the two media involved. In Figure 333, let *O* represent a luminous object located, let us say, in water. Consider the rays proceeding from *O* to the surface of the water *MM*. Evidently the ray *a* which falls vertically upon the surface *MM* will be transmitted without change of direction to *a′*. The ray *b* will be refracted upon entering the rarer medium and bent farther away from the vertical. The ray *c* falling at a still greater angle upon the interface *MM* will be bent still farther from the vertical. It will be evident that a certain ray proceeding still farther to the right, for example *f*, will fall upon the interface at such an angle that the refracted ray will coincide with the surface *MM*. The angle α which the ray *f* makes with the perpen-

dicular to the surface *MM* is called the critical **angle (evidently** this angle is the angle of incidence for that **portion** of the wave front which travels in the direction *Of*). **Any ray,**

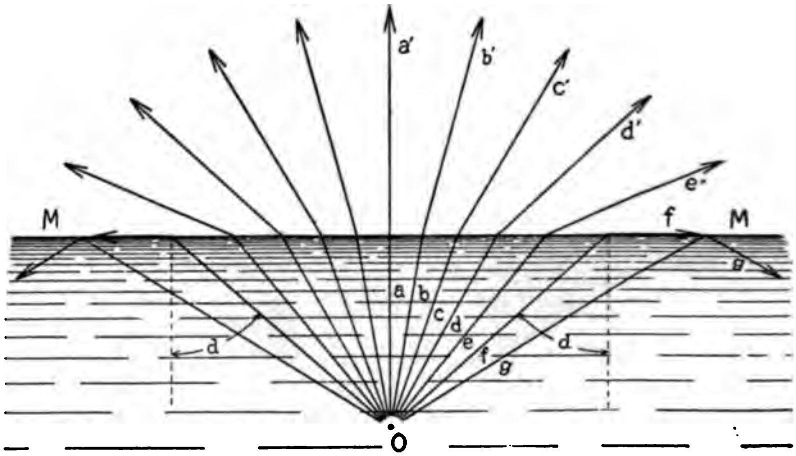


FIG. 333. — The Critical Angle.

for example *g*, for which the angle of incidence is **greater than** *a* will be totally reflected. It will be evident from the construction used in this figure that **total reflection is possible only when the ray is proceeding from the denser toward the rarer medium.**

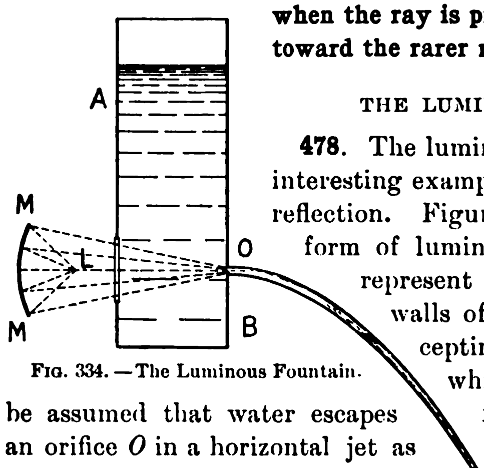


FIG. 334. — The Luminous Fountain.

THE LUMINOUS FOUNTAIN

478. The luminous fountain **affords** an interesting example of the **effect** of total reflection. Figure 334 shows **a** simple form of luminous fountain. Let *AB* represent a water-tight **tank,** the walls of which **are** opaque, excepting a small portion at *L*, which is of glass. Let it be assumed that water escapes from this tank through an orifice *O* in a horizontal jet as indicated in the figure. Let it be assumed that a powerful light is placed at the point *L* beyond which is a concave mirror *MM*. This mirror is so placed that the light which falls upon the mirror from

the source L is focused at the orifice O. **The rays of light which enter the stream of water fall upon the surface of the jet at angles which are greater than the critical angle.** They are therefore reflected back and forth within the jet, being unable to escape into the rarer medium which surrounds it. Owing to the small air bubbles and particles of foreign substance with which the stream is filled, a certain amount of this light is scattered (diffused), thus rendering the stream luminous. That part of the light which is not scattered in this manner is reflected back and forth until it reaches that portion of the jet which breaks into drops.

THE CONVEX LENS

479. A simple convex lens is a piece of glass, one or both faces of which are spherical, the lens being thicker at the center than at the edges. Such a lens produces a modification in a wave front which passes through it. The simplest case is that of the plane wave. The effect of the simple convex lens upon a plane wave is shown in Figure 335. LL is the lens and WW

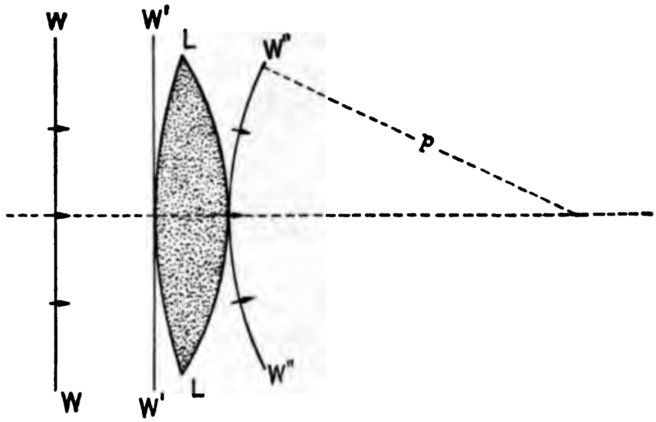


FIG. 335.

the approaching plane wave front. Consider the wave at the moment it reaches the position $W'W'$. When in this position the central portion of the wave is just entering the glass. The edge portions are still traveling in air. It will be evident that

the edge portions will run ahead of the central portion, since the velocity of light in glass is less than the velocity of light in air. Consider the wave just as the central portion is emerging on the opposite side from the glass. Call the thickness of the lens at the center s and let the index of refraction of the glass as referred to air be μ. Then evidently the edge portions of the wave will have traveled to $W''W''$ such that $W'W''$ is equal to μs. Evidently the edge portions will be ahead of the central portion by the distance $\mu s - s = s(\mu - 1)$. This distance is the sagitta of the modified wave, but since the radius of curvature of a circular arc is given by

$$R = \frac{d^2}{8h}$$

in which R is the radius of the arc, d the chord, and h the sagitta, we have, therefore,

$$p = \frac{d^2}{8s(\mu - 1)}$$

in which p is the radius of the modified wave.

Let it be assumed that the radius of curvature of one side of the lens is R_1 and the radius of curvature of the other side of the lens R_2. Then,

$$s_1 = \frac{d^2}{8R_1}$$

and,

$$s_2 = \frac{d^2}{8R_2}$$

in which s_1 and s_2 are the sagittas of the arcs forming the two sides of the lens. But evidently,

$$\begin{aligned} s &= s_1 + s_2 \\ &= \frac{d^2}{8R_1} + \frac{d^2}{8R_2} \\ &= \frac{d^2}{8}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \end{aligned}$$

Substituting this value of s in the above expression for p, we obtain

$$p = \frac{1}{(\mu - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)} \tag{126}$$

Evidently p represents the distance from the lens to the point at which the wave $W''W''$ is concentrated. This distance is known as the **principal focal length** of the lens. The point at which the plane wave is concentrated after passing the lens is known as the **principal focus.**

If one face of the lens is plane, its radius of curvature is infinite, and $\frac{1}{R}$ for this face is zero. The principal focal length of such a lens is given by,

$$p = \frac{R}{(\mu - 1)} \tag{127}$$

(put $R_2 = \infty$ in Equation 126).

THE CONCAVE LENS

480. The simple concave lens consists of a piece of glass one or both faces of which are spherical, the center of the lens being thinner than the edges. The effect of a concave lens upon a plane wave front is shown in Figure 336. Let WW represent the approaching wave, and LL the lens. Let it be assumed, for the sake of simplicity, that the thickness of the lens at the center is O. **Evidently as the wave passes such a lens its center portion will get ahead of the edge portions.** The distance by which the center portion is ahead of the edge portions after the wave passes the lens is given as before by the expression $s(\mu - 1)$ where s represents the thickness of the lens at the edge. The wave as it leaves the lens will be convex, the curvature of the wave being given by the same expression as that derived in the last section. **This wave, after passing the lens, appears to be proceeding from a point O which**

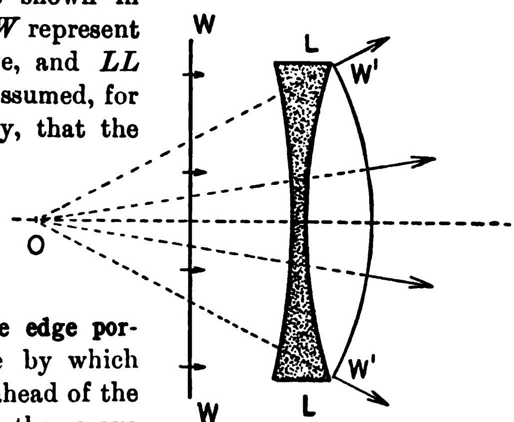


FIG. 336.

is the center of curvature of the modified wave. The distance of the point O from the center of the lens is called the principal focal length of the lens and the point O is known as the principal focus.

CONJUGATE FOCI

481. **Spherical waves proceeding from a point on the one side of a convex lens are in general brought to a focus at a point on the opposite side of the lens.** The second point is known as the conjugate of the first. The distance of the conjugate point from the lens is determined as follows. In Figure 337 let LL represent the convex lens, O the source of light from which

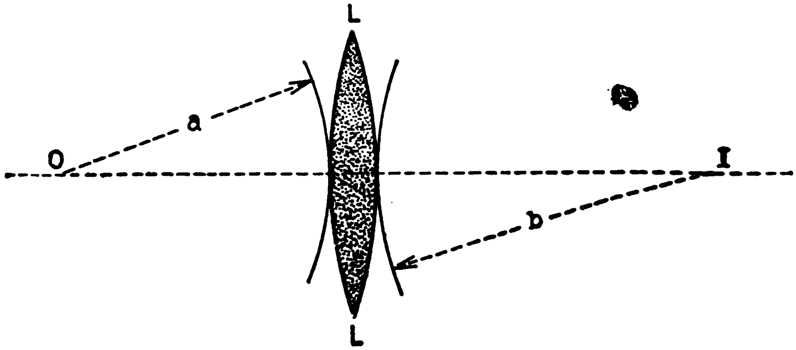


FIG. 337.

the spherical waves proceed toward the lens LL, and I the conjugate of the point O. Call the radius of the approaching wave as it falls upon the lens, a, and the radius of the modified wave front just as it leaves the lens, b. Call the sagitta of the approaching wave h and the sagitta of the receding wave k. Evidently the sum of these sagittas, that is, $h + k$, is equal to the distance which the edge portions of the wave have gained over the central portion in passing the lens. We have, therefore,

$$h + k = s(\mu - 1)$$

where s is the thickness of the lens at the center. Substituting the value of h and k in terms of a and b and of s in terms of R_1 and R_2 as above, we have,

$$\frac{d^2}{8a} + \frac{d^2}{8b} = (\mu - 1)\left(\frac{d^2}{8R_1} + \frac{d^2}{8R_2}\right)$$

That is, $$\frac{1}{a}+\frac{1}{b}=(\mu-1)\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$$

But the expression on the right is evidently equal to $\frac{1}{p}$ (Equation 126). We have, therefore, finally,

$$\frac{1}{a}+\frac{1}{b}=\frac{1}{p} \tag{128}$$

THE IMAGE FORMED BY A CONVEX LENS

482. The conjugate points O and I, Figure 337, being on the axis of the lens, their positions are best determined by the discussion given above. Points in the neighborhood of the point O, but not on the axis, are found to have conjugate points in the neighborhood of I on the opposite side of the axis. Conjugate points which are not on the axis are most readily determined by the construction shown in Figure 338. Let L

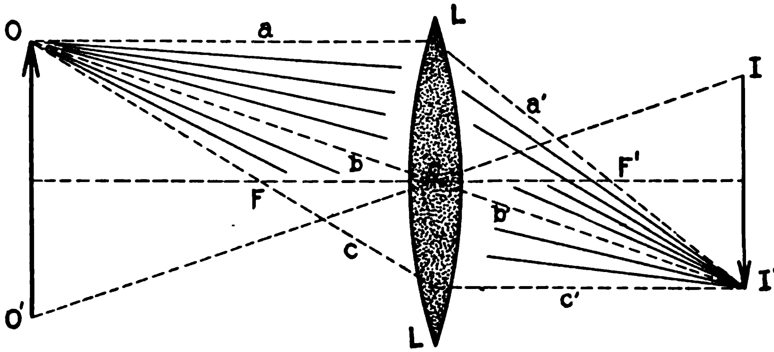


FIG. 338.

represent a convex lens of which F and F' are the principal foci, that is to say, **F' is the point at which plane waves from the left would be focused; F is the point at which plane waves from the right would be focused by the lens.** Let OO' be a luminous object. Consider the rays proceeding from the point O. The directions in which three of these rays are traveling after being refracted by the lens are readily determined. First, that ray a which passes from O toward the lens L parallel to the axis, passes, after refraction, through the point F'. Second,

that ray b which passes directly towards the center of the lens continues unchanged in direction after passing the lens. Third, that ray c which passes through F will, after refraction, pass parallel to the axis. It is seen that these three rays a, b, c are brought together at the point I'. Experiment shows that all other rays proceeding from the point O to the lens L are brought to a focus at the point I'. Thus, I' is the conjugate of O. In other words, an image of the point O is formed at the point I'. In like manner it may be shown that I is the image of O'. Also that each point lying between the points O and O' has a corresponding conjugate or image-point lying between I and I'. Thus, at II' is formed a complete image of the object OO'. It should be noted that **this image is inverted and real.**

From the construction of Figure 338, it will be evident that the size of the image is to the size of the object as the distance of the image from the center of the lens is to the distance of the object from the center of the lens. This is evident from the fact that the triangles $O'CO$ and $I'CI$ are similar by construction.

NOTE. The construction used in Figure 338 assumes that the lens is very thin, so that the rays a and c extend without change in direction to the middle of the lens, and the rays a' and c' extend without change in direction from the middle of the lens to the point I. Also that the ray b passes through the center of the lens without change in direction. The conditions assumed are approximately realized in thin lenses.

THE IMAGE FORMED BY A CONCAVE LENS

483. The principle applied in the foregoing paragraph may be used for determining the position and size of the image formed by a concave lens as follows: Let L, Figure 339, be a concave lens and F and F' its principal foci. Consider the rays from a luminous object OO'. As in the preceding case, the rays a, b, c, proceeding from the luminous point O, may be determined in their refracted positions by remembering that **the ray a which passes in the direction parallel to the axis of the lens, appears after passing the lens to come from the point F; that the ray b which passes toward the center of the lens, continues unchanged in direction after passing the lens; and the ray c which**

passes in the direction OF' will be parallel to the axis of the lens after refraction. The three rays a', b', and c' appear to come from the same point I. This is the image of the point O. In the same manner it may be shown that all rays proceeding from

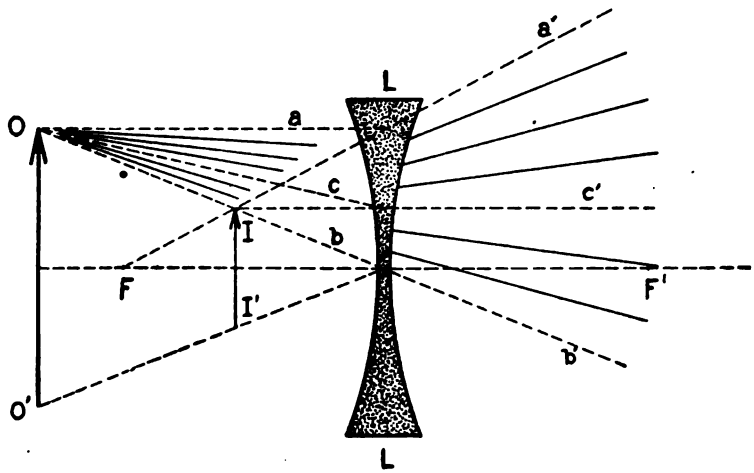


Fig. 339.

O' to the lens will, after passing the lens, appear to come from the point I'. Points intermediate between O and O' have their conjugate points between I and I'. Thus II' is an image of the object OO'. It should be observed that **this image II' is a virtual image**, that is to say, unlike the image formed by the convex lens, it exists only in the sense that the image formed by a plane mirror exists. It should be observed, further, that it is an erect image and smaller than the object.

Problems

1. If the velocity of light is altered in passing from one medium to another, does the frequency change? Does the wave length change?

2. The water in a certain vessel is 12 in. deep. What is its apparent depth to an eye looking vertically down upon its surface?

3. A lens has two convex spherical faces, the radius of one is 15 cm., that of the other, 20 cm. The index of refraction of the glass is 1.5. What is the principal focal length of the lens?

4. A luminous object is placed in front of the lens of problem 3, at a distance of 100 cm. What is the position and relative size of the image?

5. What is the velocity of light in water? $\mu = \frac{4}{3}$.

6. A glass cube is placed on the bottom of a vessel filled with water. The angle of incidence of a beam of light on the water is 60°. What is its direction in the glass? Index of refraction of water $= \frac{4}{3}$, of glass $= \frac{3}{2}$.

7. Under what conditions does light travel in a curved line? Explain how the sun is visible after it has passed below the horizon.

8. Show by diagram the path of a beam of light passing through a glass prism submerged in CS_2. Index of refraction of glass, $\frac{3}{2}$; of CS_2, 1.63.

9. The curved surface of a plano-convex lens has a radius of curvature of 10 cm. $\mu = \frac{3}{2}$. What is its principal focal length when submerged in water? $\mu = \frac{4}{3}$.

10. The radii of curvature of a biconvex lens are 20 and 30 cm. Its principal focal length is 24 cm. What is the index of refraction of the glass?

OPTICAL INSTRUMENTS

CHAPTER XLI

THE SIMPLE MICROSCOPE

484. In the discussion of the formation of an image by a convex lens given in Section 482, it was assumed that the distance between the object and the lens was greater than the principal focal length of the lens. **If an object is placed between a convex lens and its principal focus, the image of the object is virtual and magnified** by an amount which depends upon the principal focal length of the lens as shown in the following discussion. In Figure 340 let *L* represent a simple convex lens

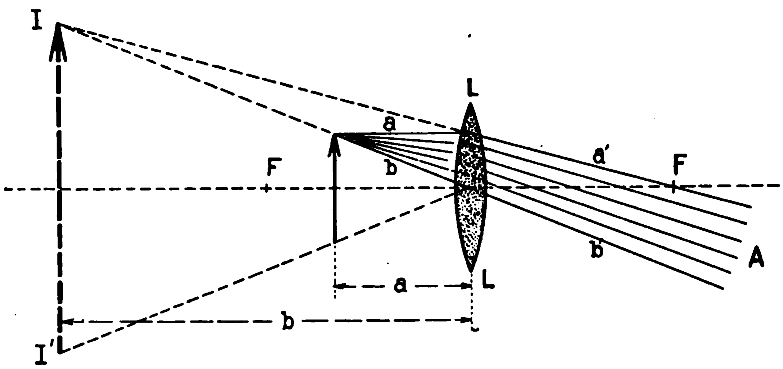


FIG. 340.

of which *F* and *F'* are the principal foci. Let the small arrow represent a luminous object placed between the lens and its principal focus *F*. Applying the principles employed in the construction of Figure 338, the image *I* of the arrow point is found to lie on the same side of the lens and at greater distance from the lens than the point on the luminous object. Thus *II'* is the magnified image of the luminous object. **An eye placed at *A* receives the rays *a'b'* as if they were proceeding from *I*.**

Thus the object appears magnified. A lens used in this way is called a simple microscope. The magnification secured by the use of the simple microscope is determined in the following manner: The magnification may be defined as the ratio of the apparent size of the image to the size of the object when placed at the same distance from the eye. The normal eye sees objects most distinctly when at a distance of about 25 cm. In using the simple microscope one unconsciously adjusts the position of the lens with respect to the object until the image is apparently at a distance of about 25 cm. from the eye. The ratio of the size of the image under these circumstances to the actual size of the object is the magnifying power of the instrument. Referring to Figure 340, it will be evident from similar triangles that the magnifiying power m is given by

$$m = \frac{\text{size of image}}{\text{size of object}} = \frac{b}{a}$$

where b is the distance of the image from the center of the lens and a is the distance of the object from the center of the lens. Now Equation 128 may be made applicable to this case providing b is regarded as a negative quantity, since, under the conditions assumed in the development of this equation, b and a were oppositely directed. We have, therefore, for the case of the simple microscope,

$$\frac{1}{a} - \frac{1}{b} = \frac{1}{p}$$

or multiplying through by b,

$$\frac{b}{a} = \frac{b}{p} + 1 = m$$

since $\frac{b}{a}$ is the magnifying power as given above. But for distinct vision, $b = 25$ cm. We have therefore for m, the magnifying power of the simple microscope,

$$m = \frac{25}{p} + 1 \tag{129}$$

THE COMPOUND MICROSCOPE

485. It is found impracticable to secure by means of the simple microscope a magnification of more than about 100 diameters. In order to secure higher magnifying powers the compound microscope is used. This instrument contains two lenses. The one, called the **object glass**, represented by A in Figure 341, forms a real and inverted image of the object OO' at

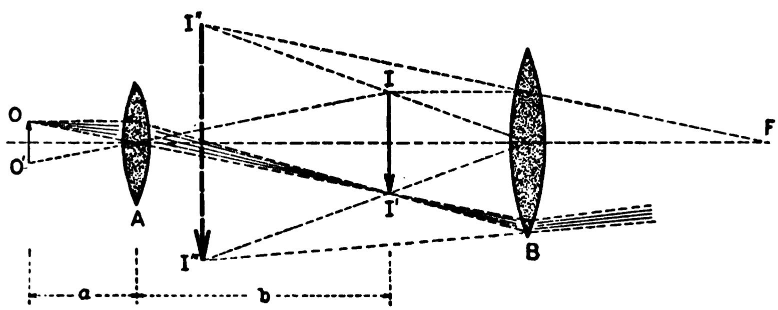


FIG. 341.

II'. The other, B, called the **eyepiece**, plays the part of a simple microscope, giving a magnified image $I''I'''$ of the image produced by the first lens. The magnifying power of the compound microscope is evidently equal to the magnifying power of the eyepiece multiplied by the ratio of the size of II' to that of OO'.

But,
$$\frac{II'}{OO'}=\frac{b}{a}$$

We may, therefore, write for the magnifying power of the compound microscope

$$M=\frac{b}{a}\left(\frac{25}{p}+1\right) \qquad (130)$$

THE TELESCOPE

486. The telescope in its simplest form contains two lenses arranged somewhat like those of the compound microscope. In the telescope, however, the object glass is designed to give a real image of small size of a distant object, whereas in the microscope the object glass is designed to give a magnified image

of a near object. In other words, the object glass of the telescope is a lens of long focal length, while the object glass of the compound microscope is a lens of short focal length. The magnifying power of a telescope is given by the ratio of the focal length of the object glass to the focal length of the eyepiece. This may be demonstrated as follows: In Figure 342, let A

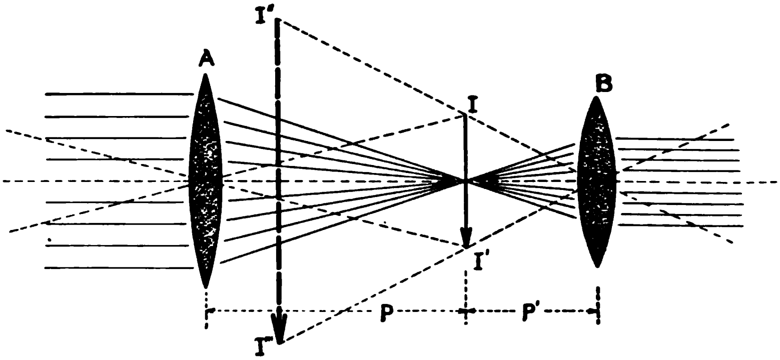


FIG. 342.

represent the object glass and B the eyepiece of a telescope. Let II' be the image formed by the object glass of the distant object which it is supposed is being viewed through the telescope. Let $I''I'''$ be the magnified image of II', formed by the eyepiece. The angle subtended by II' at the center of A is evidently the angle which the distant object subtends at the object glass of the telescope, or since the length of the telescope may be neglected in comparison with the distance of the object. this may be taken as the angle which the distant object subtends at the eye. Call this angle β. β is thus the angular diameter of the distant object as viewed by the naked eye. The angle subtended by $I''I'''$ at the center of B is the apparent angular diameter of the object as viewed through the telescope. Call this angle α. The magnification secured by the use of the instrument is therefore given by

$$m = \frac{\alpha}{\beta}$$

Now if the object is at a great distance then II' is at a distance from A which is practically equal to the principal focal length

of the object glass; and if the eye is adjusted for nearly parallel rays, then II' lies for practical purposes at the principal focus of B. But since the angular diameter of an object as seen from a given point is inversely as its distance from that point, therefore,

$$\frac{\alpha}{\beta}=\frac{P}{p}$$

where P is the principal focal length of the object glass and p is the principal focal length of the eyepiece. Therefore,

$$m=\frac{P}{p} \tag{131}$$

THE PROJECTION LANTERN

487. The projection lantern consists essentially of a source of light and two lenses. One lens, called the **condenser**, is used to secure a uniform and intense illumination of the object.

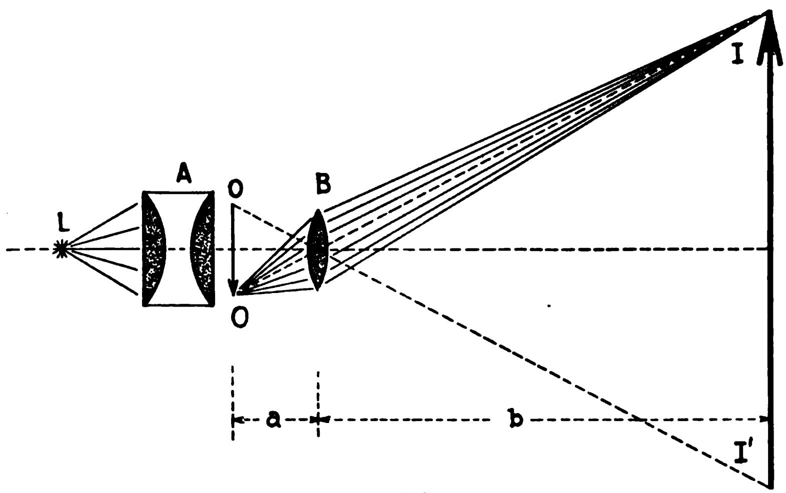


FIG. 343.

The second lens, called the **projecting lens**, is used to form a magnified image of the brightly illuminated object. The arrangement of the different parts is shown in Figure 343. A is the condenser, which usually consists of two convex lenses, as indicated in the diagram; B is the projecting lens. OO is

the object, a magnified image of which is "projected" upon the distant screen II'. If it is desired to increase or decrease the distance of the image from the lantern, the position of the lens B is changed. This changes the distance a of the object from the lens, which of course results in a change of b, the distance of the image from the lens. L is a brilliant source of light, for example, an arc lamp.

THE PHOTOGRAPHIC CAMERA

488. In the photographic camera a lens is employed for forming a sharp image of more or less distant objects upon a photographic plate. The camera proper consists of a light tight box, at one side of which is fixed the photographic plate. Opposite this is the lens. Evidently, if the distance between the lens and the photographic plate is fixed, the object of which it is desired to form an image on the plate would necessarily have to be at a fixed distance in front of the lens. It is convenient to be able to secure upon the plate sharp images of objects at different distances. This is effected by attaching the lens to a bellows, which makes it possible to alter the distance between the lens and the photographic plate, and at the same time to prevent the entrance of extraneous light. The essential parts of the camera are shown in Figure 344. L is

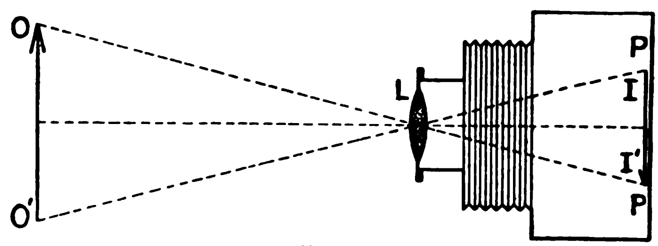


FIG. 344.

the lens, PP the photographic plate at the back of the camera. The bellows enables the lens to be pushed forward or drawn back, so as to secure upon the plate a sharp image of the object to be photographed. This adjustment of the lens for securing a sharp image upon the photographic plate is called focusing.

If a camera lens is of short focal length the distance between lens and plate changes but slightly in focusing upon objects at widely different distances. If such a camera is focused upon an object at a distance say of 50 feet, it will give fairly sharp images of all objects in the field of view whose distances may vary from a few feet to infinity. A camera having a short focus lens mounted at a fixed distance from the plate, is called a *universal focus* camera.

THE EYE

489. Optically the eye is very much like the photographic camera, the retina performing the same function in the eye that the photographic plate does in the camera. That is to say, the retina receives the image formed by the lens of the eye. Just as in the camera it is found necessary to be able to change the distance *LP*, Figure 344, in order that sharp images may be secured of objects at various distances, so in the eye a similar adjustment is necessary. The eye is focused, not by changing the distance between the lens and the retina, but by changing the thickness of the eye lens. When the eye is directed to near-by objects, in which case, if the curvature of the lens were constant, the image would tend to be formed too far from the lens, the curvature of the lens is involuntarily increased; that is to say, its thickness at the center is made greater, so as to bring upon the retina a sharp image of the near-by object. When the eye is directed toward a distant object, the lens is flattened so that the image of the distant object falls properly upon the retina. This involuntary adjustment of the curvature of the lens of the eye is called **accommodation.**

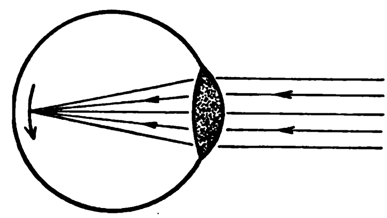


FIG. 345.—Nearsighted Eye.

Sometimes the accommodation of the eye is limited to such an extent that it is impossible for the eye lens to form a sharp image upon the retina. Thus it may be impossible in a given eye to flatten the lens sufficiently to make distant objects distinctly visible. An eye so affected is called a near-

sighted eye. **In a nearsighted eye, the image of a distant object is formed in front of the retina,** as shown in Figure 345. Nearsightedness may be corrected by the use of a concave lens. Such a lens, placed in front of the eye, has the effect of increasing the principal focal length of the eye lens.

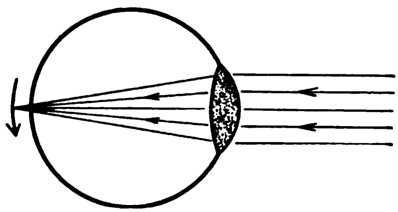


FIG. 346. — Farsighted Eye.

In another eye it may be impossible to thicken the lens sufficiently to make near-by objects distinctly visible. Such an eye is called farsighted. **In a farsighted eye, the image of a near-by object tends to form behind the retina,** as shown in Figure 346. Farsightedness may be corrected by the use of a convex lens. Such a lens placed in front of the eye has the effect of decreasing the principal focal length of the eye lens.

Problems

1. What is the magnifying power of a simple lens used as a microscope, the principal focal length of the lens being 2.5 cm.?

2. Two lenses are used in combination as a compound microscope. The focal length of the objective is 0.5 cm., that of the eyepiece 5 cm. If the instrument is focused on an object 0.52 cm. from the objective, what is the magnifying power of the microscope?

3. What is the magnifying power of a telescope, the focal length of its objective being 200 cm. and that of its eyepiece 5 cm., when focused on a very distant object? Assume observer's eye to be adjusted for parallel rays.

4. The focal length of the objective of a projecting lantern is 15 in. The lantern slide is 3 × 4 in. and the distance between the slide and the screen upon which the image is to be formed is 40 ft. What is the size of the image? How far is the projecting lens from the slide?

5. The focal length of a photographic lens is 10 in. If the camera is focused sharply upon an object 6 ft. away, how far will the lens have to be moved to give a clear image of an object whose distance is 100 ft.?

DEFECTS OF MIRRORS AND LENSES

CHAPTER XLII

CHROMATIC ABERRATION

490. In discussing the effect of the simple lens in changing the form of a light wave it has been assumed for the sake of simplicity that all parts of the disturbance are equally affected in passing through the lens. Now, **white light is complex in its nature, in that it consists of a large number of light waves of different wave lengths,** and experiment shows that these component parts are not equally affected in passing through a lens.

The effect of a lens in changing the curvature of a wave front depends, as we have seen, upon the index of refraction of the glass of which the lens is made. It is found by experiment that **the index of refraction of a given kind of glass is different for light of different wave lengths, being greater for short waves than for long ones.** The shortest waves in white light are violet waves. The red waves are the longest.

It follows, therefore, that when white light passes through a convex lens, its component wave lengths tend to become separated, the curvature of the shorter waves being more affected than that of the longer waves. This effect is known **as chromatic aberration.**

The effect described above is shown in Figure 347. Let A be a convex lens receiving white light from a source O on the left. Because of the dispersive action of the lens A, the violet rays will be focused at some such point as V, while the red rays will be focused a point farther away such as R. If, therefore, a screen is placed at V as represented by the dotted line, there will be formed upon this screen a violet image of the source O surrounded by a red fringe. If the screen is placed at

R, there will be formed upon the screen a red image surrounded by a violet fringe, the violet rays after passing the point *V* having diverged so as to surround the point *R*.

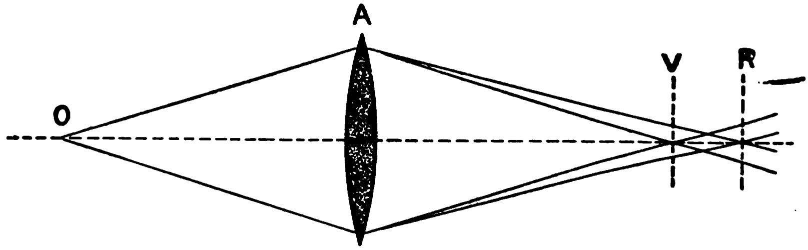


FIG. 347.—Chromatic Aberration in a Convex Lens.

The effect of a concave lens upon converging rays is just the reverse of that of a converging lens. For example, let *B*, Figure 348, represent a concave lens. The converging rays *a*, *b* are rendered less convergent upon passing the lens *B*, and as in the case above, the effect of the lens upon the violet waves is greater than upon the red waves, so that the red waves after

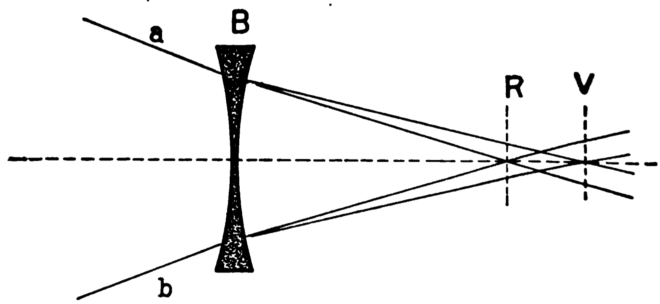


FIG. 348.—Chromatic Aberration in a Concave Lens.

passing the lens will be focused at some such point as *R*, the violet waves at a point farther from the lens, the waves of the intermediate colors falling between *R* and *V*.

It is possible by combining a convex lens with a concave lens to bring the violet and the red waves to a common focus. Lenses combined in this manner are made of different kinds of glass. For example, one of the lenses may be made of crown glass and the other of flint glass. A pair of lenses which brings the violet and the red waves to the same focus, does not

altogether prevent the dispersion of the other colors. Generally speaking, however, it is possible by placing two lenses together in this manner to render the combination sufficiently free from dispersive action for most practical purposes.

SPHERICAL ABERRATION

491. A light wave after passing through a simple lens is not quite spherical. Evidently such a wave will not be sharply focused. In other words, a simple lens does not form a perfect image of a luminous object. If such an image is examined, it will be found to be "fuzzy" or blurred. This is due to the fact that **those rays of light which pass through the edges of the lens are focused at a point nearer the lens than those which pass**

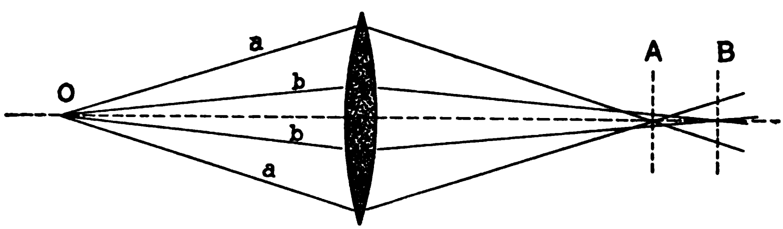


FIG. 349.—Spherical Aberration.

through the center of the lens. This effect is distinct from and independent of chromatic aberration. If a simple lens is used for forming an image of an object from which light of one color only is proceeding, for example, red light, evidently the effects of chromatic aberration will be absent. Nevertheless, the image will be defective, since those light waves which pass through the edges of the lens are focused nearer the lens than those which pass through the center. The effect is illustrated in Figure 349. Let O be a luminous object sending out waves of light of one color only, let us say red. Those portions of a wave which pass through the extreme edges of the lens will be brought to a focus at some such point as A. Those which pass through the lens near its center will be focused at a point farther away from the lens, for example B. This is known as **spherical aberration.**

Various means are employed for reducing spherical aberra-

tion. In photographic cameras "stops" are sometimes used which cover the edges of the lens and confine the beam of light to the central portions. The spherical aberration of such lenses is also reduced by presenting the surface of greater curvature to the incident light. It is also possible to grind the surfaces of a lens so that they differ slightly from the spherical form and thereby, for a given pair of conjugate focal distances, eliminate spherical aberration. A lens corrected for spherical aberration is called an **aplanatic lens.**

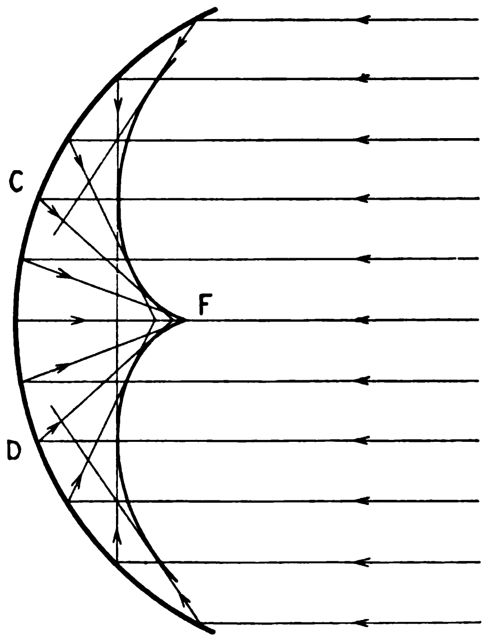


FIG. 350. — Caustic Curve formed by a Spherical Mirror.

Spherical aberration is also noticeably present in a spherical mirror if the width of the mirror is comparable to its radius of curvature. Figure 350 shows the rays reflected from a wide spherical mirror. The incident rays are assumed to be parallel. It will be seen from the figure that if a limited portion, say CD, is used, the reflected rays will all be concentrated at F. Such a mirror is contemplated in the above discussions of concave and convex mirrors. When a larger portion of the mirror

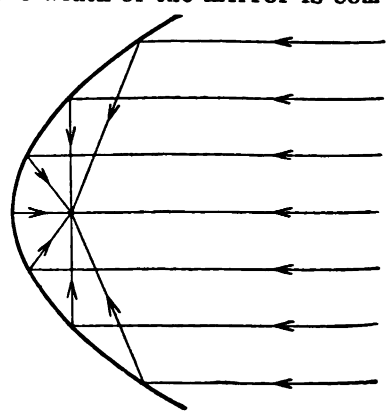


FIG. 351. — Parabolic Mirror.

surface is used, the reflected rays are not all focused at *F*. The majority of them fall behind *F*, as shown in the figure. The effect of this crossing of the reflected rays is to form a region of intensified brightness called a **caustic**. The caustic curve is a curve drawn tangent to these crossing rays. It has a cusp at *F*.

If the section of the mirror is a parabola instead of a circle, the reflected rays are focused at one point. A parallel beam of light falling upon such a mirror is concentrated at the focus of the parabola. Conversely, light rays proceeding from a source placed at the focus of a parabolic mirror are rendered parallel after reflection, Figure 351.

ASTIGMATISM

492. A lens is said to possess **astigmatism** or to be astigmatic when it is incapable of giving an image in which all lines that pass through the center of the image are equally in focus. For example, the horizontal lines of an image, formed by a lens possessing this defect, may be sharply focused, while the vertical lines of the image are blurred, or *vice versa*. This defect is present in a lens the faces of which are not truly spherical, but have different curvatures in different directions. Astigmatism due to this cause is often found in the lens of the eye. The simple lens does not possess this defect to any extent except for rays entering the lens obliquely, and at a considerable angle to the axis.

Astigmatism may be corrected by employing two lenses in which the astigmatic effects are opposite. Astigmatism of the eye is corrected by the use of a cylindrical spectacle lens so placed before the eye that its convexity is, as it were, added to that of the eye in the direction of its least curvature. A lens corrected for astigmatism is called an **anastigmatic lens**.

DISTORTION

493. The image formed by a simple lens, of an object made up of straight lines, is imperfect in that straight lines in the object are reproduced as curved lines in the image. Let it be imagined, for example, that the lens is used to form the image

of an object like that represented at *A*, Figure 352. Then the image may be like the figure shown at *B* or at *C*. This defect, known as distortion, is due to the fact that the magnifying power of a simple lens depends upon the angle at which the rays

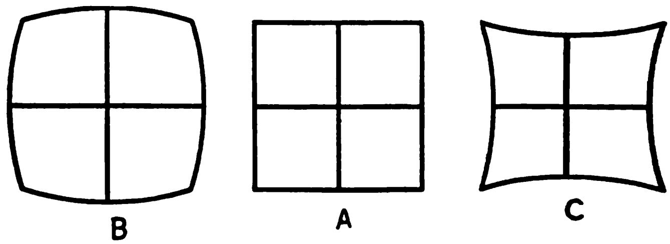


FIG. 352.— Barrel and Pincushion Distortion.

enter the lens. When the magnifying power is less for the oblique rays than for the direct rays, the distortion shown at *B*, called "barrel distortion," is the result. When the magnifying power for the oblique rays is greater than for the direct rays, the effect shown at *C*, called "pincushion distortion," results.

This defect of the simple lens is obviated by the use of two lenses placed a short distance apart and on opposite sides of a screen having an opening at its center. A lens corrected for distortion is called a **rectilinear lens.**

CURVATURE OF FIELD

494. When a simple lens is used to form an image of a plane object, the plane of the object being perpendicular to the axis of the lens, it is found that the images of the different points on

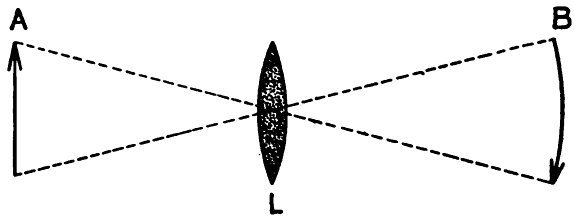


FIG. 353. — Curvature of Field.

the object are not exactly in the same plane, the image of edge points in the object being formed nearer the lens than the images of central points on the object. This effect is illustrated

in Figure 353, in which *A* is an object all points of which lie in a plane perpendicular to the axis of the lens. *L* is a simple lens which forms an image of the object at *B*. The image *B* is curved in the manner shown. This effect is known as **curvature of field.** This defect in a simple lens may be corrected by combining it with a suitably proportioned concave lens, the principal focal lengths of the two lenses of the combination being so chosen that they give, when acting together, the desired focal length. With such a combination it is possible to produce a "flat field." Evidently this correction in a photographic lens for reproducing drawings, etc., is of the highest importance.

DISPERSION

CHAPTER XLIII

THE PRISM

495. A **prism** is a piece of glass or other transparent medium having a triangular cross section. **When a beam of parallel rays of light falls upon a prism, two effects are observed. First, the beam as a whole is changed in direction. This is known as *deviation*. Second, the rays after passing the prism diverge to a certain extent and exhibit color. This effect is known as *dispersion*.** The general results obtained in this experiment are shown diagrammatically in Figure 354, in which *ABC* is sup-

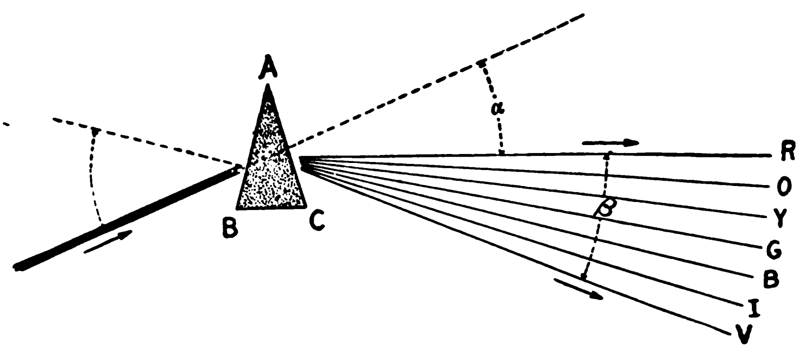


FIG. 354. — Effect of a Prism upon a Beam of White Light.

posed to represent a prism of glass. A beam of parallel rays of white light falls upon the face of the prism *AB*, as shown. It will be found under these circumstances that the rays marked *R*, *O*, *Y*, *G*, *B*, *I*, *V*, are in color, red, orange, yellow, green, blue, indigo, and violet, respectively. The experiment shows among other things that white light is really a compound of a number of different colors. One of the effects of the prism is

to disperse the light waves corresponding to these different colors. It will be observed that the violet rays are most strongly deviated, while the red suffer the least deviation. The angle marked α is the **angle of deviation** for the red rays; the angle β is the **angle of dispersion** for the red and violet rays.

The deviation of a ray in passing through a prism is effected in precisely the same manner as that of a ray passing through the edge of a lens. **The deviation of any given ray is determined by the index of refraction of the glass of which the prism is made.** But the deviation of the red ray is different from that of the violet, since the index of refraction of the glass is different for light waves of different colors. It has been pointed out that light travels more slowly in glass than in air. It is because of this fact that the direction in which the wave is traveling changes as the wave passes from the air into the glass. Evidently if the velocity of light in glass is a great deal less than the velocity of light in air, the change in the direction of propagation will be correspondingly great. It thus appears that violet light travels less rapidly in glass than red light does. In other words, **the index of refraction of glass is greater for violet than for red light.**

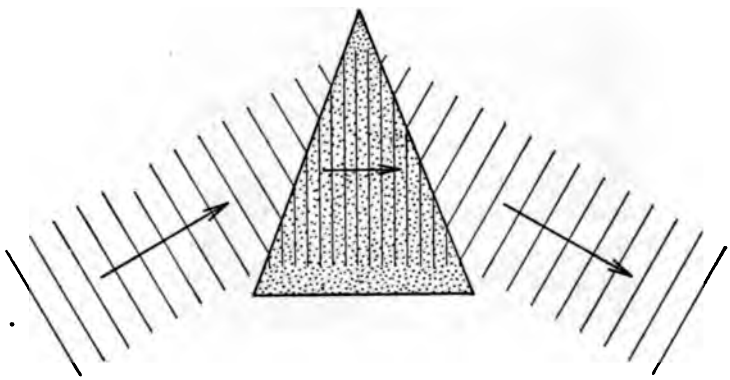


FIG. 354 *a*.

The change in direction of a wave front as it enters the prism, and again as it emerges from the prism, is shown in Figure 354 *a*. (Compare Figure 283.)

THE SPECTRUM

496. The band of light with the succession of colors, red, orange, yellow, etc., which appear in the above experiment upon a screen held at *RV*, constitutes what is known as a **spectrum**. The red shades gradually into the orange, and the orange gradually into the yellow, and so on, so that it is impossible to distinguish where one color leaves off and another be-

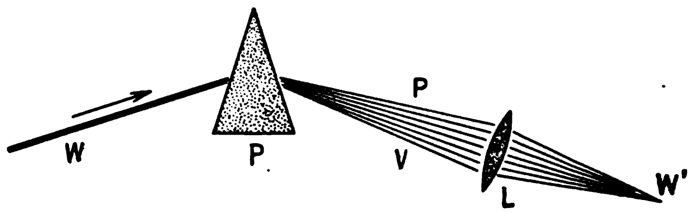


FIG. 355.

gins. In other words, there is between *R* and *V* an infinite number of colors. We may, however, distinguish the groups red, orange, yellow, etc. enumerated above, and use their names in referring to the different parts of the spectrum.

That the spectrum is due to the composite nature of white light rather than to any transformation occurring in light which passes the prism may be shown by recombining the colors of the spectrum. When this is done, it is found that the combination of the separate colors produces white. The separate colors of the spectrum may be combined by means of a lens as shown in Figure 355. *P* is a prism upon which falls a beam of white light *W*. This beam is dispersed, forming a spectrum at *L*. A lens *L* placed as shown will recombine the various colors in the colored beam, forming white light at *W'*. Another way of recombining the colors of the colored beam is to use a second prism, so placed with respect to the prism *P* that it tends to deviate the beam in the opposite direction. This arrangement

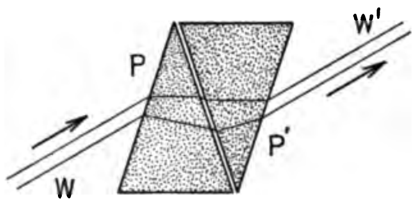


FIG. 356.

is shown in Figure 356. The white light W which is dispersed by the prism P is recombined by the prism P'. Under these conditions W' will be white light, and the direction in which it is traveling will be parallel to that of W. It is here assumed that P and P' are prisms of the same form and the same kind of glass.

DEVIATION WITHOUT DISPERSION

497. **The deviation produced by a prism, *i.e.* the angle a, Figure 354, depends upon (*a*) the angle of incidence i, Figure 354 : (*b*) the refracting angle A, *i.e.* the prism angle opposite the base of the prism ; and (*c*) the index of refraction of the glass of which the prism is made.** The dispersion produced by a prism depends upon these same things, but the dispersion produced by a given prism is not proportional to the deviation. It is, therefore, possible by using two prisms made of different kinds of glass and arranged as shown in Figure 356, to neutralize by means of the second prism the dispersive action of the first prism without entirely correcting its deviation. That is, with such a combination of prisms it is possible to deviate a beam of white light without dispersing it. Such a combination of prisms is known as an **achromatic** combination. This is the principle employed in correcting for chromatic aberration in lenses (Section 490).

DISPERSION WITHOUT DEVIATION

498. Evidently from the statements made in the preceding paragraph, it is equally possible to combine two prisms of different kinds of glass in the manner indicated in Figure 356 so as to neutralize, by means of the second prism, the deviation produced by the first without entirely correcting for its dispersion. A beam of white light passing through a combination of prisms like that shown in Figure 357 will be dispersed

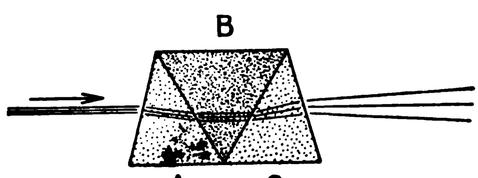


FIG. 357. — Dispersion Without Deviation.

THE SPECTRUM

496. The band of light with the succession of colors, red, orange, yellow, etc., which appear in the above experiment upon a screen held at *RV*, constitutes what is known as a **spectrum.** The red shades gradually into the orange, and the orange gradually into the yellow, and so on, so that it is impossible to distinguish where one color leaves off and another be-

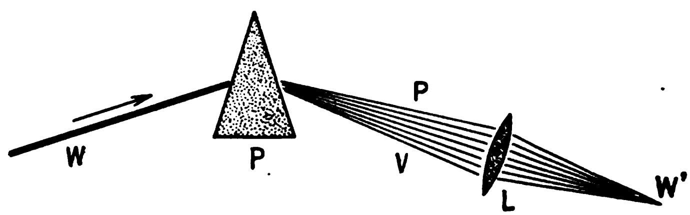


FIG. 355.

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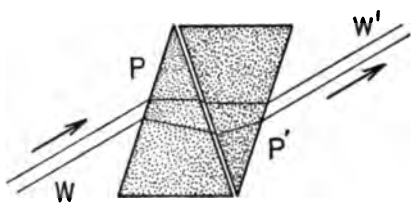

FIG. 356.

is shown in Figure 356. The white light *W* which is dispersed by the prism *P* is recombined by the prism *P'*. Under these conditions *W'* will be white light, and the direction in which it is traveling will be parallel to that of *W*. It is here assumed that *P* and *P'* are prisms of the same form and the same kind of glass.

DEVIATION WITHOUT DISPERSION

497. The deviation produced by a prism, *i.e.* the angle *a*, Figure 354, depends upon (*a*) the angle of incidence *i*, Figure 354 : (*b*) the refracting angle *A*, *i.e.* the prism angle opposite the base of the prism ; and (*c*) the index of refraction of the glass of which the prism is made. The dispersion produced by a prism depends upon these same things, but the dispersion produced by a given prism is not proportional to the deviation. It is, therefore, possible by using two prisms made of different kinds of glass and arranged as shown in Figure 356, to neutralize by means of the second prism the dispersive action of the first prism without entirely correcting its deviation. That is, with such a combination of prisms it is possible to deviate a beam of white light without dispersing it. Such a combination of prisms is known as an **achromatic** combination. This is the principle employed in correcting for chromatic aberration in lenses (Section 490).

DISPERSION WITHOUT DEVIATION

498. Evidently from the statements made in the preceding paragraph, it is equally possible to combine two prisms of different kinds of glass in the manner indicated in Figure 356 so as to neutralize, by means of the second prism, the deviation produced by the first without entirely correcting for its dispersion. A beam of white light passing through a combination of prisms like that shown in Figure 357 will be dispersed

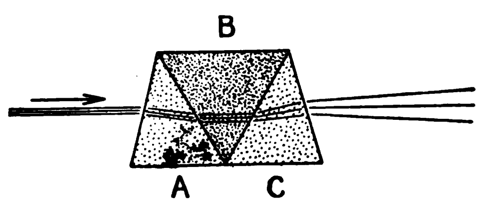

FIG. 357. — Dispersion Without Deviation.

without, as a whole, suffering any deviation. *A* and *C* are made of the same kind of glass. *B* is made of a different glass, so chosen that it completely neutralizes the effects of *A* and *C* so far as deviation is concerned.

CONDITIONS NECESSARY FOR THE PRODUCTION OF A PURE SPECTRUM

499. As pointed out above, when a beam of parallel rays of white light passes through a prism, a dispersion of the rays of different wave lengths takes place. If the beam, after passing the prism, is allowed to fall upon a screen, a **spectrum** is formed. A spectrum formed in this manner is, in general, impure, because of the overlapping of certain colors. This

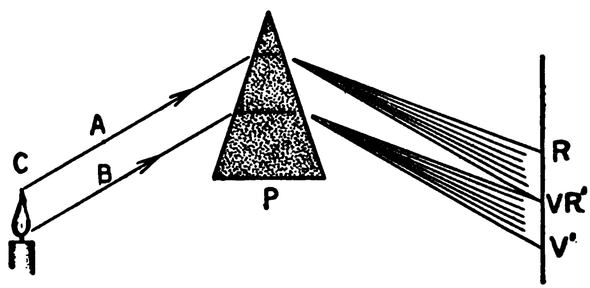

FIG. 358.

effect will be understood from a consideration of Figure 358. *C* is a source of light, *P* a prism; *A* and *B* are rays from the upper and lower parts of the source. *R* and *V* are the red and violet of the ray *A* and *R'*, and *V'* the red and violet of *B*. As indicated in the figure, *V* and *R'* fall together upon the screen; in other words, there is an overlapping of the spectra from *A* and *B*. **It is evident, therefore, that for producing a pure spectrum a narrow source of light must be used.**

THE SPECTROSCOPE

500. For the purpose of facilitating the study of spectra the spectroscope is employed. The essential parts of the spectroscope are shown in Figure 359. A narrow, adjustable slit *S*, which is strongly illuminated by the light to be ex-

amined, is used as the source. This slit is located at one end of a closed tube *AB*. At the opposite end is a converging lens. The length of the tube *AB* is equal to the principal focal length of the lens *B*. Evidently, with this

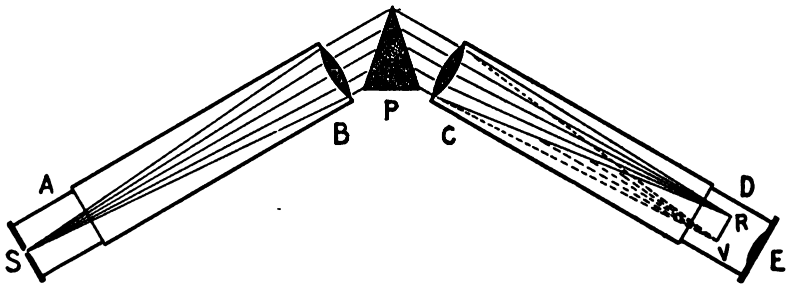

FIG. 359. — The Spectroscope.

arrangement, the rays of light which fall upon the prism *P* will be parallel. The tube *AB* is called the **collimator.** The rays, after passing the prism, are received upon the the converging lens *C*, and form a pure spectrum at *RV*. **This spectrum is in reality formed of a series of images of the slit S.** Evidently at *R* there is a red image of the slit, at *V* a violet image, and images in the other spectrum colors at points intermediate. A simple microscope *E* is used for viewing this spectrum. The converging lens *C* and the simple microscope *E* are contained in the same tube *CD*, which is called the **telescope.**

When the spectroscope is provided with an arrangement for measuring the angle of deviation it is called a **spectrometer.**

THE DIFFERENT KINDS OF SPECTRA

501. The Continuous Spectrum.—If a white-hot, solid body is placed before the slit of the spectroscope, the spectrum formed in the instrument will be a continuous one; that is to say, the succession of images between *R* and *V*, Figure 359, will be so complete that a continuous band of light is formed. A continuous spectrum is also given by an incandescent liquid or by an incandescent gas under very high pressure.

The Bright Line Spectrum.—The light given off by a gas under low pressure, when heated to incandescence, forms in

the spectroscope a bright line spectrum; that is to say, if such light is caused to illuminate the slit of the spectroscope, the spectrum formed at *RV*, instead of being one continuous band shading imperceptibly from the red through the orange and the yellow, etc., on to the violet, will be found to be discontinuous, there being in general but comparatively few colored images of the slit present. For example, if the light from incandescent sodium vapor is used, there will be but a single line or image of the slit in the spectrum. This line is in the orange-yellow. If the light used is that which is given off by lithium vapor, there will be but two colored images of the slit. Other incandescent gases have greater numbers of lines in their spectra. It is found, however, that **each substance, when heated to incandescence in the vapor state, gives a bright line spectrum which is characteristic of that substance.** This fact is taken advantage of in spectrum analysis, and constitutes a very sensitive test for the presence or absence of a substance in a given compound.

The Dark Line Spectrum.—If the slit of the spectroscope is illuminated with sunlight, we obtain at *RV*, Figure 359, what is known as the solar spectrum. It will be observed that the solar spectrum is crossed by a great many dark lines. It is as if the colors corresponding to the positions of these dark lines were not present in the sunlight. This being the explanation commonly accepted for the dark lines, it becomes necessary to account for the absence of the particular wave lengths corresponding. Undoubtedly the central portion of the sun sends off light waves of all lengths, ranging from the extreme red to the extreme violet. Since some of those wave lengths do not reach the earth, we must conclude that they have been lost or absorbed on the way. The central or hot portion of the sun is surrounded by an atmosphere of various gases at high temperatures. The light proceeding from the central portion of the sun must pass through this atmosphere on its way to the earth. In its passage through the sun's atmosphere, as well as through the earth's atmosphere, the sun's light loses some of its wave lengths. **It is found that the light waves absorbed in this manner are those**

light waves which would be given off by the elements of the sun's atmosphere and the earth's atmosphere as bright line spectra if they were heated to incandescence.

That this absorption effect is sufficient to account for the dark lines observed in the solar spectrum is easily demonstrated by placing an incandescent solid before the slit of the spectroscope and securing all adjustments for a continuous spectrum. If then incandescent sodium vapor is interposed between the source and the slit, a dark line in the orange is immediately observed. Finally, if the slit is screened from the source and only the light from the sodium flame is allowed to fall upon it, a bright orange-colored line will be found in the position formerly occupied by the dark line.

FRAUNHOFER'S LINES

502. The dark lines in the solar spectrum were first observed by Wollaston in 1802. They were studied by Fraunhofer in 1814. Fraunhofer called a number of the principal dark lines

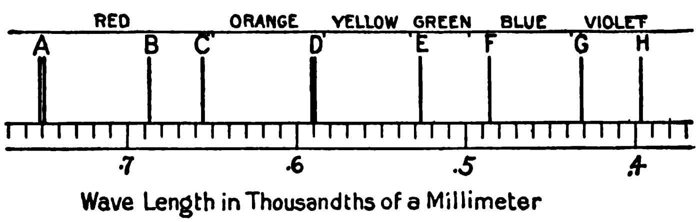

FIG. 359 *a*. — Fraunhofer's Lines.

of the solar spectrum *A*, *B*, *C*, *D*, etc., and his classification is in use at the present day. The principal Fraunhofer lines are given in Figure 359 *a*, together with the corresponding wave length of light measured in thousandths of a millimeter.

Problems

1. A narrow beam of white light passes from air to water, the angle of incidence being 60°. Is there a tendency for the components to become separated? Explain.

2. The index of refraction of CS_2 for red (C) is 1.6336, for yellow (D) is 1.6433, for blue (F) is 1.6688. What are the velocities of red, yellow, and blue light in CS_2?

3. The index of refraction of water for red (C) is 1.3318, for blue (F) is 1.3377. What is the difference in cm./sec. between the velocities of red and blue light in water?

4. The index of refraction of a certain kind of glass for red (C) is 1.5826, for yellow (D) is 1.5867, and for blue (F) is 1.5967. A plano-convex lens is made of this glass. $R = 50$ cm. What is the principle focal length of this lens for red? for yellow? for blue?

INTERFERENCE

CHAPTER XLIV

CONDITIONS UNDER WHICH LIGHT WAVES INTERFERE

503. In discussing the subject of interference of wave trains (Section 432) it was pointed out that two sound waves may be so related as to completely destroy one another so that their combined effect would be silence, and that two water waves passing over the same surface may be so related as to leave the surface under their combined influence undisturbed. If light is really of the nature of a wave disturbance, it ought to be possible by combining two light waves, properly related, to produce darkness. This is found to be the case. In order that this effect may be brought about, it is only necessary to have two light waves of the same wave length traveling in the same direction, the one wave being half a wave length behind the other so that the crests of one wave train will fall opposite the troughs of the other. Attempts to secure interference between waves of light from two different sources are unsuccessful, the explanation being that the phase of the disturbance proceeding from any source is continually changing, so that if at a given instant the phase relation existing between the two trains is such as to cause interference, a very short interval of time later they may be so related as to add their effects. It is therefore necessary, in making a study of this phenomenon, to secure **two beams of light from the same source.** One of the simplest devices for securing this is that due to Fresnel.

FRESNEL'S BIPRISM

504. For the purpose of securing interference effects Fresnel employed what is known as a biprism. This is a double prism, the two refracting angles of which, *A* and *B*, Figure 360, which

are small, are turned away from one another. **Imagine a source** of light, for example a slit, placed at S and brightly illuminated by **monochromatic** light, that is, by light of but one wave length like that given off by incandescent sodium vapor. The light will reach the point P on the right of the prism along two paths as indicated. The one ray passing toward the side A is refracted by the prism A. The other ray is refracted by the

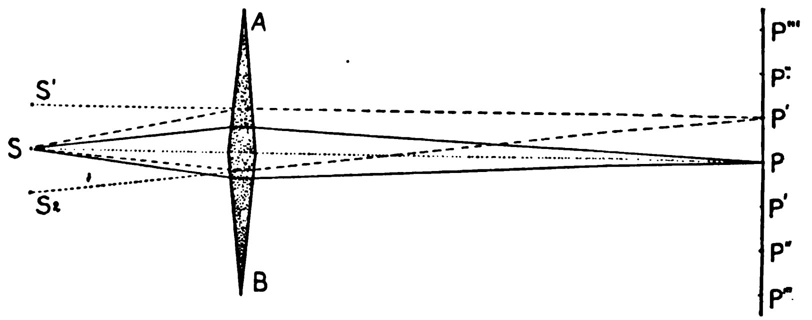

FIG. 360.— Interference produced by Fresnel's Biprism.

prism B. **Since the two paths SP are equal,** it will be evident that the waves coming to the point P along the two paths will be in the same phase, that is to say, crest will correspond with crest and trough with trough so that the two disturbances will be added at this point. Consider, however, a point such as P' a short distance above or below the point P. Rays of light will reach this point P' from the source S along two paths. It will be evident, however, from the construction of the figure that **the two paths SP' are unequal** in length. Let it be assumed that the point P' is so situated that the difference in the lengths of these two paths is half a wave length of the light proceeding from S. Then the wave train proceeding to the P' along the shorter path will get ahead of that which travels along the longer path by half a wave length, so that of the two wave trains arriving at P' the crests of one will fall opposite the troughs of the other. The result is that the two wave trains are in condition to interfere at the point P', and the effect of one of the wave trains is destroyed by that of the other. Hence there is darkness at the point P'. Again, con-

sider a point P'' a short distance farther from the central point P such that the distances SP'' measured along the two paths as before differ by one whole wave length. For this point, since the one wave train gains over the other a complete wave length, evidently the waves will be in a condition to add their effects together at this point and P'' will be a point of maximum illumination. If P''' is so located that the two paths SP''' differ in length by $\frac{3\lambda}{2}$ ($\lambda =$ the wave length) then P''' will be a region of interference and darkness. If P^{IV} is so located that the two paths SP^{IV} differ in length by $\frac{4\lambda}{2}$, P^{IV} will be a region of brightness.

The general statement covering all points on the screen CD is as follows: Let the difference in length of the two paths be d, and put

$$d = n \cdot \frac{\lambda}{2} \tag{132}$$

where λ is the wave length of the light under consideration and n is any whole number odd or even. **When n is odd, interference effects will be present. When n is an even number, interference effects will be absent.**

THE COLORS OF THIN PLATES

505. Brilliant color effects are observed in very thin plates or layers of transparent media; for example, in films of oil on water, in thin layers of oxide on polished metal, in soap bubbles, etc. **These colors are the result of the interference of light waves.** The brilliant colors sent to the eye from the soap bubble, for example, are white light minus one or more of its colors which have been destroyed by the interference effect in the film. The manner in which this interference effect takes place will be understood from the following discussion: Let AB, Figure 361, represent a thin film upon which a beam of white light a is falling at the angle indicated. This beam of light is broken up at m, one part b being reflected, a second part c being refracted and passed on to the point n at the opposite face of the film. At this point the ray c is broken into two parts, the one being reflected to f and the other d passing into the surround-

ing medium below. That component which is **reflected back** to the point *f* is again divided, a part *e* being **refracted, a second part being reflected, and so on.** The rays *e* and *b* **are parallel, and** if the film *AB* **is very thin, they** proceed practically from the same point. Evidently the **ray** *e* has traveled over a longer path than the ray *b* and **if in traversing this greater distance it has fallen behind an odd number of half wave lengths it will be in a condition to interfere with the ray** ***b.***

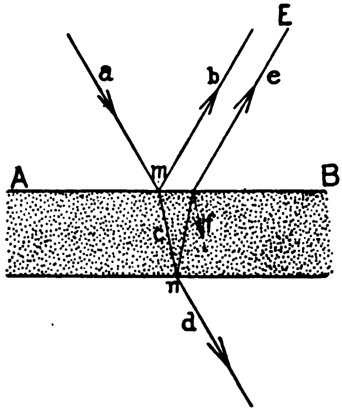

FIG. 361.—Interference in a Thin Plate.

The fact that the **ray** *e* **falls behind** the ray *b* is **sufficient** to account for the **interference** colors observed in thin plates, **but various accompanying phenomena** indicate that this **explanation** is not complete. For example, if with this **explanation** in mind we imagine that the plate or film *AB* is made **extremely** thin, then the interference effect should disappear since under these circumstances the ray *e* would not fall appreciably behind the ray *b*. The fact is, however, that the interference effect is very marked for an extremely thin film. Such a film appears black by reflected light. It will therefore be evident that **in the mere process of reflection there is a loss of half a wave length** by one of these trains of waves. This is explained in the following manner: It will be noted that the **ray** ***b*** **has been reflected in the rarer medium,** while **the ray** ***e*** **has been reflected in the denser.** Now it is not a difficult matter to show that when a wave is reflected at an interface on the side of the rarer medium, it suffers a change of phase of half a wave length, while if it is reflected on the side of the denser medium, no such change of phase is brought about. Thus the ray *b* loses (or gains) half a wave length in the process of reflection at *m*. Therefore in order that interference effects may take place, *e* **must lose in virtue of its greater path an even number of half wave lengths.**

Let it be assumed that the film AB is of such thickness that the conditions for interference of violet waves are present. Then the eye placed at E will receive by reflection from the film **white light minus violet light.** The film at this point will therefore appear to be brilliantly colored.

A film slightly thicker than the one discussed above would cut out the blue light by interference, a still thicker film would extinguish the green, and so on.

DIFFRACTION GRATING

506. An important experiment demonstrating the wave nature of light is the formation of spectra by what is known as a **diffraction grating.** A diffraction grating consists of a

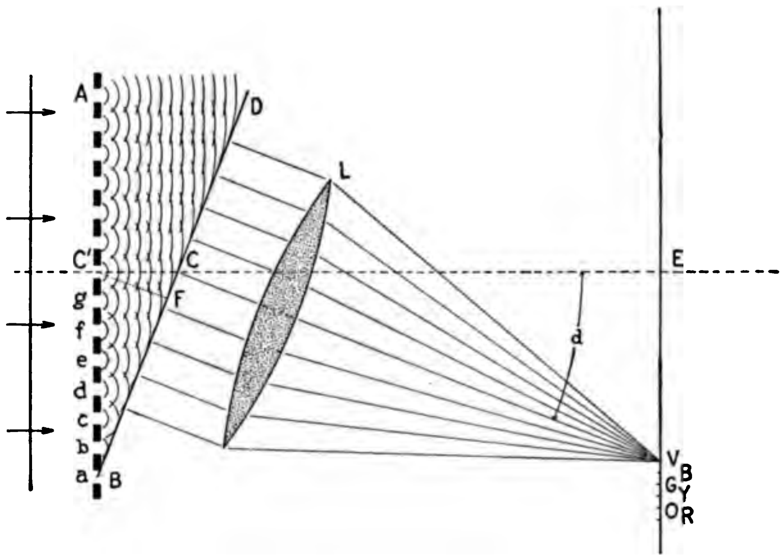

FIG. 362. — Diffraction Grating.

large number of very narrow parallel openings placed close together. One of the simplest ways in which a grating may be formed is by ruling lines with a diamond point upon a glass plate. The scratches produced in this manner may be regarded as opaque. The spaces left between are the portions which transmit the light. Let the broken line AB, Figure 362, rep-

resent such a grating. The dashes, let us say, correspond to the opaque portions, while the spaces between the dashes represent the portions which transmit light. Let it be imagined that plane waves parallel to the grating are falling upon it from the left as indicated in the figure. Each of the openings a, b, c, d, etc., will constitute a separate source of disturbance so far as the medium on the right of the grating is concerned. Let it be imagined then that trains of secondary wavelets have been for some time proceeding toward the right from these several secondary sources. Consider the condition of affairs at the moment a crest of the wave is just passing the opening a. Evidently there is also a crest of a wave at the point b and a second crest at a distance λ from the point b. Let it be imagined that a line is drawn from the center point of a, tangent to the crest which is at a distance λ from b. This line will also be tangent to a crest which has spread from c to a distance 2λ and to a crest which has proceeded from d to a distance 3λ, and so on. In other words, it will be evident that **the secondary wavelets proceeding from the sources a, b, c, d, etc., add their effects together on the line BD and in effect produce a plane wave BD which is traveling in the direction CV.** Let α represent the angle ECV. Evidently, then, since $ECV =$ angle ABD, therefore,

$$\sin\alpha = \frac{C''F}{C''B}$$

$$= \frac{7\lambda}{7s}$$

or,

$$\sin\alpha = \frac{\lambda}{s} \qquad (133)$$

in which s is the distance from a to b, that is to say, from the center of one opening to the center of the next or the distance from one line to the next on the grating. Evidently, therefore, **the angle α depends upon the wave length of the light.** The wave front BD, if violet, will make a different angle with the face of the grating from that which is made by the wave front for red light. In other words, the grating produces dispersion, sending the violet rays in one direction, the red rays in a different direction. Evidently the angle α is greater for

the longer waves. Therefore, a spectrum will be formed on the screen EV having the violet on the inner edge or that part which is nearest to the point E, and the blue at a greater distance, and so on. The order of colors in the spectrum formed in this manner is indicated in the figure.

THE MEASUREMENT OF THE WAVE LENGTH OF LIGHT

507. While it is impossible to measure the wave length of light directly, there are various ways in which it may be calculated. One of the most accurate and convenient is that afforded by the diffraction grating. For example, imagine a diffraction grating to be set up as represented in Figure 362. Let the angle α be measured. Let it be further supposed that the "grating constant," s, that is to say, the distance from the center of one opening to the next, is known. Then from the equation,

$$\sin \alpha = \frac{\lambda}{s}$$

the value of λ may be at once calculated. In the use of such a grating as is represented in Figure 362 a converging lens L must be employed. This lens focuses the rays of the different colors at the points, Y, B, G, etc. The lens is placed at a distance from the screen equal to its focal length.

Problems

1. Two glass gratings have respectively 4000 and 6000 lines per centimeter. Compare the deviation (diffraction) of yellow light ($\lambda = 6050 \times 10^{-8}$ cm.) produced by these gratings.

2. A glass grating has 4250 lines to the centimeter. When yellow light falls perpendicularly upon this grating, it is found that the spectrum of the second order is deviated 30°. What is the wave length of the yellow light?

PHOTOMETRY

CHAPTER XLV

LIGHT STANDARDS

508. The illuminating power of any source of light is determined by comparing its effect with that of a standard source of light. The values of standards used in this way are necessarily more or less arbitrary. There are a number of different standards in use at the present time. Among the most prominent and most widely used are the British standard candle, the Hefner lamp, which is used as a standard in Germany, and the Carcel standard lamp, which is used in France.

The British standard candle is defined as a candle made of pure spermaceti, weighing six to the pound, and burning 120 grains per hour.

The Hefner lamp in form is very much like an alcohol lamp. The wick is held in a tube 8 millimeters in diameter, and is so adjusted that the flame has a height of 40 millimeters. The lamp burns the acetate of amyl.

The Carcel lamp is a modification of the common argand lamp, having a wick of circular form. It burns Colza oil, which is kept at a constant level by small pumps operated by clockwork.

The International candle, recently adopted by the Standards laboratories of the United States, Great Britain, and France, is defined as follows: —

$$\begin{aligned} 1 \text{ International Candle} &= 1 \text{ American Candle} \\ &= 1.11 \text{ Hefner unit} \\ &= 0.104 \text{ Carcel unit} \end{aligned}$$

Candle Power and Illumination

509. The candle power of a light source is thus a specification of its illuminating power in terms of the standard candle.

Thus, when we say the candle power of an incandescent lamp is 16, we mean that it possesses 16 times the illuminating power of a standard candle.

The illuminating power of a source of light is, in general, found to be different in different directions. We therefore come to speak of the "mean horizontal candle power," meaning by this term the average illuminating power of a given source in the horizontal direction. In the same way the term "mean

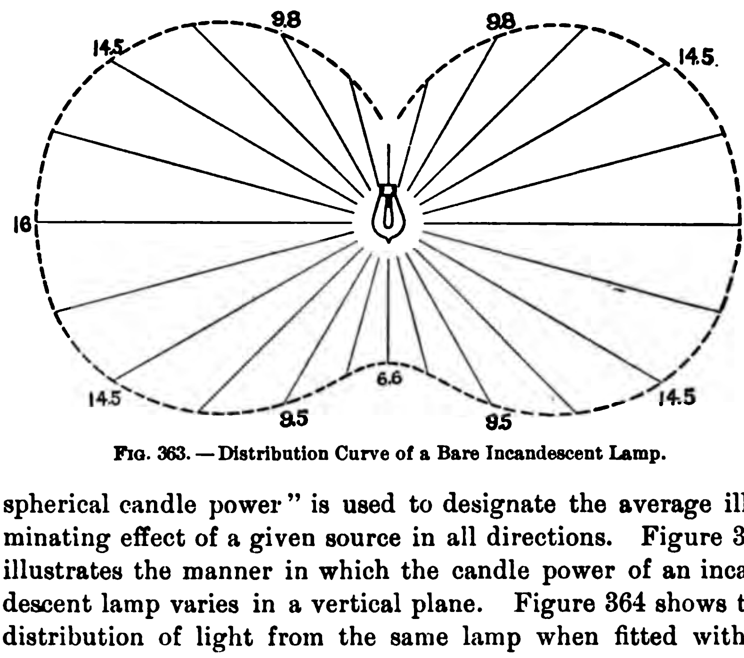

FIG. 363. — Distribution Curve of a Bare Incandescent Lamp.

spherical candle power" is used to designate the average illuminating effect of a given source in all directions. Figure 363 illustrates the manner in which the candle power of an incandescent lamp varies in a vertical plane. Figure 364 shows the distribution of light from the same lamp when fitted with a reflector. The candle power curves shown in these figures are called "**distribution curves.**" The character of the illumination afforded by a lamp, upon a table, or other receiving plane beneath, is determined largely by the form of its distribution curve. It is, therefore, possible to control such illumination, within limits, by the proper choice of lamp and attached reflector.

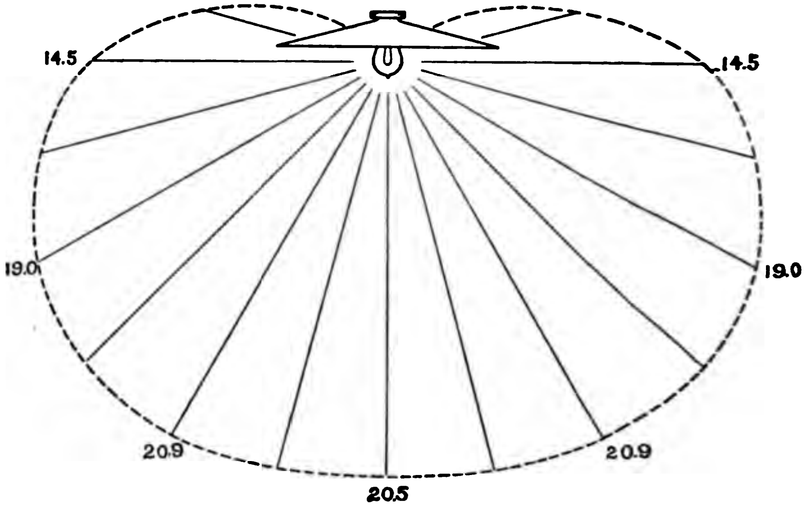

Fig. 364. — Distribution Curve of an Incandescent Lamp with Reflector.

THE LAW OF INVERSE SQUARES

510. Since the spherical wave front which proceeds from any small source of light continues to widen as it advances, it must be evident that the amplitude of the disturbance gradually grows less. This will be evident from a consideration of the fact that the total energy represented by the wave front remains constant, assuming that the medium through which it is being transmitted absorbs none of it; and, therefore, since the area over which the energy is spread is all of the time increasing, the amount of energy per unit area of the wave front must be steadily decreasing. Let it be assumed that the total quantity of light emitted by a given source is Q. Let it be imagined that this light is received upon a spherical surface, the center of which is at the source from which the light is proceeding. See Figure 365. If the radius of this sphere is R_1, its area is $4\pi R_1^2$. The amount of light received by each unit area of this spherical surface is, therefore,

$$q_1 = \frac{Q}{4\pi R_1^2}$$

in which q_1 stands for the amount of light received on each

unit area of the spherical surface. **Let it be imagined now that this spherical surface is removed, and the light is allowed to fall**

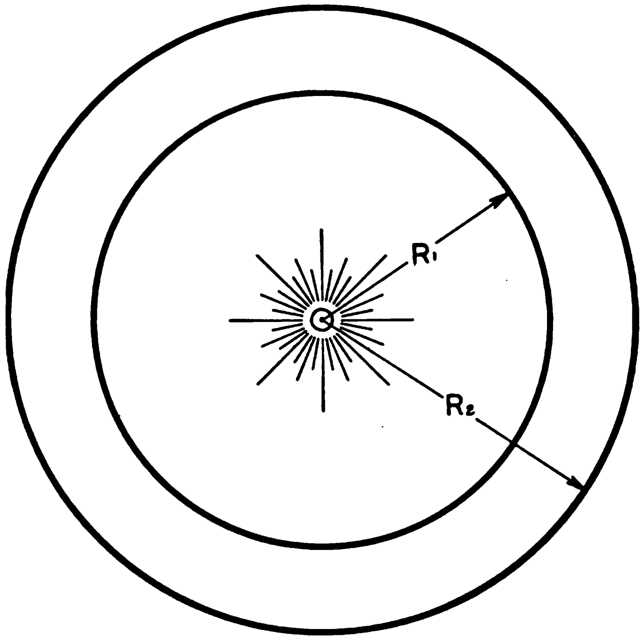

FIG. 365.

upon a second spherical surface of larger radius, let us say, of radius R_2. Evidently the amount of light received per unit area on this spherical surface is

$$q_2 = \frac{Q}{4\,\pi R_2^2}$$

Dividing this expression into the one above, we have

$$\frac{q_1}{q_2} = \frac{R_2^2}{R_1^2} \tag{134}$$

That is to say, the light received per unit area of the small sphere is to the light received per unit area of the large sphere as the square of the radius of the larger sphere is to the square of the radius of the smaller sphere. In other words, the **illumination of a given surface as it is moved farther and farther**

from a given source decreases in the ratio that the square of the distance increases.

BOUGUER'S PRINCIPLE

511. In comparing the candle powers of two different sources of light, it is usual to make use of some device which depends for its action upon Bouguer's Principle. The principle is as follows: The illuminating powers of two sources of light may be compared by so placing them that they give the same illumination upon a given screen. When this condition has been secured, **their illuminating powers are directly as the squares of their distances from the screen.** This is demonstrated as follows: The amount of light received by any surface from a given source is proportional to the candle power of the source and inversely as the square of its distance from the source. Let us call the candle power of the first source A, and its distance from the screen d_1. The intensity of illumination upon the screen due to this source is, therefore,

$$I_1 = k \cdot \frac{A}{d_1^2}$$

in which I_1 stands for the intensity of illumination. Calling the candle power of the second source B and its distance from the screen d_2, we have for the intensity of illumination produced by this source

$$I_2 = k \cdot \frac{B}{d_2^2}$$

Now if the sources A and B are so placed that they produce the same illumination upon the screen, then $I_1 = I_2$. Hence,

$$\frac{A}{B} = \frac{d_1^2}{d_2^2}$$

or, to express A in terms of B, we have

$$A = B \cdot \frac{d_1^2}{d_2^2} \qquad (135)$$

THE RUMFORD PHOTOMETER

512. A photometer is a device for determining the quantity of light emitted by a given source. The shadow photometer is

one of the earliest devices employed for the comparison of the illuminating powers of different sources of light. It consists essentially of a screen and an opaque obstacle which is so placed, between the two sources to be compared and the screen, that **two shadows are thrown upon the screen side by side.** For example, in Figure 366, let A and B represent the sources of light to be compared, O the opaque object, and SS the screen. Under this arrangement, the shadows cast by the sources A and B fall upon different parts of the screen. The shadow b

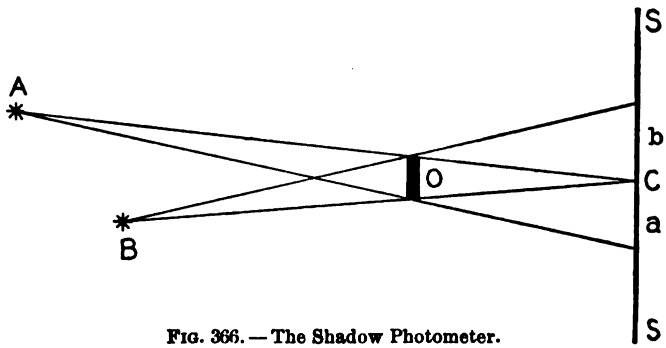

FIG. 366. — The Shadow Photometer.

is illuminated by the source A, while the shadow a is illuminated by the source B. By varying the distance between B and the screen **the illumination of the shadow a may be varied at will and hence may be made to equal that of b.** When this condition is reached, then, by measuring the distances d_1 and d_2, of A and B from the screen SS, substitution may be made in Equation (135) and the ratio of the candle powers of the sources A and B determined.

THE BUNSEN PHOTOMETER

513. The Bunsen photometer is a device whereby the two sources whose candle powers are to be compared may be placed upon opposite sides of the same screen, and their illuminating effects upon the screen compared by means of suitably placed mirrors. The arrangement of the mirrors and the screen is shown in Figure 367. A and B are the sources to be compared; SS_1 is a screen usually made of white paper with a grease spot

at its center. MM are two mirrors placed at the back of the screen in the positions indicated; the angles between the mirrors and the screen being such that to an eye placed at S_1, the grease spot is visible in each mirror. The screen and mirrors are rigidly fastened together, and are free to move in the direction AB. Let it be assumed that the conditions are such that the intensity of illumination on the screen due to the source A

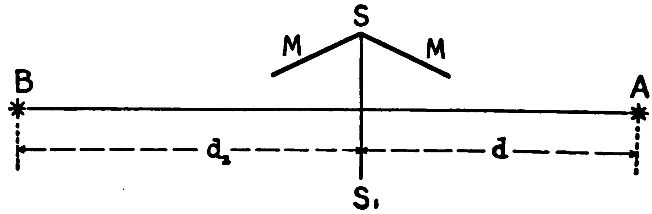

FIG. 367. — Bunsen Photometer.

exceeds that due to the source B. This condition will be indicated to the eye placed at S_1 as follows: As seen in the mirror on the right the screen will appear bright with a dark spot in the center. As seen in the mirror on the left the screen will appear dark with a bright spot in the center. If, now, the screen is moved in the direction of the source B when a certain point is reached, equality of illumination on the two sides of the screen will be secured. Under these circumstances the central spot will almost, if not quite, entirely disappear. When this adjustment has been made, the distances d_1 and d_2 are measured, and by substituting in the formula given above, the relative illuminating powers of the sources A and B may be obtained.

THE LUMMER-BRODHUN PHOTOMETER

514. The Lummer-Brodhun photometer, which is represented in Figure 368, is widely used in accurate photometric work. The white, opaque screen SS is illuminated upon the one side by one of the sources to be compared, and upon the other side by the other source. The diffusely reflected light from the surfaces of this screen falls upon the mirrors M and M_1, and is thence regularly reflected to the prisms PP_1. These prisms are right-angle prisms cemented together with Canada balsam

in the manner indicated in the figure. The prism P is so ground that the two prisms are in contact over a limited area at the center only. This being the case, it will be understood that the light coming from the mirror M which passes through the central portions of the compound prism will pass directly through to a. That light which falls upon the central portions of the compound prism from M_1 will pass directly through to c. **That light which, coming from M_1, falls near the edge of the compound prism, will be totally reflected and pass out in the**

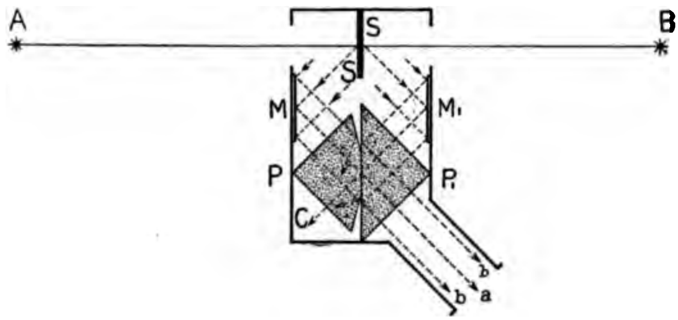

FIG. 368. — Lummer-Brodhun Photometer

direction b. The light coming from the mirror M and falling upon the edges of the compound prism, will be totally reflected to the left. It will, therefore, be evident **that an eye placed at ab will, in effect, see a part of each side of the screen SS**, since the ray a comes from the left-hand side of the screen, while the ray b comes from the right-hand side of the screen. It will also be evident that when the two sides of the screen are equally illuminated, the intensities of the rays a and b will be the same, since the paths over which these rays travel from the screen to reach the eye are the same in every respect. In the use of this photometer the adjustment is made as with the Bunsen photometer, and the determination of the relative intensities is carried out in the same manner.

THE FLICKER PHOTOMETER

515. In the flicker photometer, arrangement is made whereby the eye is enabled to view the effects of two sources

of light in rapid succession. The arrangement of apparatus will be understood from Figure 369. A and B are the sources to be compared. S is a stationary screen placed at some convenient angle θ to the line AB. S_1 is a disk made of the same material that S is made of, which is placed at the same angle θ to BA and caused to rotate rapidly about the axis aa. This disk has the form shown at C, Figure 369. The radii of the two portions of the disk are so chosen that to an eye placed at E the screen S is covered for one half of each revolution of S_1, while it is visible during the other half of the revolution. With this arrangement it is evident that during one

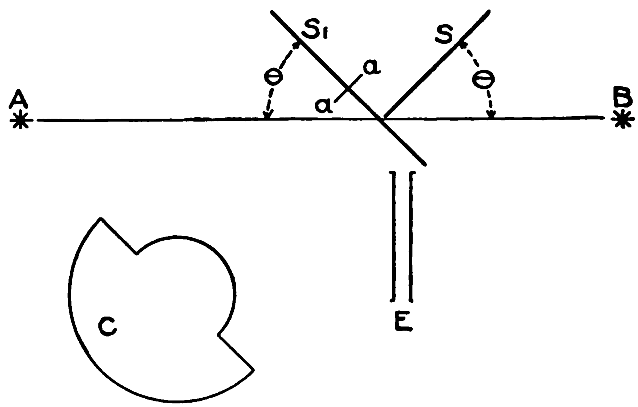

FIG. 369. — The Flicker Photometer.

half of each revolution of the disk S_1 light will reach the eye E from the source A, having been diffusely reflected from the rotating disk S_1. During the other half of the revolution of S_1 light will reach the eye E from the source B, having been diffusely reflected from the stationary screen S. Thus, when S_1, is caused to rotate rapidly, the eye at E is enabled to compare the illumination of the screen S by the source B, and that of S_1 by the source A. When these screens are unequally illuminated, the light which reaches the eye will appear to flicker. When the illuminations are equal, the flicker disappears, and in this way it is known when the the adjustment is reached.

It is found that the flicker photometer is especially useful

in comparing the candle powers of two sources of different color. It is impossible to make accurate settings on either the Lummer-Brodhun or the Bunsen photometer when the color of the light falling upon one side of the screen is different from that falling upon the other. In the use of the flicker photometer it is found that the **color flicker** is distinct from the **intensity flicker**, and that by properly choosing the speed of the disk the color flicker may be caused to disappear while the intensity flicker still remains. When this speed has been secured, the screen may be moved to and fro to secure a balance of intensities, exactly as in the use of the Bunsen or Lummer-Brodhun screen. The effect upon the eye of the color difference is thus avoided.

ILLUMINATION

516. The intensity of illumination of any surface is defined as the ratio of the light received by the surface to the area of the surface upon which the light falls. A unit of intensity which is oftentimes employed is known as **the foot candle,** and **is defined as the intensity of illumination which would be present upon a screen placed at a distance of one foot from a standard candle.** The meter candle is a unit of intensity which is employed to some extent.

The table below gives a number of values of illumination such as are commonly observed, the intensity of illumination being expressed in foot candles.

Suitable for drafting table	5 to 10
Suitable for library table	3 to 4
Suitable for reading table	1 to 2
Required for street lighting	0.05 to 0.60
Moonlight (full moon)	0.025 to 0.03

Problems

1. What is the intensity of illumination at a distance of 4 ft. from a 16 candle power lamp?

2. At what distance from a 32 candle power lamp is the intensity of illumination 1 foot candle?

3. A photometer is in adjustment with a standard 16 candle power lamp at a distance of 1 m. and an unknown source at 80 cm. from the screen. What is the candle power of the unknown source?

4. Two sources of light of 16 and 48 candle power respectively are placed 16 ft. apart. At what point will the illuminations produced by them be equal?

5. What intensity of illumination is produced by each source at the point determined in problem 4?

6. At what distance from an arc lamp of 3000 candle power is the intensity of illumination 3 foot candles?

COLOR

CHAPTER XLVI

THE ORIGIN OF COLOR

517. We have seen that colors may be produced by **refraction**, as exhibited in the prism, or by **interference**, as exemplified by the soap film. In nature many color effects are produced in this manner. There are also certain bodies to be found in nature which exhibit marked color, whose colors are not to be explained as due to either of these causes. Such bodies appear colored, because of **absorption effects** which take place in them. That is to say, **the surfaces of some bodies appear to possess the property of reflecting certain colors readily, while other colors are more or less completely absorbed. This property is known as *selective absorption*.** We may therefore say that, in general, there are three principal modes of color production; namely, by refraction, by interference, and by selective absorption.

THE COLOR OF TRANSPARENT BODIES

518. If a transparent body transmits with equal readiness all of the various colored components of white light, the body is colorless. If, however, it transmits any part of the spectrum more readily than another, the transparent body will appear colored when seen by transmitted light. **The color which it exhibits is a mixture of those colors which it transmits.** This color is evidently white light minus those colors which have been absorbed. Thus, a piece of red glass, when held before the eye, appears red, not because the glass changes the light which passes through it in any way, but because it has sifted out of the white light which falls upon it certain of its component wave lengths, and has allowed only the red light to pass

freely through it. A piece of blue glass appears blue because it absorbs the red and the yellow, and allows only the blue to pass, or, at least, allows the blue to pass most readily.

The color exhibited by two transparent objects when so placed that a beam of white light is allowed to traverse both of them in succession is evidently determined by those colors or wave lengths which pass through both bodies. If, for example, a solution of copper sulphate in a narrow vessel is placed before the slit of a spectroscope, and white light is used, the spectrum will be observed to consist of green together with some of the more refrangible colors, that is to say, colors of shorter wave length, the red and yellow having been completely absorbed. If a similar vessel filled with a solution of potassium bichromate is employed, the spectrum observed will consist of the green together with the longer wave lengths, yellow and orange, while the shorter wave lengths, blue and violet, will be entirely absent. If, now, both solutions are placed before the slit of the spectroscope, then, evidently, the only color found in the spectrum will be green, since green is the only color which is transmitted by both solutions. This will be evident from the following table, in which the colors absorbed by each solution are indicated by underscoring the corresponding letters:

Copper Sulphate Solution . . .	$\underline{R}\ \underline{O}\ \underline{Y}\ G\ B\ V$
Potassium Bichromate Solution .	$R\ O\ Y\ G\ \underline{B}\ \underline{V}$

In this example, therefore, the color of these two solutions, as exhibited by transmitted light, is green, while the color of the copper sulphate solution alone is blue, and that of the potassium bichromate solution is yellow. The effect is evidently a differential one.

THE COLOR OF OPAQUE BODIES

519. When white light falls upon the surface of an opaque body it is, generally speaking, diffusely reflected. If the surface of the body is of such nature that it reflects with equal facility all of the various wave lengths which enter into the composition of white light, the body will appear in this reflected light, white in color. A sheet of white paper or a white screen reflects equally well all of the various colors of the spectrum,

and hence, when placed in white light, it appears white. If the surface of the opaque body is of such nature that it reflects the waves corresponding to one part of the spectrum more readily than those corresponding to the other parts of the spectrum, its color will be something other than white. Suppose, for example, that the surface is of such nature that everything but the red light is reflected. Then the color of the surface, under these circumstances, will be white minus red. It is the color that would be obtained by combining all of the colors of the spectrum excepting red. Such a color is said to be **complementary** to red. **Complementary colors are colors which, combined, will give the effect of white.**

Most objects absorb from the white light which falls upon them certain wave lengths in larger proportion than others. Such objects exhibit what is known as **body color.** Body color is due to the same effect which gives rise to color of transparent bodies, or colors by transmitted light, since it is found that the white light incident upon such bodies penetrates to a certain depth into the surface layers, is then irregularly reflected and again traverses the surface layer. In thus passing twice through the superficial layers of the body, the same absorption effect upon the white light takes place as that which accompanies the transmission of light through transparent bodies. Thus, when a building is painted red, its surface is covered with a pigment which possesses the property of reflecting red light and absorbing in large measure the other colors of the spectrum. An opaque body which appears green is one which reflects the green light most readily. Such a body probably absorbs all of the red and the violet which falls upon it.

MIXING PIGMENTS

520. Since the body color of an opaque object is determined by this absorption effect, it is not difficult to predict the effect of spreading two pigments of different color upon the same surface. If, for example, the colors of the pigments chosen are yellow and blue, the resultant will be green, since the experiment would be essentially the same as that of placing the blue and yellow solutions before the slit of the spectroscope (Section 518).

MIXING COLORED LIGHTS

521. It has been seen that in the mixing of pigments a color effect is obtained which is **differential** and is determined by those wave lengths which are transmitted by both pigments. In the mixing of colored lights the result is very different, since in this case the effect is a **summation** of the effects due to the colored lights individually.

There are various ways in which colored lights may be mixed for making a study of this kind. One method employed for this purpose is to form a spectrum of the light given off by a white-hot body, thus securing a continuous spectrum. The colors of this spectrum may be recombined, as pointed out

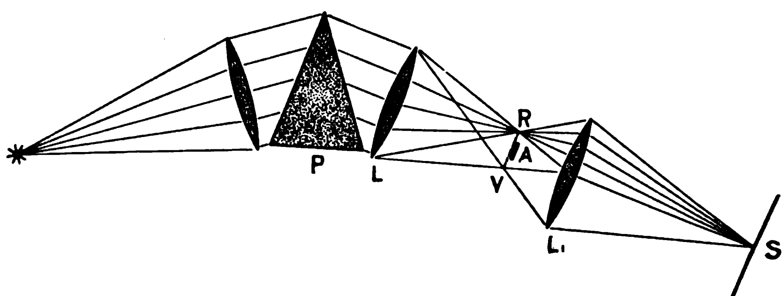

FIG. 370. — Apparatus for mixing Colored Lights.

above (Section 496), to form white light. By the use of suitable screens placed in the plane in which the spectrum is formed, it will be possible to remove from the beam of light such colors as are not desired in the experiment, leaving only those which it is desired to place in combination. These may be recombined by the use of a suitable lens. For example, in Figure 370, let P represent the prism of a spectroscope, L the lens upon which the light falls after passing the prism, RV the position of the spectrum formed by the instrument. L_1 is a lens by means of which the colors of the spectrum may be recombined forming a bright spot of light on the screen S. If, now, an opaque obstacle A is placed so as to receive a part of the spectrum, the wave lengths corresponding will be obstructed in their passage toward the right. Therefore, a portion of the spectrum only will pass to the lens L_1 and be combined at S. The result is a

colored spot of light at S which is the sum of all those colors which reach the screen. If, for example, the obstacle is of such width and placed in such position that it obstructs all but the red and the violet, the resultant color effect on the screen S will be that due to the addition of red and violet (purple).

Another method for studying the effects of combining various colored lights is by the use of colored disks of paper. As employed for this purpose the disks are slit radially so that they may be placed together and the amount of each disk exposed varied at will. When such a combination of two disks is caused to rotate rapidly while illuminated with white light, the effect upon the eye is the same as that secured by mixing two beams of light, the colors of which correspond to the colors of the disks. Let it be imagined, for example, that blue and yellow disks are employed, the adjustment being such that one half of each disk is exposed. When this combination of two disks is rotated rapidly in white light, the result (white) is the same as that secured by combining, by the process described above, beams of blue and yellow light. This result is due to what is known as **persistence of vision**, *i.e.* the retention of an impression by the retina of the eye for a certain length of time after the stimulus (light) has been removed. Thus, as the disks rotate, alternate flashes of blue and yellow light reach the eye. The corresponding impressions persist, and in effect the blue and yellow are added.

PRIMARY COLORS

522. It is customary to call the colors violet, indigo, blue, green, yellow, orange, and red, primary colors, since all parts of the spectrum are thereby included and are described in terms with which we are all familiar. The term "primary," however, as used in this connection has but little significance. It is supposed that the presence of each primary color is necessary to the production of white light. But it is very easily demonstrated that **white light may be secured by the combination of three or even but two of the spectrum colors.**

Because of certain phenomena which manifest themselves in the study of color vision it is thought that there are three primary color sensations, namely, green, blue, and red. For

this reason these colors are often spoken of as primary colors. The study of pigments has led to the conclusion that the three primary colors proper are red and yellow and blue, since a pigment of any one of these colors is found to absorb all of the light transmitted by the other two. Again, it is found possible to match any color by combining any three spectrum colors providing they are somewhat separated in the spectrum. In this sense, therefore, there is a number of groups of primary colors.

COMPLEMENTARY COLORS

523. If, with the arrangement of apparatus represented in Figure 370, an opaque obstacle is placed at *R V* in such position as to intercept the red light only, the resultant color upon the screen *S* will be complementary to red, that is to say, it is that color which combined with red will produce white light. It is thus apparent that it is possible to combine two colors and secure as a result white light. The experiment also shows that complementary colors are not necessarily simple colors, that is to say, they do not necessarily consist of a single wave length only, but each of the two complementary colors may be a compound of several wave lengths. For example, if the opaque obstacle in the experiment referred to is placed so as to intercept the yellow and all of the longer wave lengths, a certain color will result at *S*, which is of course a compound of violet, blue, and green. If, now, the obstacle is shifted in position so as to intercept the green and all of the shorter wave lengths, the color intercepted upon the screen will be complementary to that obtained in the first experiment, and will be a mixture of yellow, orange, and red.

THE CHARACTERISTICS OF A COLOR

524. For the complete description of any color three things must be stated:

1. Hue.
2. Saturation.
3. Luminosity.

The **hue** of a color is a specification of the wave length of the color, for example, red, orange, blue, etc.

The **saturation** of a color is a specification of the amount of white light it contains. If a beam of red light is allowed to fall upon a screen which is already illuminated with white light, the red which appears upon the screen is non-saturated, that is to say, it is red plus a certain amount of white. **A color is said to be saturated when it is free from the admixture of white light.** The pure spectrum colors are examples of saturated colors.

The **luminosity** of a color is a specification of its brightness. If the spectrum formed by a prism is allowed to fall upon a printed page, it will be observed that the portion illuminated by yellow is much more easily read than the other portions.

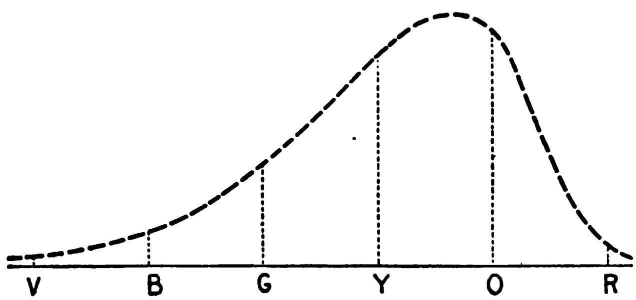

FIG. 371.—Luminosity Curve.

This is expressed by saying that the yellow is the most luminous of all the colors of the spectrum. In the same sense violet is the least luminous. The ordinates of the curve shown in Figure 371 represent the relative luminosities of the corresponding spectrum colors.

NON-SPECTRAL COLORS

525. Among the more prominent colors aside from those found in the spectrum of white light are the following: **purple,** which consists of a mixture of violet and red; **magenta,** which consists of a mixture of blue and red; and **brown,** which is a red or a yellow of low luminosity. These three colors are saturated colors. As examples of non-saturated colors might be mentioned pink, lavender, etc.

MAXWELL'S COLOR DIAGRAM

526. Maxwell's color diagram affords a convenient means of specifying a given color in terms of its components and its saturation. This diagram is represented in Figure 372. It consists of a triangle, the corners of which are supposed to represent the colors, red, blue, and green, as indicated by the letters. Since in the spectrum orange and yellow are found between red and green, these two colors will be represented by points on the line *RG*. Since

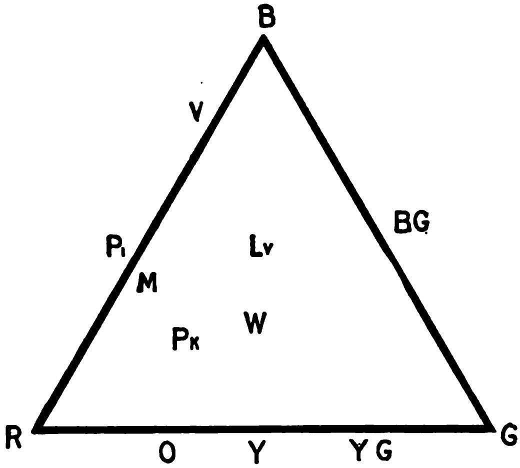

Fig. 372. — Maxwell's Color Triangle.

the green in the spectrum gradually shades into blue, a point midway between *B* and *G* might be called blue-green. Violet is to be found on the line *BR*, but close to *B*. Purple and magenta are found near the middle of this line *BR* since they consist of combinations of red and violet, and red and blue, as pointed out above. The point *W* corresponds to white, the various colors of the spectrum being arranged symmetrically about it.

A saturated red is represented, of course, by the corner of the triangle. A non-saturated red, that is to say, a red having an admixture of white, would be represented by a point between *R* and *W*. If close to *R*, the red is nearly saturated. If close to *W*, it is nearly white.

The complementary colors of this diagram are obtained by drawing lines through *W*. For example, yellow and blue are complementary, yellow-green and purple, orange and green-blue, red and blue-green.

THE DEPENDENCE OF BODY COLOR UPON THE CHARACTER OF THE INCIDENT LIGHT

527. We have seen that the body color of any object is due to the fact that it reflects certain wave lengths readily, while others are more or less completely absorbed. It is therefore evident that **an object can show its true body color only when the light in which it is viewed contains those wave lengths which it most readily reflects.** Suppose, for example, that the body color of an object is red, and that it absorbs from white light all of the various wave lengths except red. **Such an object will appear black when illuminated by any of the spectrum colors except red.** In white light it will show its true body color, since white light contains red. It will be seen, therefore, that **in white light all objects exhibit their true body colors,** and white light is the only kind of illumination of which this is true.

This is of great importance in comparing the various sources of light used in artificial illumination. For example, the gas flame and in some cases the incandescent lamp give a light which is decidedly yellowish. The mercury vapor lamp gives a greenish light. The flaming arc gives a reddish light, and so on. Evidently from the principle stated above **neither of these forms of light is capable of exhibiting all objects in their true body color.**

YOUNG-HELMHOLTZ THEORY

528. Various theories have been advanced to account for color perception and the various characteristic phenomena related thereto. The theory which is most commonly accepted at the present day is known as the Young-Helmholtz theory. This theory may be briefly outlined as follows: It is assumed that there exists in the retina of the eye three sets of nerve terminals. One of these sets is particularly sensitive to red light, and the corresponding nerve terminals are usually referred to as the **red nerve terminals.** The second set is particularly sensitive to blue light, and the nerve terminals of this set are called the **blue nerve terminals.** The third set, consisting of the **green nerve terminals,** are especially sensitive to green light. It has been determined that each of the three

nerve terminals is affected by any color of the spectrum. The name "blue nerve terminal" is not to be understood as meaning that the corresponding nerve terminal is sensitive to blue alone, but that it is more sensitive to blue than to any other color. It is fairly sensitive to those colors of the spectrum which lie in the neighborhood of the blue, for example, the green and the violet. The blue nerve terminal is least sensi-

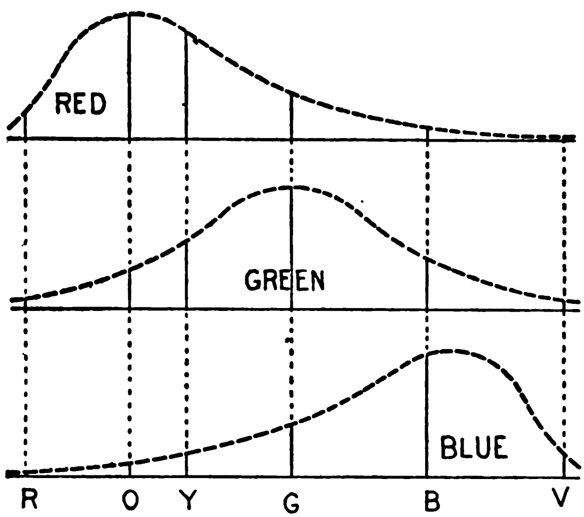

FIG. 373.

tive to those colors which are the farthest removed from the blue in the spectrum, that is to say, the red. The curves shown in Figure 373 are drawn to indicate the sensitiveness of each of the three nerve terminals to the various colors of the spectrum. For example, the curves show that orange light affects the **red** terminals very strongly and produces considerable effect upon the **green** terminals, while upon the **blue** terminals its effect is almost negligible.

SUBJECTIVE COLORS

529. The Young-Helmholtz theory enables us to explain satisfactorily most of the phenomena of color vision. One of the most important of these is the development of color by

what is called **after effect**, or the phenomenon of subjective colors. This effect will be understood from the following simple experiment: If a brilliantly colored object, let us say, a blue card, is placed against a gray background and strongly illuminated with white light, and the eyes are directed toward the card for a few (about 20) seconds and then turned aside to a gray wall, there will immediately appear in the field of vision an image of the card **colored yellow**. If a bluish-green card is employed, the after image will be red in color, that is to say, **the colors of the object and the after image are complementary**. The explanation of the phenomenon under the Young-Helmholtz theory is as follows: When the eye is directed steadily toward a blue object, the blue nerve terminals gradually become fatigued. If, after this effect has set in, the eye is turned to a gray object, which in the normal condition of the eye would affect all three nerve terminals equally, the effect upon the red and green nerve terminals will predominate. In other words, the eye will perceive gray (or white) minus the blue, that is to say, the color which is complementary to the blue, namely, yellow.

COLOR BLINDNESS

530. In the normal eye, it is possible, as we have seen, to produce any color sensation by combining the effects of what we might call the three primary sensations, namely, red, green and blue. For the normal eye it is therefore possible to **match** any color by combining red, green, and blue. In matching colors in this way it would of course be necessary to have the luminosity of each of the primary colors under control.

For certain eyes it is found possible to match every color perceived by combining green and blue. Such an eye is said to be red-blind. In other cases it is found possible to match all colors for a given eye by combining red and blue. Such an eye is said to be green-blind. Under the Young-Helmholtz theory color blindness is explained by assuming that **in the color-blind eye one of the three sets of nerve terminals described above is either entirely wanting or much less sensitive than the others.**

A study of Figure 373 will enable us to determine in a general way how various colors would appeal to the red-blind eye.

For such an eye the upper curve would be lacking, and evidently if the green and the blue are the only sensations possible, then when these two sets of terminals are equally excited, the result will be white or gray just as the normal eye receives the impression of white when all three nerve terminals are affected equally. It will therefore be evident that for the red-blind eye that portion of the spectrum which lies about midway between the green and the blue will appear **white**, since the ordinates of the green and blue curves are equal in this region. Colors lying near this region will evidently be very pale, since they have in effect a large amount of white mixed with them. The peculiarities of color vision for the green-blind eye may be determined in the same way.

A TEST FOR COLOR BLINDNESS

531. Since all of the various systems of signaling, both on railways and at sea, require the use of colored lights, it is evidently of the greatest importance that those who are supposed to interpret such signals be able to distinguish colors in their proper values. Railway companies subject their employees to a test for color blindness. A test which is quite commonly employed for this purpose is known as the **Holmgren test**. For making this test a number of samples of colored worsteds are employed. The samples used consist largely of worsteds of green, blue, purple, and brown in various degrees of saturation, and a number of skeins of neutral tint. In addition there are three samples known as the **confusion samples**, one of which is a very **pale green**, the second a **brilliant red**, and the third a **magenta** which is not very near saturation. The test is made in the following manner. The group of colors is placed before the individual whose color vision is to be tested, and one at a time the confusion samples are placed before him and he is asked to select those colors from the general group which match the confusion sample.

To the red-blind eye the magenta confusion sample appears blue. For such an eye, therefore, it will be found that the blues will be placed with the magenta. The browns will also be placed with the grays. To the green-blind eye a green is

a gray, as we have seen above. Therefore, a person possessing this defect in color vision will place the grays with the pale green confusion sample. In this manner, by observing the selections made to the different confusion samples, it is an easy matter to detect color blindness when it exists, and to determine which of the three sets of nerve terminals is defective.

POLARIZATION

CHAPTER XLVII

LIGHT WAVES ARE TRANSVERSE WAVES

532. As we have seen, there are various phenomena which lead us to believe that the disturbance we call light is of the nature of a wave motion. We would not, however, be able to determine from any of the phenomena thus far discussed whether light waves are transverse or longitudinal. **There are certain phenomena which afford conclusive proof that light waves are transverse waves**; that is to say, that the ether particles which transmit light vibrate at right angles to the direction in which the disturbance is being propagated. The experiment described in the following paragraph affords evidence of this kind.

THE EXPERIMENT WITH CROSSED TOURMALINES

533. By taking advantage of the natural cleavage of the mineral it is possible to separate tourmaline into crystals of the form shown in Figure 374. These crystals are quite transparent, and if one of them is held in the path of a narrow beam of light as indicated in the figure, a large percentage of the light will be transmitted. So far as the unaided eye is able to discover the transmitted beam is in no way different from the incident beam. Upon careful examination, however, it is found that the transmitted beam possesses peculiar properties. If the transmitted beam is allowed to fall upon a second tourmaline, it will be transmitted, providing the second tourmaline is placed

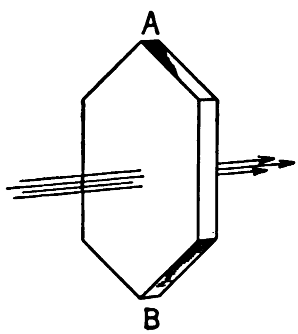

FIG. 374. — Tourmaline Plate.

with its greatest length parallel to the corresponding dimension of the first tourmaline. That is, if the two tourmalines are arranged as shown in Figure 375, the beam which is transmitted by the first tourmaline *AB* will pass almost undiminished in intensity through the second tourmaline *A'B'*. **If, however, the tourmalines are "crossed," that is to say, arranged as shown in Figure 376, the beam which is transmitted by the tourmaline *AB* will be completely intercepted by the tourmaline *A'B'*.**

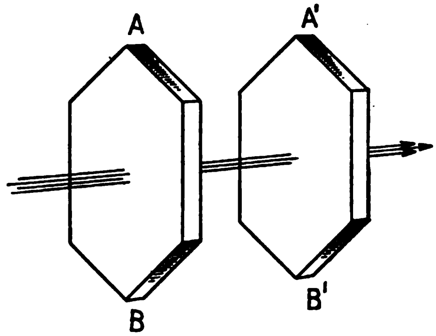

FIG. 375. — Parallel Tourmalines.

Referring again to the experiment represented in Figure 374, let it be imagined that the single tourmaline *AB* is rotated about the beam of light as an axis. Under these circumstances, no change in the intensity of the light transmitted will be observed. If, however, two tourmalines are employed, a rotation of the second tourmaline **is accompanied by a change in the intensity of the transmitted beam.** When the tourmalines are parallel, Figure 375, the maximum amount of light is transmitted. When they are crossed, Figure 376, the minimum amount of light is transmitted. For positions intermediate between these two, the amount of light varies depending upon the angle between the tourmalines. Since no change in the intensity of the transmitted beam was observed in the first case, that is, when a single tourmaline was used, it will be evident that **the light in passing the tourmaline *AB* acquires a**

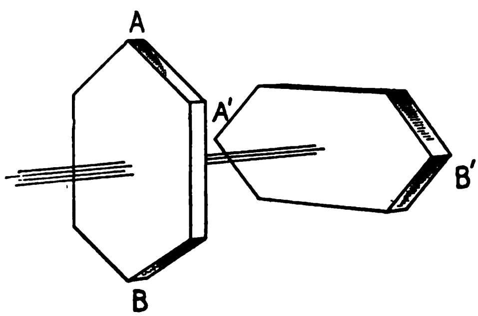

FIG. 376. — Crossed Tourmalines.

property which it before did not possess. It evidently has different properties as seen from different sides, since the position of the tourmaline $A'B'$ determines the amount of light which is transmitted. Such a beam of light is said to be **polarized.**

The phenomena of polarization are best understood by considering the following mechanical analogy. Imagine that a long flexible rubber tube AB, Figure 377, has one end fastened to the wall and the other is held in the hand. By moving the end of the tube which is held in the hand to and fro it is possible to cause transverse waves to travel down the length of the tube. If a block of wood with a slot cut in it is placed

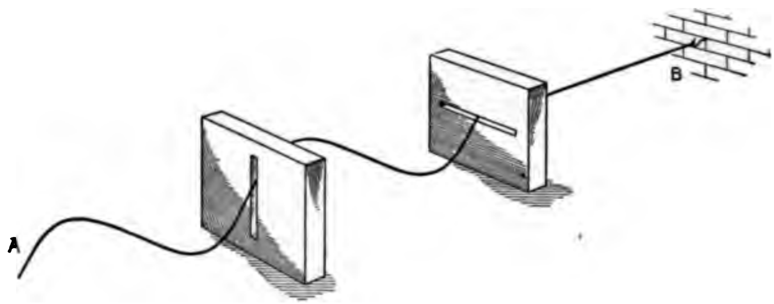

FIG. 377. — Mechanical Analogue of Polarizing Apparatus.

over the tube, it will be evident that the motion of the tube will not be interfered with so long as the slit is parallel to the direction of motion. If, however, we attempt to cause the tube to vibrate at right angles to the slot, evidently the vibratory motion will not be able to pass the block of wood. Let it be further imagined that the end of the tube which is held in the hand is caused to vibrate in a number of different directions, horizontally, vertically, and at various angles to the horizontal. Let it be assumed at the same time that the slot is in a vertical position. **Then of all these vibratory motions which are imparted to the tube only those which are in a vertical direction will be transmitted or passed beyond the block.** If now a second slotted block is placed over the tube, those vibrations which pass the first slot will pass the second providing the second slot is parallel to the first. If, however, the second slot is placed at right angles to the first, no vibrations will pass.

The inference is that ordinary light consists of a transverse wave motion, the vibrations taking place in many different directions. When such a beam is caused to pass through a tourmaline crystal, only certain vibrations, that is to say, vibrations in a certain direction, are allowed to pass so that the transmitted beam **differs from ordinary light** in that the **vibratory motion of the ether particles are all in the same plane.** This being the case, it is very evident that this beam of light can pass a second tourmaline only when it is parallel to the first.

The beam which passes the first tourmaline plate is said to be **plane-polarized.** Evidently in the experiment the second tourmaline acts as a sort of detector of the polarized condition. It is customary, therefore, to refer to the first tourmaline as the **polarizer** and the second tourmaline as the **analyzer.**

THE PLANE OF POLARIZATION

534. It is assumed that in the beam of light which passes the polarizer, the vibratory motion takes place parallel to the length of the plate. The plane which extends through the beam of light at right angles to this vibratory motion is called the plane of polarization, that is, **the ether particles are supposed to vibrate at right angles to the plane of polarization.**

POLARIZATION BY REFLECTION

535. Light may be polarized by reflection from a non-metallic surface. Thus it is found that when light falls upon a glass plate the reflected beam is more or less completely polarized depending upon the angle of incidence. It has been determined by experiment that for each substance there is a definite angle of incidence for which the polarization of the reflected beam is most complete. This angle is known as the **polarizing angle.**

For example, the incident beam, Figure 378, after reflection at the mirror M is found to be plane-polarized, the plane of polarization being the same as that of incidence, that is to say, it contains the incident ray and the perpendicular to the mirror M. Since, as we have seen above, an analyzer is not essentially dif-

ferent from a polarizer, evidently a second mirror might be employed as an analyzer. It is found that when a second mirror M' is placed in the position shown in the figure that the polarized beam of light which falls upon it is reflected exactly as an ordinary beam of light is reflected from such a mirror. **If, however, the mirror M' is turned through an angle of 90° about the line $M'M$ as an axis, no part of the polarized beam will be reflected from the second mirror.** If M' is turned in the same direction through another 90°, the polarized beam will once more be reflected in its full value, and so on. In other words, it is possible to make up a complete polarizing apparatus of two plates of glass. In the use of glass plates for this purpose the angle of incidence ϕ should be made equal to the polarizing angle. The polarizing angle for glass is between 57° and 58°.

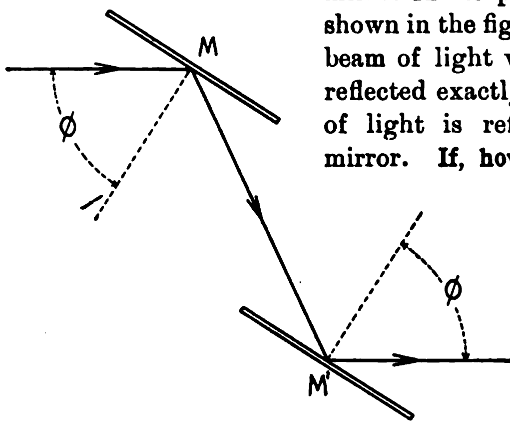

FIG. 378. — Apparatus for demonstrating Polarization by Reflection.

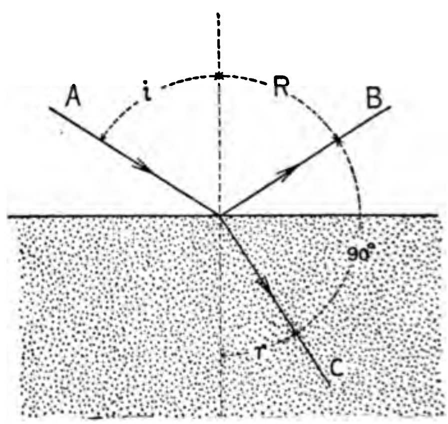

FIG. 379.

BREWSTER'S LAW

536. Brewster found that **polarization by reflection is most complete when the angle between the *reflected ray* and the *refracted ray* is 90°.** From this circumstance it follows that the polarizing angle for any medium may be very simply expressed in terms of its index of refraction.

Let A, Figure 379, represent a ray of light in air incident upon the surface of a denser medium. Let i be the angle of **incidence**, R the angle of **reflection**, and r the angle of **refraction**. When the angle between the reflected ray B and the refracted ray C is $90°$ as shown, then i is the polarizing angle. For these conditions

$$r + 90° + R = 180°$$

$$\therefore r + R = 90°$$

i.e., $$R = i = \text{the complement of } r \qquad (a)$$

Now $$\frac{\sin i}{\sin r} = \mu \qquad \text{(Section 475)}$$

but $$\sin r = \cos i \qquad (\text{from } a)$$

$$\therefore \frac{\sin i}{\cos i} = \mu$$

or $$\tan i = \mu \qquad (136)$$

That is to say, the tangent of the polarizing angle of any medium is equal to its index of refraction.

DOUBLE REFRACTION

537. In considering the various phenomena of refraction we have assumed that the media considered were isotropic, that is, that they had the same physical properties in all directions. There are certain substances in nature which are transparent and which at the same time have decidedly different physical properties in different directions. When a beam of light is refracted into a crystal of such a substance, certain phenomena are observed which are not present when refraction takes place in an isotropic medium.

If, for example, a narrow beam of light is caused to pass through a crystal of Iceland spar (crystalline calcium carbonate) it will be found that in general the beam becomes divided into two beams. A study of these two beams of light will show that one of them obeys the ordinary laws of refraction. This beam is called the **ordinary ray.** The other beam, known as the **extraordinary ray,** does not obey the ordinary laws of refraction. A further examination will show that both the ordinary and extraordinary rays are polarized, their planes of

polarization being at right angles to one another. Figure 380 represents a beam of ordinary light *a* falling in perpendicular direction upon one face of a crystal of Iceland spar *AB*. A part of the transmitted ray passes unchanged in direction to *o*. This is known as the ordinary ray. Another part of the disturbance is refracted and passes out of the crystal in the direction *ee'*. This is known as the extraordinary ray. The ordinary ray is plane-polarized, the plane of polarization being that of the paper. The extraordinary ray is plane-polarized at right angles to this plane. The short cross lines on *ee'* are drawn to indicate the direction in which the ether particles are supposed to be vibrating. The ether particles in the beam *oo'* are vibrating in a plane perpendicular to the page.

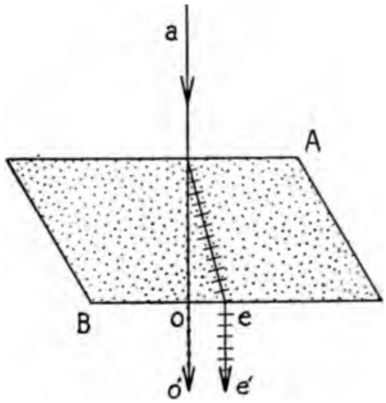

Fig. 380.—The Ordinary and Extraordinary Rays.

There are many crystalline substances which exhibit the phenomena of double refraction of which we may mention the following:

Iceland Spar	Selenite
Tourmaline	Mica
Quartz	Sugar

As was pointed out above, when a beam of light passes into a double refracting medium, it is usually separated into two beams. It is found, however, that there are certain directions in double refracting media along which bifurcation does not take place. These directions are known as the **optic axes** of the crystals. **A crystal of Iceland spar is in the form of a rhombohedron, two opposite solid angles of which are bounded by three obtuse angles. The optic axis of this crystal is parallel to a line drawn through one of these solid angles equally inclined to all three faces.** Let *ABCD*, Figure 381, represent a crystal of Iceland spar. Let *B* and *D* represent the solid angles which are bound by three

obtuse angles. Then a line drawn in the crystal through *B* or *D* making equal angles with the three planes containing these points is the optic axis of the crystal. When the edges of the crystal are all equal, the optic axis coincides with the diagonal *BD*.

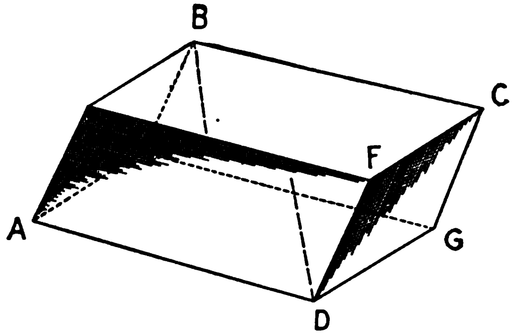

FIG. 381. — Rhomb of Iceland Spar.

THE DOUBLE IMAGE PRISM

538. When an ordinary beam of light is passed through a crystal of Iceland spar, the opposite faces of which are parallel, the ordinary and extraordinary rays which emerge from the crystal are parallel. If the opposite faces of the crystal are not parallel, the ordinary and extraordinary rays diverge and become more and more separated the farther they pass from the crystal. Such a crystal is of great service in the study of double refraction and polarization. Such a crystal of Iceland spar would act as an ordinary prism, giving rise to dispersion. To obviate this difficulty, it is customary to place against the Iceland spar a prism of glass with its refracting edge turned in the opposite direction from that of the prism of Iceland spar. The refracting angle of this glass prism is so chosen as to make of the two prisms an achromatic combination. This combination is represented in Figure 382, in which *I* is the prism of Iceland spar and *G* the prism of glass.

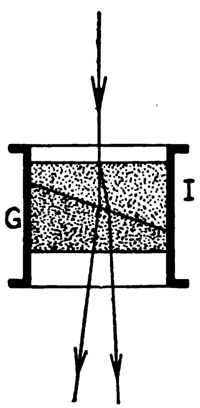

FIG. 382. — Double Image Prism.

NICOL'S PRISM

539. One of the best means of obtaining a beam of plane-polarized light is by the use of what is known as a Nicol's prism.

Referring to Figure 381, let it be imagined that the crystal of Iceland spar, *ABCD*, is cut by a plane passing through the line *DB* parallel to the diagonal *FG*. Let it be further imagined that the two faces which are thus formed are carefully ground and polished and cemented together by means of a thin layer of Canada balsam. Let *ABCD*, Figure 383, represent such a crystal, the diagonal *AC* representing the plane along which the crystal was cut. That is to say, the crystal as represented in the figure is so placed that the plane in which it has been cut is perpendicular to the page. Let it be imagined that a beam of ordinary light *a* is caused to enter this crystal from the left as

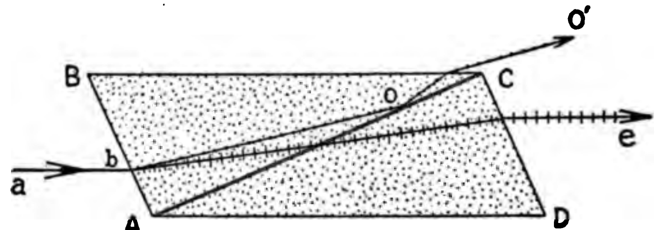

Fig. 383.—Nicol Prism.

indicated in the figure. The crystal being double refracting, this beam will be broken into two beams, the ordinary beam passing off in the direction *bo* and the extraordinary ray traveling in the direction *be*. Now the index of refraction of Canada balsam is less than that of Iceland spar for the ordinary ray and greater for the extraordinary ray. The values of these indices are given below:

Canada balsam	1.55
Iceland spar ordinary ray	1.658
Iceland spar extraordinary ray . . .	1.468

It will therefore be evident that if the angle at which the ordinary ray falls upon the interface *AC* is greater than the critical angle (Section 477), this ray will be totally reflected, since for this ray Canada balsam is optically less dense than the Iceland spar. Thus the ordinary ray will be turned aside. The extraordinary ray, however, will pass practically without change of direction across the interface *AC* and emerge as indicated in the figure. Thus the nicol prism separates

ordinary light into the ordinary ray, which is suppressed as indicated above, and the extraordinary ray, which is transmitted. The transmitted ray is polarized in a plane perpendicular to the page as indicated by the short cross lines, which are placed to represent the direction in which the ether particles vibrate.

THE POLARISCOPE

540. A polariscope consists essentially of a device for polarizing light and a second device used for analyzing the polarized beam. Evidently a polariscope might be made of two plates of tourmaline, two mirrors, or two Nicol prisms, or of combinations of these various devices.

THIN PLATE OF A DOUBLE REFRACTING SUBSTANCE IN POLARIZED LIGHT

541. An instructive experiment is the following: Let it be assumed that in a polariscope the analyzer is so turned as to completely suppress the plane-polarized light which proceeds to it from the polarizer. Under such conditions the analyzer is said to be "crossed." If, now, a thin plate of a double refracting substance, for example mica, is placed between the polarizer and the analyzer, it will be found that in general light will pass the analyzer. Furthermore, it will be observed that the light which passes the analyzer under these circumstances is more or less brilliantly colored.

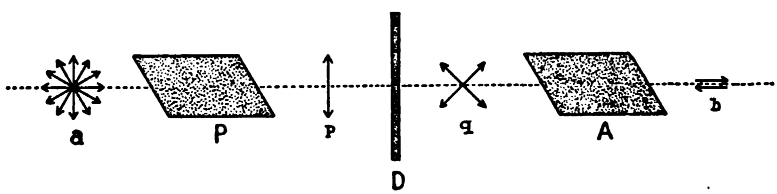

FIG. 384. — Thin Plate of Double Refracting Medium between Crossed Analyzer and Polarizer.

These phenomena are explained as follows: Let P, Figure 384, represent the polarizer of the polariscope used in the experiment and A the crossed analyzer. Let it be assumed that the polarized light which passes P is polarized in the plane

perpendicular to the paper so that the vibratory motion of the ether particles will be represented by the double-headed arrow *p*. The double-headed arrows *a* represent the vibratory character of the light before it falls upon the polariscope. It is assumed that *A* is so placed as to entirely suppress the polarized beam which passes *P*. Let *D* represent the thin plate of double refracting substance. Let it be assumed that this plate is in such position that, if ordinary light were passing, it would be broken into two beams whose vibrations are at 45° to the vertical as shown at *q*. Under these circumstances it is evident that the polarized beam *p* will pass the plate *D*, being separated into two beams as represented at *q*. These two beams, whose vibrations are at right angles to one another, will each be broken into two beams upon passing *A*, but *A* is supposed to be so placed that it will allow nothing but horizontal vibrations to pass. The result is, the horizontal components of both vibrations shown at *q* will pass the analyzer. It will thus be understood how the presence of *D* enables the polarized beam *p* to pass the analyzer *A*.

The color effects observed are to be explained in the following manner: The two components *q* into which the plane-polarized beam *p* is separated by the plate *D* pass through the plate *D* with different velocities, since, as has been pointed out above, the index of refraction of a double refracting medium is different for the ordinary and extraordinary rays. Thus, one of the components *q* falls behind the other component in passing the plate *D*. **If the amount by which the one component is retarded is equal to an even number of half wave lengths of the light under consideration, the horizontal parts of these components will be in condition to interfere.** Hence they will neutralize one another. Under these circumstances no light would pass the analyzer *A*, providing light of one wave length only were employed. If white light is made use of, it may happen that the above effect will take place for some one wave length, for example red, which will therefore be extinguished. The other wave lengths will pass the analyzer, since the retardation for these is not such as to bring them into proper relation for complete interference. Hence the light which passes the analyzer

will be colored. It will be white light minus those wave lengths which have been extinguished by interference.

If, now, the analyzer A is rotated about the beam of light as an axis through 90°, evidently that wave length which was extinguished by interference in the first position of the analyzer will now pass the analyzer, since the vertical components of q act together. The vertical parts of the components q, Figure 384, extend in the same direction, while the horizontal parts of these components are opposed. With the analyzer in this position those wave lengths will be extinguished for which the retardation in the plate D is an odd number of half wave lengths. It is, therefore, evident that such wave lengths as are quenched by the analyzer in the first position will predominate in the transmitted beam when the analyzer is in the second position, and *vice versa*. **We thus obtain complementary colors in the two positions of the analyzer.**

If in the experiment described above the plate D is rotated about the beam of light as an axis, evidently there will be certain positions in which the above described phenomena will not take place. For example, if the plate D is rotated 45° from the position which it is supposed to occupy in the above discussion, the polarized beam p will be transmitted by the plate D without alteration. It will, therefore, not be able to pass the analyzer. There is another position for the plate D 90° from this one for which the same statement is true.

ROTATION

542. If a thin plate of quartz cut perpendicular to the optic axis is placed between the polarizer and crossed analyzer of a polariscope, light will pass the analyzer. **This effect is distinct from that described in the last section, since it is found that a rotation of the quartz plate about the beam of light as an axis will produce no effect upon the intensity of the transmitted beam.**

Evidently, therefore, the passage of light through the analyzer under these circumstances is to be explained in some other way. The beam of light which enters the quartz plate under these circumstances is broken up into two polarized beams, but the character of the vibratory motion in each of these beams is

different from that which has been considered in the preceding sections.

These beams of light within the quartz are said to be **circularly polarized**, that is, the ether particles instead of moving to and fro in straight lines are thought of as whirling in circles whose planes are perpendicular to the ray of light. These circular components pass through the quartz with different velocities. They recombine upon emerging into the air to form a beam of plane-polarized light, but **since one of these circularly polarized beams has been retarded in passing the plate, when they recombine it will be in a plane different from that in which the beam *p* is vibrating.** Thus, in effect, the quartz plate rotates the plane of polarization of the beam *p*. The beam which passes the quartz plate is in no respect different from that which is incident upon it, except that its plane of polarization is different. By rotating the analyzer a position is found for which the beam is completely quenched.

Double refracting substances which are capable of producing this effect are said to be "optically active." Certain solutions are found to be optically active. **The rotation** produced by a solution of an optically active substance **is proportional to the mass of the substance contained in the solution and the thickness of the solution in the direction in which the light is passing.** If the thickness of the solution is kept constant it thus becomes possible to estimate the amount of the optically active substance dissolved, by measuring the angle through which a beam of plane-polarized light is rotated in passing the solution. This method is employed in a practical way for the determination of the percentage strengths of sugar solutions.

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